Dynamics of Resonances in Strongly Interacting Matter

(Resonance transport)

J. Knoll[1], F. Riek[1], Yu.B. Ivanov[1,2], D. Voskresensky[1,3]

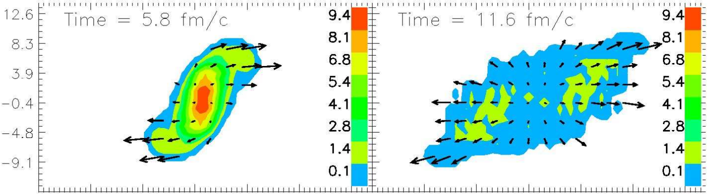
[1]GSI [2]Kurchatov Inst. (Moscow)

[3]Moscow Ins. for Physics and Engineering

Outline

Towards transport of broad resonances

Description of high energy nuclear collisions



3 fluid hydro: Ivanov, Russkikh & Toneev

- typical thermodynamics properties:
  densities $\gg 2\rho_0$ (nucl. saturation)
  $T \in [50 - 150]$ MeV
- typical resonance width (Δ-res., ρ-meson): $\Gamma \geq 100$ MeV
- typical collision rates: $\geq 1/2\text{fm/c} \sim \Gamma \geq 100$ MeV
- $\Rightarrow$ On-shell concepts questionable!

Boltzmann Ühling Uhlenbeck Eq.:

$$\underbrace{\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\frac{\partial}{\partial \vec{x}}\right)}_{\text{free motion}} f(\vec{x},\vec{p},t) - \underbrace{\{U_{\text{pot}}, f\}}_{\text{mean field}} = \underbrace{C(f(\vec{x},\vec{p},t))}_{\text{Collision Term}}$$

$$\omega = \frac{p^2}{2m} - U_{\text{pot}}(\vec{x},\vec{p})$$

$$\int \mathrm{d}p_1 \mathrm{d}p_2 \mathrm{d}p_3 \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(1-f)f_1(1-f_2)f_3$$

+ loss term

Off-Shell Propagation?:

$$\underbrace{f(\vec{x},\vec{p},t)}_{\text{3 Phase Space}} \Longrightarrow \underbrace{F(\vec{x},t;\vec{p},\omega)}_{\text{4 Phase Space}}$$

- which equation(s)?
- is the knowledge of F sufficient?
  $\Longrightarrow$ Spectral Fct.: $A(\vec{x},t;\vec{p},\omega)$

Boltzmann Ühling Uhlenbeck Eq.:

$$\underbrace{\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\frac{\partial}{\partial \vec{x}}\right)}_{\text{free motion}} f(\vec{x},\vec{p},t) - \underbrace{\{U_{\text{pot}}, f\}}_{\text{mean field}} = \underbrace{C(f(\vec{x},\vec{p},t))}_{\text{Collision Term}}$$

$$\omega = \frac{p^2}{2m} - U_{\text{pot}}(\vec{x},\vec{p}) \qquad \int \mathrm{d}p_1\mathrm{d}p_2\mathrm{d}p_3 \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(1-f)f_1(1-f_2)f_3$$

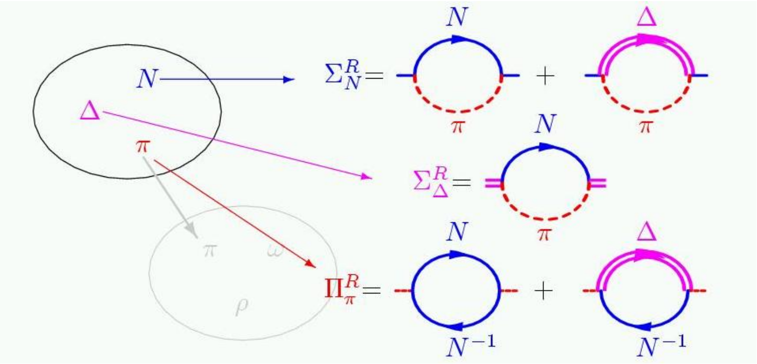
+ loss term

Off-Shell Propagation?:

