B. L. Friman & J. Knoll, Lectures WS 2000-01

Problem set 6, date 24.11.00 11:00h Seminarraum Theorie

1 **One level Boson gas:** Consider a system of n bosons that can occupy one one single particle state. The system is described by the Hamiltonian

$$\underline{\mathrm{H}} = \varepsilon \underline{\mathrm{a}}^\dagger \underline{\mathrm{a}} + \frac{1}{2} U \underline{\mathrm{a}}^\dagger \underline{\mathrm{a}}^\dagger \underline{\mathrm{a}}\underline{\mathrm{a}}$$

where $\underline{\mathrm{a}}^\dagger$ creates and $\underline{\mathrm{a}}$ annihilates a particle.

a) Show that for large n ($>> 1$), the energy can be approximated by the mean-field energy

$$E_{m.f.}(n) = \varepsilon n + \frac{1}{2} U n^2$$

and that the single particle energy is

$$\varepsilon + Un \simeq E(n+1) - E(n).$$

b) Show that in the mean-field approximation, the thermodynamic potential of this system is given by

$$\Omega(T, \mu, V) = -\frac{1}{2} U n^2 + k_B T \ln(1 - e^{-\beta(\varepsilon + Un - \mu)}),$$

where $n = 1/(e^{\beta(\varepsilon + Un - \mu)} - 1)$.

c) Show that the mean number of particles is

$$\langle N \rangle = -\frac{\partial \Omega}{\partial \mu} = n$$

and the entropy

$$S = -\frac{\partial \Omega}{\partial T} = k_B((1+n)\ln(1+n) - n \ln n)$$