Continuous decoupling and freeze-out*

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The decoupling and freeze-out of energetic nuclear collisions is analysed in terms of transparent semi-classical decoupling formulae. They provide a smooth transition and generalise frequently employed instantaneous freeze-out procedures. Simple relations between the damping width and the duration of the decoupling process are presented and the implications on various physical phenomena arising from the expansion and decay dynamics of the highly compressed hadronic matter generated in high energy nuclear collisions are discussed.

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1. Introduction

Dynamically expanding systems may pass various stages, where different dynamical concepts or description levels are appropriate. For nuclear collisions a possibly formed quark gluon plasma (QGP) converts to a dense hadronic medium. Subsequently the chemical components (i.e. the abundances of the different hadrons) decouple and finally the system kinetically freezes out releasing the particles that reach the detectors.

All such transitions share that they need quite some time, be it in order to cope with strong rearrangements of the matter, in part accompanied with a significant change in entropy density and corresponding release of latent heat (e.g. for the QGP $\rightarrow$ hadron matter transition), or simply that the processes happen probabilistically. Some of these transitions can adequately be formulated in terms of micro- or macroscopic transport equations, other ones such as e.g. phase transitions or composite-particle formation involve changes of degrees of freedom, which are complicated in microscopic terms and mostly not yet well formulated. Therefore in many cases recipes are applied, mostly of instantaneous nature. These concern coalescence pictures,

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which combine nucleons to composite nuclei at freeze-out [1] or coalesce quarks to hadrons in the deconfinement-confinement transition [2]. Also the general freeze-out is mostly treated as a sudden transition happening at a suitably chosen three dimensional freeze-out hyper-surface in space-time [3]. In many cases such prescriptions violate general principles as detailed balance, unitarity, conservation laws or entropy requirements [4, 5]. In particular the instantaneous freeze-out picture is widely used to analyse nuclear collision data in terms of thermal models. The achieved fits in temperature, chemical potentials and parametrised flow effects to the observed particle abundances and kinetic spectra, are then frequently used as measured data that are taken as clues on the physics of the collision dynamics, cf. A. Andronic, and W. Broniowski on this workshop or Refs. [6, 7].

In this contribution I'll present some simple analytical considerations which illustrate the space-time dynamics of transition processes in didactic terms at the example of the freeze-out. The results are well in line with recent progress reported by Y. Sinyukov and S. Pratt at this meeting, see also [8–11]. Earlier attempts towards a continuous freeze-out description as derived by the Brazilian group [12, 13] and later efforts [14–16] directly focussed on a global decoupling scheme for fluid-dynamic approaches. Here we investigate these processes micro-dynamically similar to the conceptual progress and clarifications given in [8, 9] on the basis of classical kinetics.

2. Decoupling formulae

The decoupling properties of an observed particle are given by its interaction with the particles in the source. The latter is encoded in the corresponding current-current correlation function or polarisation function Π. It provides the following local decoupling rate in four phase space [17, 18]

$$\frac{dN(x,p)}{d^4p dt d^3x} = \frac{1}{(2\pi)^4} \underbrace{\Pi^{\rm gain}(x,p) A(x,p)}_{C^{\rm gain}(x,p)} \mathcal{P}_{\rm escape}(x,p). \tag{1}$$

in the context of the gradient expanded Kadanoff-Baym equations [19, 20]. Here the first two factors (gain rate encoded in $\Pi^{\rm gain}$ times local spectral function A) determine the gain part of the local collision term $C^{\rm gain}(x,p)$ [20]. The last factor $\mathcal{P}_{\rm escape}(x,p)$ captures the probability that particles, created or scattered at space-time point x into a momentum p, can escape to infinity without further being absorbed by the loss part of the collision term. This formulation thus restricts the emission zone to the layer of the last interaction. Semi-classically in the small damping width limit one obtains

$$\mathcal{P}_{\rm escape}(x,p) = {\rm e}^{-\chi(x,p)}, \quad \text{where} \quad \chi(x,p) = \int_{(x,\vec{p})}^{\infty} \Gamma(x',p') dt' \tag{2}$$

with $\Gamma(x',p') = -\mathrm{Im}\Pi^{\mathrm{R}}(x',p')/p'_0$. The time integration defining the optical depth χ runs along the classical escape path starting at $(x,\vec{p})$, the latter determined by the real part of the retarded polarisation function Π^{R} [17,18]. The above rate causes drains in particle number and energy and a recoil momentum from the source which in a fluid dynamical description lead to