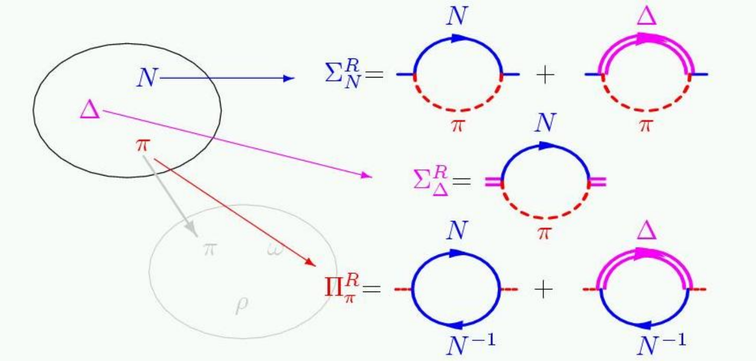
$$\underbrace{f(\vec{x},\vec{p},t)}_{\text{3 Phase Space}} \Longrightarrow \underbrace{F(\vec{x},t;\vec{p},\omega)}_{\text{4 Phase Space}}$$

- which equation(s)?
- is the knowledge of F sufficient?
  $\Longrightarrow$ Spectral Fct.: $A(\vec{x},t;\vec{p},\omega)$

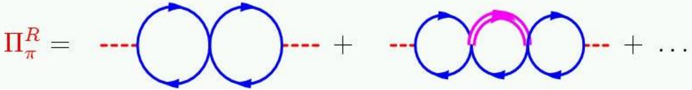
with Felix Riek



with Felix Riek

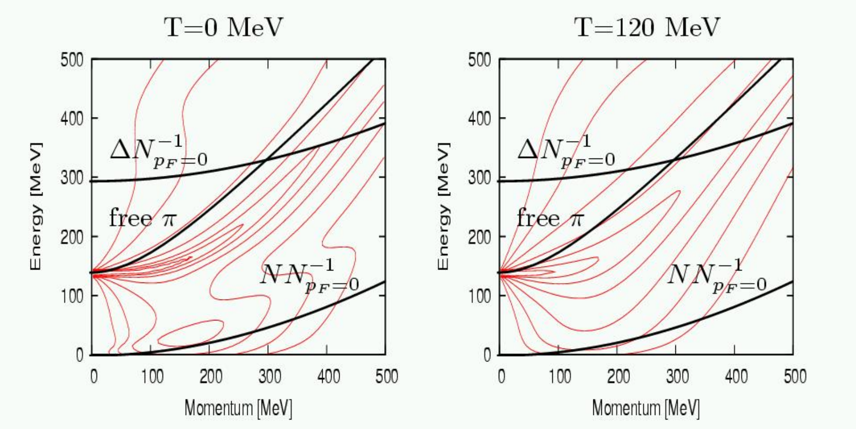


- Migdal's short-range repulsions



Pion spectral-function

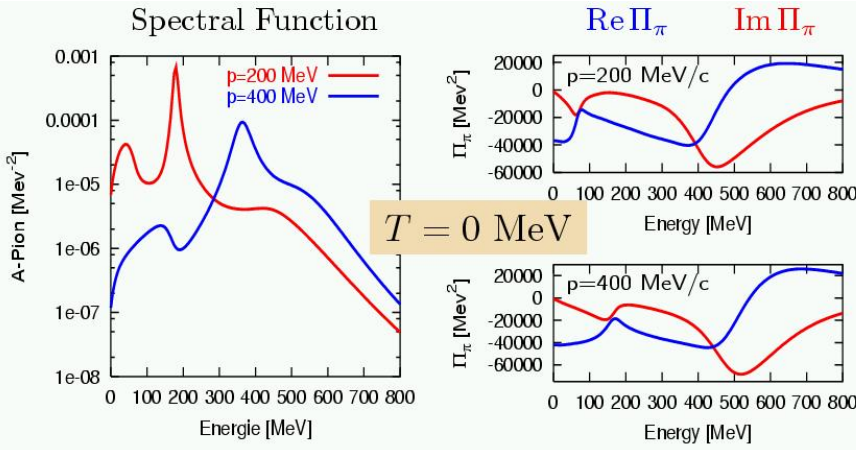
Nucl. Phys. A 740(2004)287
with Felix Riek



