Friman & Knoll, Lectures SS 2000

Problem set 1, date 10.5.00

1 Show that $\delta^{(4)}(p)$ is Lorentz invariant.
Hint: Show that $\delta(E')\delta p'_{\|} = \delta(E)\delta p_{\|}$, where E' and p' are obtained from E and p by a Lorentz transform in the $\|$ direction.

2 Write $\delta^{(3)}(\vec{p} - \vec{p'})$ in cartesian, polar and spherical coordinates.

3 Show that

$$\int d^4k\, \Theta(k_0)\delta(k^2 - m^2) f(k) = \int \frac{d^3k}{2\varepsilon(k)} f(k)\bigg|_{k_0=\varepsilon(k)},$$

where $\Theta(k_0)$ is the unit step function. What is $\varepsilon(k)$?

4 Show that for massless particles the invariant phase-space integral $\Phi_{1,\ldots,N}(s)$ is proportional to s^α, where α depends on N. Determine α and discuss the consequences for the single-particle distribution

$$W(\vec{p}) \propto \Phi_{1,\ldots,N-1}((\sqrt{s} - \varepsilon)^2 - (\vec{p})^2)$$

also for large N, where $\varepsilon^2 = (\vec{p})^2$.

5 Show that, for a three-body system at a given s, the differential phase space distribution

$$\frac{d\Phi_{1,2,3}(s)}{d\varepsilon_1 d\varepsilon_2}$$

is a constant inside the kinematically allowed region. Here the $(\varepsilon_1, \vec{p}_1)$ etc. are the 4-momenta of the particles in the c.m. frame of the system and $d\Phi_{1,2.3}/d\varepsilon_1 d\varepsilon_2$ is defined such that

$$\Phi_{1,2,3}(s) = \int d\varepsilon_1 d\varepsilon_2 \frac{d\Phi_{1,2,3}(s)}{d\varepsilon_1 d\varepsilon_2}$$

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