$$\partial_\mu \begin{pmatrix} j^\mu_{\alpha,\mathrm{fluid}}(x) \\ T^{\mu\nu}_{\mathrm{fluid}}(x) \end{pmatrix} = -\sum_a \int d^4p \begin{pmatrix} e_{a\alpha} \\ p^\nu \end{pmatrix} \frac{d\,N_a(x,p)}{d^4x d^4p}, \tag{3}$$

resulting from the dissipative part of the underlying transport equations (here a labels the different particles and α a conserved current). For small spectral width all spectral strength $A(x,p)$ will be guided towards the detector with on-shell momentum $\vec{p}_A$ providing the following detector yield

$$\frac{dN_a(p_A)}{d^3p_A} = \int \frac{d^4xd^4p}{(2\pi)^4}\,\Pi^{\mathrm{gain}}_a A_a\,\mathcal{P}_{\mathrm{escape}} \left(\frac{\partial \vec{p}_A}{\partial \vec{p}}\right)^{-1} \delta^3(\vec{p}-\vec{p}(x,\vec{p}_A)). \tag{4}$$

Here $\vec{p}_A(x,\vec{p})$ and $\vec{p}(x,\vec{p}_A)$ denote the corresponding mapping of the local momentum $\vec{p}$ to the detector momentum and its inverse, respectively. The corresponding Jacobi determinant accounts for the focussing or defocusing of the classical paths due to deflections.

In thermal equilibrium the source function becomes

$$\Pi^{\mathrm{gain}}_a(x,p) = -2\,f_{\mathrm{th}}(x,p^0)\,\mathrm{Im}\Pi^{\mathrm{R}}_a(x,p) = f_{\mathrm{th}}(x,p^0)\,2p^0\,\Gamma_a(x,p), \tag{5}$$

where $\Gamma_a(x,p)$ is the local damping width of particle a. This property leads to quite some compensation effect, which is frame independent. Namely, for large source extensions the integral over the Γ-dependent damping factors in (4), which define the visibility probability P_t, equates to unity

$$\int_{-\infty}^{\infty} dt\ \underbrace{\Gamma(t)\ \mathrm{e}^{-\chi(t)}}_{=\ P_t(t)} = 1, \quad \text{where} \quad \chi(t) = \int_t^\infty dt'\Gamma(t'), \tag{6}$$

if integrated along any path leading from the *opaque* interior to the outside. This compensation is independent on the structural details of Γ and on the classical paths leading to the detector, along which the decoupled particles are accumulated. When rates drop smoothly in time the visibility probability $P_t(t)$ achieves its maximum at

$$\left[\frac{d}{dt}\Gamma(t) + \Gamma^2(t)\right]_{t_{\max}} = 0, \qquad \text{where} \quad P_t(t_{\max}) \approx \Gamma(t_{\max})/\mathrm{e}. \tag{7}$$

The corresponding decoupling duration Δt_{dec} approximately follows from the normalisation of the visibility function P_t through a kind of decoupling uncertainty relation $\Gamma(t_{\max})\ \Delta t_{\mathrm{dec}} \approx \mathrm{e}$.

In the limit that $P_t(t)$ can be replaced by a δ-function one recovers an improved Cooper-Frye [3] formulae, where the freeze-out hypersurface is no longer globally defined, but individually by the posed detector momentum [8] through the features of $\Gamma(x,p)$ at peak condition (7).

3. Analytic model considerations

In Ref. [17] various consequences of the continuous decoupling formalism are discussed. As an illustration we consider the competition between chemical and kinetic (thermal) freeze-out of slow particles escaping from a spherically expanding uniform fireball. The emission is than essentially from a time-like hypersurface. Both processes go with a different pace as a function of density and/or temperature during the expansion, since inelastic processes drop much faster than the elastic scattering processes, the latter essentially determining the kinetic rates.

For example a fireball evolution with a freeze-out radius of $R_{\rm dec} \approx 6$ fm and collective velocity $\dot{R}_{\rm dec} = 0.5$ fm/c leads to a decoupling peak at $t_{\rm max} =$ 12 fm/c for both types of freeze-out. The damping widths at decoupling peak are as large as $\Gamma^{\rm chem}_{\rm max} = \Gamma^{\rm kin}_{\rm max} = 0.5$ c/fm ≈ 100 MeV providing the values given in Table 1. The typical duration of a decoupling process is thereby of less importance.

Rather the robust feature is the relative volume growth during which the system decouples, which is mostly beyond half an order of magnitude. During this time the thermodynamic properties of the system can significantly change thereby influencing the spectrum of the observed particles. Even more robust is the behaviour of the overall damping rate Γ of the decoupling particle. Coming from a completely opaque zone where the damping is strong, it decreases by about a factor $\mathrm{e}^{\mathrm{e}} \approx 15$ with values between begin, peak and end of the freeze-out of notably $\Gamma_{\rm i} : \Gamma_{\rm max} : \Gamma_{\rm f} = 390 : 100 : 26$ MeV for the above nuclear collision scenario.