- broad π spectral function
- 2 components: pion & particle-hole branches

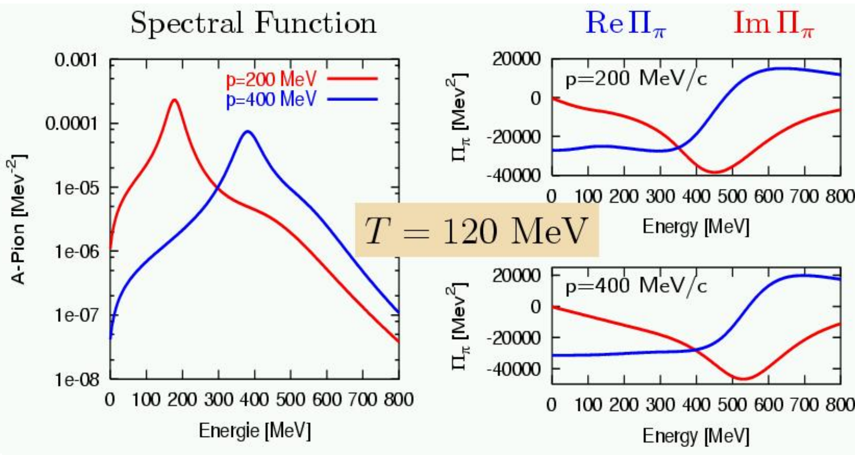
Pion spectral-function

Nucl. Phys. A 740(2004)287
with Felix Riek



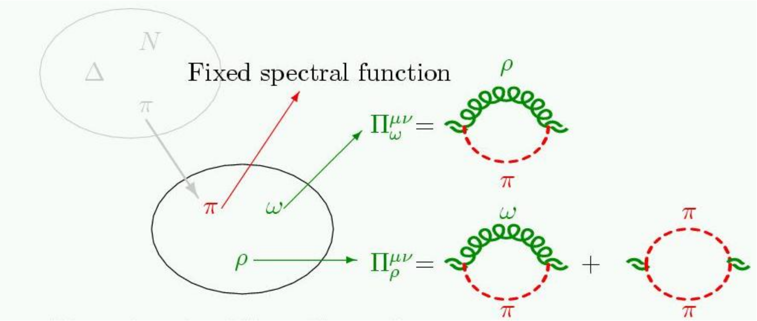
- broad π spectral function
- 2 components: pion & particle-hole branches

Pion spectral-function



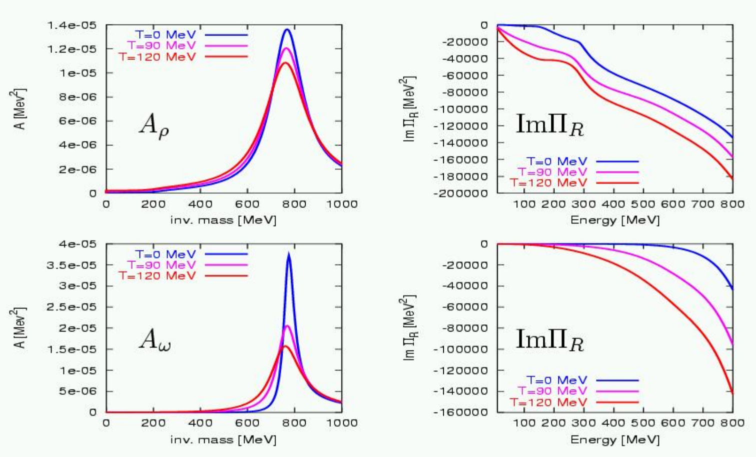
- broad π spectral function
- 2 components: pion & particle-hole branches

freezing the in-medium pion cloud of the π-N-Δ system



Vector mesons coupled to pions

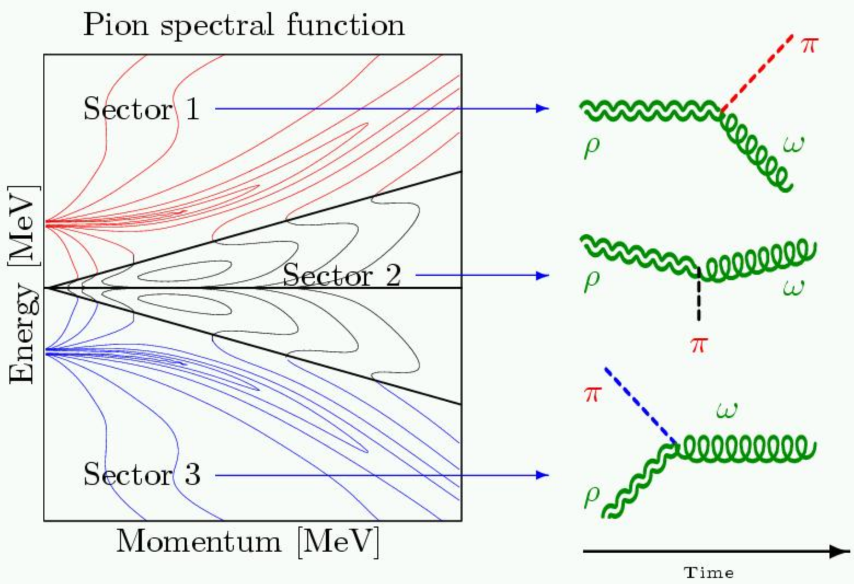
Nucl. Phys. A 740(2004)287
with Felix Riek



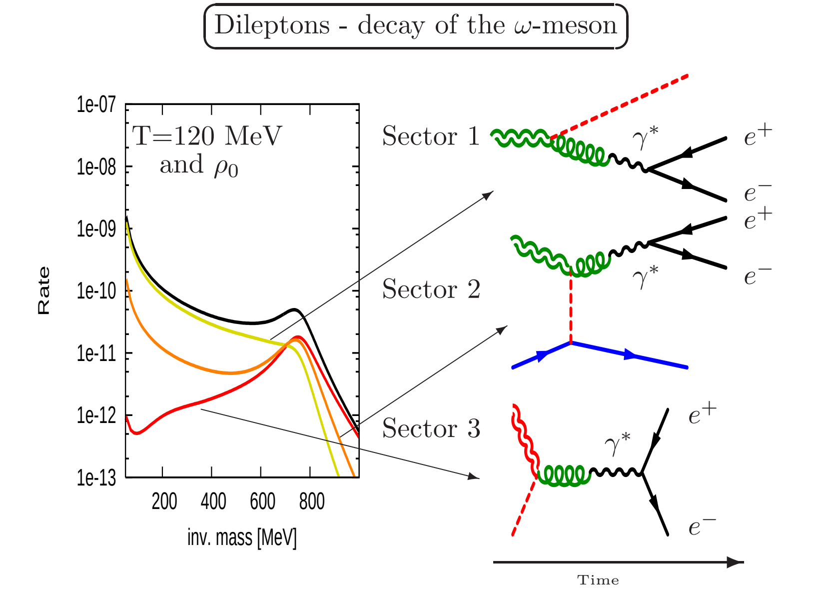
- neglecting real parts of self-energy
- $\Longrightarrow$ broadening of both vector meson spectral functions