Depending on the underlying EoS the thermodynamic properties of the matter can therefore significantly change during the decoupling time window. The resulting distributions in temperature $P_T(T) = P_t(t)(dT(t)/dt)^{-1}$ as shown in the right panel of Fig. 1 for two normally behaved example EoS show a significant spread in the T distributions. Remarkable is further that

100 GeV Au + Au	decoupl. time	vol. growth
phase transition [4,5]:	6 - 10 fm/c	> 5
chemical freeze-out:	> 5 fm/c	> 4
kinetic freeze-out:	> 8 fm/c	> 6
CMB early universe [21]:	$Z = [1300 - 800]$	$(13/8)^3 = 4.3$

Table 1. Typical decoupling durations and volume growths.

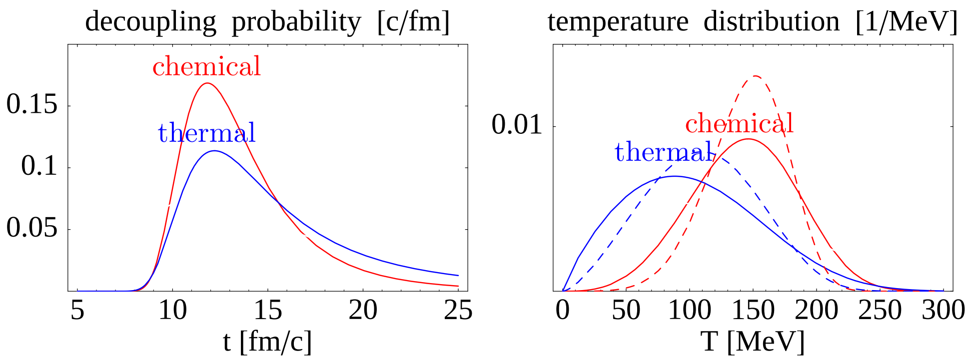

Fig. 1. Decoupling probability $P_t(t)$ as a function of time (left panel) and the resulting temperature distributuions (right panel) the latter for two simple EoS with adiabatic index $\kappa = 1.5$ (full lines) and $\kappa = 4/3$ (dashed lines) for the schematic chemical and thermal freeze-out scenarios discussed in the text [17].

although both time distributions P_t peak at the same time (left panel of Fig. 1), the slower decrease in kinetic rates leads to a considerably downward shifted and much broader T distribution for thermal freeze-out compared to that in the chemical case.

4. Summary and perspectives

For any observed probe the structural properties of the source are encoded in the current-current correlation function defining the gain part of the polarisation function Π (or self-energy). It knows about resonances and other structural properties that influence the emitted particle. Only penetrating probes, which suffer no distortion and absorption in the matter ($\mathcal{P} = 1$), have an undisturbed view on the source. Strongly interacting probes, though, have a significantly reduced view. In the opaque limit most structural effects are wiped out and one is rather left to observe statistical features, i.e. temperature effects, of the source. In particular kinematical fingerprints from decays of resonances with lifetimes less than the decoupling duration will become invisible in the kinetic spectra of the probe, deferring to include such decay modes into statistical model descriptions.

The here discussed decoupling features are generic and apply to many dynamically expanding systems. This includes the microwave background radiation released during the early universe evolution. For applications to nuclear collisions the here diagnosed long decoupling and freeze-out times are both a challenge but also a chance: a chance to map out the thermodynamic properties of the expanding collision zone during the freeze-out of various probes. An observation of a quite narrow distribution in temperature, for example, could point towards effects that significantly slow down the temperature drop during expansion, and this way provide hints towards the underlying equation of state or possible phase transition effects.

In summary I see a need to reanalyse nuclear collisions data in the light of the results and discussions given here. Promising steps towards this goal were presented at this meeting by Y. Sinyukov and S. Pratt [9, 10]. In those hybrid model calculations the entire decoupling stage is treated within kinetic transport. These model calculations do not only confirmed the here advocated long freeze-out durations (above 10 fm/c). At the same time, contrary to earlier HBT interpretations that inferred extremely short decoupling durations [22], these long durations emerged well conform with $R_{\mathrm{out}}/R_{\mathrm{side}}$ close to unity [11] as observed in the data at the CERN SPS and at RHIC, this way providing a solution to the alleged HBT puzzle discussed on this workshop, cf. also the corresponding HBT reviews of [23, 24].

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