Vector mesons coupled to pions

Nucl. Phys. A 740(2004)287
with Felix Riek

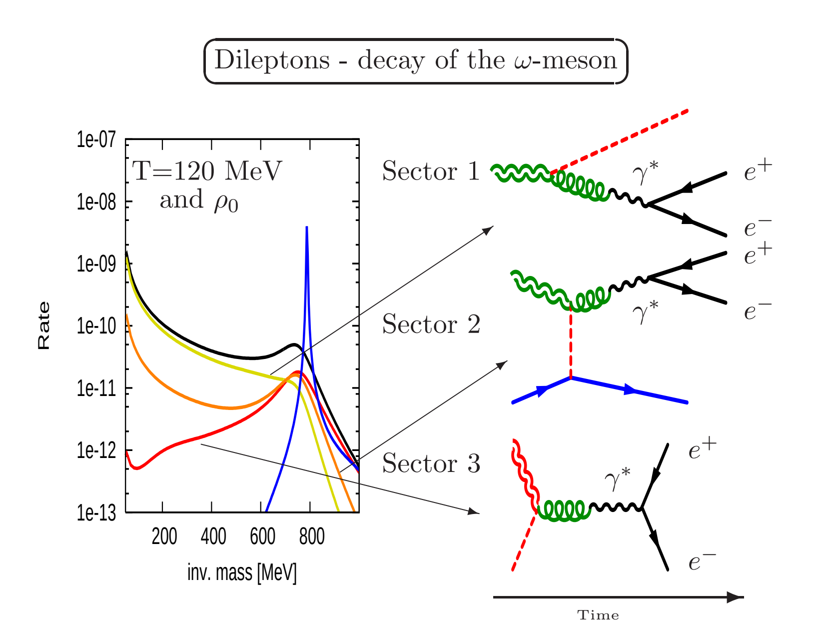


Di-leptons from vector mesons

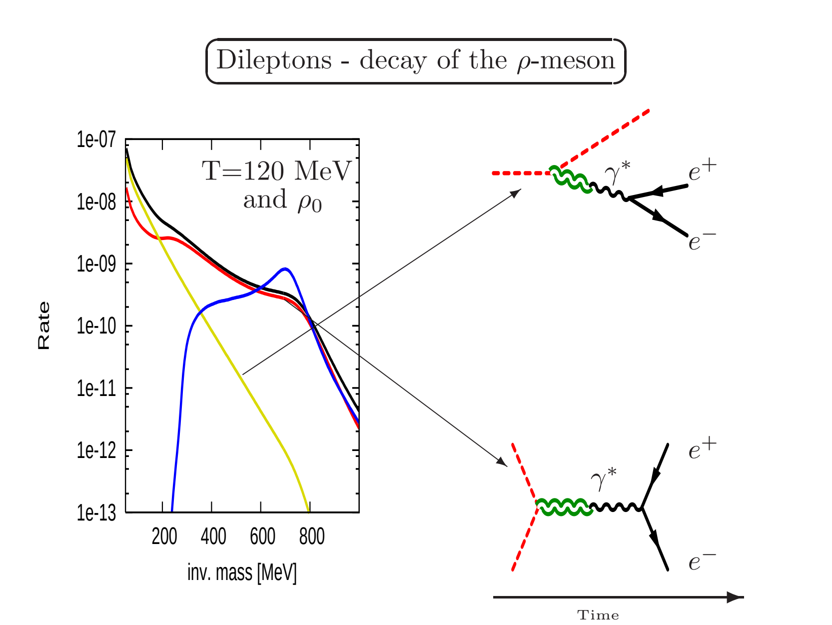


Di-leptons from vector mesons

Nucl. Phys. A 740(2004)287
with Felix Riek



Di-leptons from vector mesons



Conserving approximations

How to come to a

closed, consistent scheme?

respecting conservation laws
avoiding double counting
keeping the causality structure
the retarded relations and
detailed balance

Conserving approximations

- **Perturbation Theory fails:**
  - secular behavior at long times
  - higher order diagramms plagued by singularities
    Cure by cut-offs or appropriate resummations
- **Partial resummation schemes:**
  - Simplest: mean (classical) field
    and Dyson (Kadanoff-Baym) Eqs.

Φ-derivable method (2PI)

Luttinger-Ward '61
Baym 62'
Cornwall-Jackiw-Tomboulis '74

for classical fields
and two-point functions (Green fcts)

Conserving approximations

- General aim: equation of motion for

  $\Rightarrow$ Classical Fields ϕ_α (one-point fcts.)
  $\Rightarrow$ Propagators G_α (two-point fcts.)

  which are:
  - self consistent
  - conserving (Charge, Energy, Momentum, Symmetries, ...)
  - Thermodyn. consistent

$$\left(\partial^\mu\partial_\mu + m^2\right)\phi_\alpha = J_\alpha \qquad \text{(Cl. Field Eq)}$$

$$v^\mu\partial_\mu G_\alpha = G_\alpha \odot \Sigma_\alpha - \Sigma_\alpha \odot G_\alpha \qquad \text{(K.B. Eq)}$$

Diagrammatic generating functional $\Phi(\phi_\alpha, G_\alpha)$ with

$$J_\alpha(x) = \frac{\delta\Phi}{\delta\phi_\alpha(x)}; \qquad \Sigma_\alpha(x,y) = \mp\frac{\delta\Phi}{\delta G_\alpha(y,x)}$$

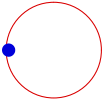
Φ: connected two-particle irred. closed diagrams
$\Rightarrow$ Conserving & Thermodyn. Consistent Apprx.

Invariances of Φ $\Rightarrow$ Conservation laws

Conserved Noether current:

$$v^\mu = \begin{cases} 2\,p^\mu & \text{rel.} \\ (1, \frac{\vec{p}}{m}) & \text{non-rel.} \end{cases}$$

$$J^\mu(X) = e \int \frac{d^4p}{(2\pi)^4} v^\mu \underbrace{f(X,p)A(X,p)}_{F(X,p)} =$$



Space-time invariance: $x \to x + \xi$: E-M-tensor

$$\Theta^{\mu\nu}(X) = \underbrace{\int \frac{d^4p}{(2\pi)^4} v^\mu p^\nu F(X,p)}_{\text{single particle energies}} + g^{\mu\nu}\left(\mathcal{E}^{\rm int}(X) - \mathcal{E}^{\rm pot}(X)\right)$$

$\Theta^{00}(X)$:

$$\mathcal{E}^{\rm int}(x) = -\langle \mathcal{L}(x) \rangle = \frac{\delta\Phi}{\delta\lambda(x)}$$ Interaction Energy Density

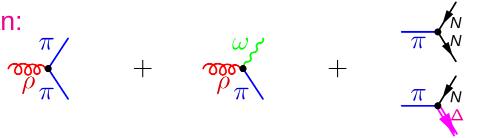
$$\mathcal{E}^{\rm pot}(x) = \frac{1}{2}\left\langle \frac{\partial\mathcal{L}}{\partial\phi(x)}\phi(x)\right\rangle$$ Single Particle Potential Energy Density

$$= \int \frac{d^4p}{(2\pi)^4} \left[\mathrm{Re}\,\Sigma^R(X,p)F(X,p) + \mathrm{Re}\,G^R(X,p)\Gamma^{in}(X,p)\right]$$

The interacting N-Δ-π-ρ-ω system

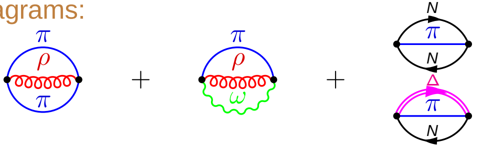
Lagrangian:

$\mathcal{L}_{\text{int}} =$



closed Diagrams:

$\Phi =$

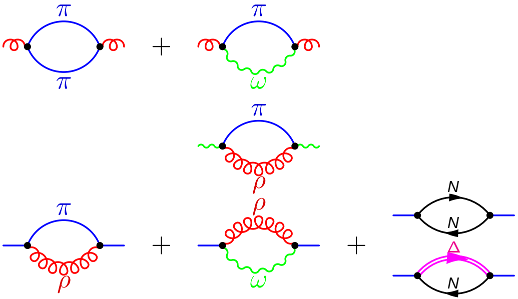


Meson self-energies:

$\Pi_\rho =$

$\Pi_\omega =$

$\Pi_\pi =$



Towards transport dynamics

How to come to a

closed and consistent
Transport scheme?

which is conserving
respects detailed balance
treats broad spectral widths
keeps the causality structure
and the retarded relations

Consistent Gradient approximation of K-B Equations
in Φ-derivable approximation

Generalized gradient apprx.

Ann. Phys. (NY) 293 (2001) 126

Taylor expanded with respect to $X = (x_1 + x_2)/2$

$$G\left(\frac{x_i + x_j}{2}, p\right) \approx \underbrace{G(X,p)}_{\text{local}} + \underbrace{\frac{1}{2}\left[(x_i^\mu - x_1^\mu) + (x_j^\mu - x_2^\mu)\right]\frac{\partial}{\partial X^\mu}G(X,p)}_{\text{gradient terms}}$$

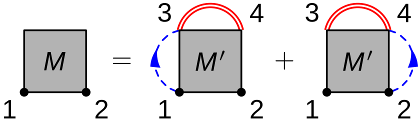
$$= \tfrac{1}{2}(\partial_i + \partial_j)\,G(i,j) \longrightarrow \partial_X G(X,p)$$

$$= -\mathrm{i}\,(x_i - x_j) \longrightarrow -(2\pi)^4 \frac{\partial}{\partial p}\delta(p)$$

For any two-point function:

$$M(X,p) \approx \left(1 + \frac{\mathrm{i}}{2}\Diamond\right) M^{\mathrm{local}}(X,p)$$

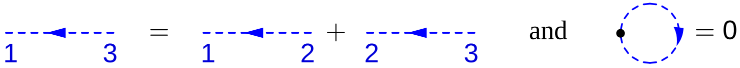
$$\Diamond\{M(1,2)\} = \Diamond$$



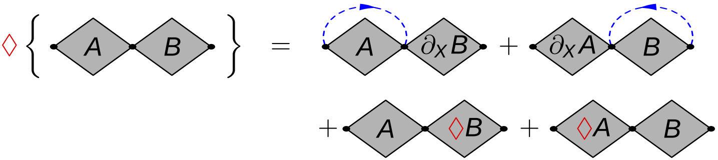
$$M'(1,2;3,4) = \mp\frac{\delta M(1,2)}{\delta\,\mathrm{i}G(4,3)}.$$

Gradient Diagram Rules

Addition rule:



Convolution rule:



$$= \{A(X,p), B(X,p)\}$$

$$+A(X,p)\Diamond[B(X,p)] + \Diamond[A(X,p)]B(X,P)$$

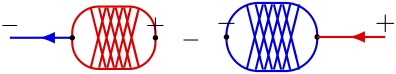
Gradient expanded K-B Eqs.

Ann. Phys. (NY) 293 (2001) 126

$$v^\mu \partial_\mu F(X,p) = \left(1 + \tfrac{\mathrm{i}}{2}\Diamond\right)\left\{C^{-+}_{(\mathrm{loc})}(X,p)\right\}$$

local (non-gradient) right side: Collision term $\to$ detailed balance!

$$\mp C^{-+}_{(\mathrm{loc})} \overset{\text{diagram}}{=}$$



$$\overset{\text{value}}{=} \underbrace{\Gamma_{\mathrm{in}}(X,p)\tilde{F}(X,p)}_{\text{gain}} - \underbrace{\Gamma_{\mathrm{out}}(X,p)F(X,p)}_{\text{loss}}$$

3-momentum		4-momentum
$f(X,\vec{p})\dfrac{\mathrm{d}^3\vec{p}}{(2\pi)^3}$	$\Longrightarrow$	$F(X,p)\dfrac{\mathrm{d}^4p}{(2\pi)^4} = f(X,p)A(X,p)\dfrac{\mathrm{d}^4p}{(2\pi)^4}$

$$\Gamma(X,p) \equiv -2\mathrm{Im}\,\Sigma^R(X,p) = \Gamma_{\mathrm{loss}}(X,p) \pm \Gamma_{\mathrm{gain}}(X,p)$$

$$A(X,p) \equiv -2\mathrm{Im}\,G^R$$

Retarded eq.

$$G^R(X,p) = \frac{1}{p^2 - m^2 - \mathrm{Re}\,\Sigma^R(X,p) + \mathrm{i}\Gamma(X,p)/2}$$

Gradients & conservation laws

Ann. Phys. (NY) 293 (2001) 126

$$v^\mu \partial_\mu F(X,p) - \{\mathrm{Re}\,\Sigma^R, F\} + \{\mathrm{Re}\,G^R, \Gamma^{\rm in}\} - C^{(\rm non-loc)} = C^{(\rm loc)}$$

dragflow
group velocity

backflow from
fluctuations, gain & non-local terms
from internal gradients in Σ

- $\Rightarrow$ conserved Noether currents and E-M-tensor ($\to$ EoS)

$$\partial_\mu \sum_a \int \frac{d^4p}{(2\pi)^4} e_a v^\mu F(X,p) = \partial_\mu J^\mu(X) = 0$$

$$\Theta^{\mu\nu}(X) = \int \frac{d^4p}{(2\pi)^4} v^\mu p^\nu F(X,p) + g^{\mu\nu}\left(\mathcal{E}^{(\rm loc)}_{\rm int}(X) - \mathcal{E}^{(\rm loc)}_{\rm pot}(X)\right)$$

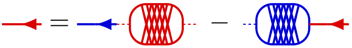
$$\mathcal{E}^{\rm pot}(x) = \int \frac{d^4p}{(2\pi)^4}\left[\mathrm{Re}\,\Sigma^R(X,p)F(X,p) + \mathrm{Re}\,G^R(X,p)\Gamma^{in}(X,p)\right]$$

- Relation to Delay Times (P. Danielewicz)
  drag flow: forward delay   other gradients: scattering delay

Towards Quantum Transport

Nucl.Phys.A 672 (2000) 313
Ann. Phys. (NY) 293 (2001) 126

Kadanoff-Baym-Dyson Eq.

$2p^\mu \partial_\mu$ 

$\Leftarrow$ Φ-derivable / conserving consistent

$\Downarrow$

Consistent Gradient Approx.

$\Downarrow$

1 Quantum Transport Eq.
1 Algebraic Eq. for Spectral Fnct.

Merits:

- ♣ A self-consistent & conserving transport scheme
- ♣ Allows to include Classical Fields (Soft Modes)
- ♣ Includes all QM Effects that are included in Equilibrium
- ♣ No Limitation to small Widths
- ♣ Delay-time, Drag & Back Flow, Memory & non-local Effects
- ♣ Non-equilibrium Entropy-current & H-Theorem
- ♠ Limitation to slow Space-time variations inherent to **all** transport schemes

Limitations:

- ♠ Test-particle simmulation unsettled
  Problem: backflow;
  approx. treatment: Botermans-Malfliet
  used by W. Cassing & S. Leupold
- ♠ Problems with Symmetries on Correlator Level
  - a) violation of Goldstone modes,
  - b) violation of Gauge Invariance Transversality of the polarization tensor (vector bosons)

general cure: next higher vertex eq.: Bethe-Salpeter eq.
(generally untractable)

special repair:

$\Rightarrow$ a) supplement a symmetry restoring term to Φ

Y.B. Ivanov, J.K. & F. Riek, Phys.Rev.D71:105016,2005; hep-ph/0506157

$\Rightarrow$ b) use only spatial components of $\Pi^{\mu\nu}$ (short relaxation) and construct a 4-transverse tensor by projection methods

H. van Hees & J.K., Nucl. Phys. A683(2001)369