Finite pion width effects on the rho–meson

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Abstract

We study the influence of the finite damping width of pions on the in–medium properties of the ρ–meson in an interacting meson gas model at finite temperature. Using vector dominance also implications on the resulting dilepton spectra from the decay of the ρ–meson are presented. A set of coupled Dyson equations with self–energies up to the sunset diagram level is solved self consistently. Following a Φ–derivable scheme the self–energies are dynamically determined by the self–consistent propagators. Some problems concerning the self–consistent treatment of vector or gauge bosons on the propagator level, in particular, if coupled to currents arising from particles with a sizable damping width, are discussed.

Key words:
Rho–meson, Medium modifications, Dilepton production, Self–consistent approximation schemes.
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1 Introduction and summary

The question how a dense hadronic medium changes the properties of vector mesons compared to their free space characteristics has attracted much attention in recent times. Experimentally this question is studied in measurements of the dilepton production rates in heavy ion collisions. Recent experiments by the CERES and DLS collaborations [1–3] show that the low lepton pair mass spectrum is significantly enhanced in the range between 300 MeV and 600 MeV compared to the yield that one expects from the corresponding rates in pp–collisions.

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From the theoretical side various mechanisms are proposed to explain these influences of a hadronic matter surrounding on the spectral properties of vector mesons[4–11]. With the upgrade of CERES and the new dilepton project HADES at GSI a more precise view on the spectral information of vector mesons is expected. Still in most of theoretical investigations the damping width gained by stable particles due to collisions in dense matter is either ignored or treated within an extended perturbation theory picture[12]. In this contribution we study the in–medium properties of the ρ–meson due to the damping width of pions in a dense meson gas within a self–consistent scheme.

The field theoretical model, which is inspired from vector meson dominance theories[13], is discussed in section 2. Thereby the finite pion width is modelled with a four–pion self–interaction in order to keep the investigation as simple as possible. The coupling strength is adjusted as to produce a pion damping–width of reasonable strength, such as to simulate the width a pion would obtain in a baryon rich environment due to the strong coupling to baryonic resonance channels, like the Δ–resonance. The self–consistent equations of motion are derived from Baym's Φ–functional[14–16] and the problem of renormalisation is left aside by taking into account only the imaginary parts of the self–energies but keeping the normalisation of the spectral function fixed.

This self–consistent treatment described in section 3 respects the conservation laws for the expectation values of conserved currents and at the same time ensures the dynamical as well as the thermodynamical consistency of the scheme. Especially the effects of bremsstrahlung and annihilation processes are taken into account consistently.

Finally in section 4 we discuss the principal problems with the treatment of vector mesons in such a scheme which are mainly due to the fact that within the Φ–functional formalism the vertex corrections necessary to ensure the Ward–Takahashi identities for the propagator are ignored. In our model calculations we work around this problem by projecting onto the transverse part of the propagators such that the errors of this shortcoming can be expected to be small. In the appendix we elaborate on some details of this projection method.

2 The model

In order to isolate the pion width effects we consider a purely mesonic model system consisting of charged pions, neutral ρ–mesons, and also the chiral partner of the ρ–, the a_1–meson, with the interaction Lagrangian

$$\mathscr{L}^{\rm int} = g_{\rho\pi\pi}\rho_\mu\pi^*\overleftrightarrow{\partial}^\mu\pi + g_{\pi\rho a_1}\pi\rho_\mu a_1^\mu + \frac{g_{\pi 4}}{8}(\pi^*\pi)^2 + cc. \tag{1}$$

We do not explicitely write down the free Lagrangian, but we like to mention that we consider the ρ-meson as a gauge particle. The first two coupling constants are adjusted to provide the corresponding vacuum widths of the ρ– and a_1–meson at the nominal masses of 770 MeV and 1200 MeV and widths of $\Gamma_\rho = 150$ MeV and $\Gamma_{a_1} = 400$ MeV, respectively. The four–π interaction is used as a tool to furnish additional collisions among the pions. The idea of this term is to provide pion damping widths of about 50 MeV or more as they would occur due to the strong coupling to the NN^{-1} and ΔN^{-1} channels in an environment at finite baryon density.

The Φ–functional method originally invented by Baym[15] provides a self–consistent scheme applicable even in the case of broad resonances. It is based on a resummation for the partition sum [14,16]. Its two particle irreducible part $\Phi\{G\}$ generates the irreducible self–energy $\Sigma(x,y)$ via a functional variation with respect to the propagator $G(y,x)$, i.e.

$$-{\rm i}\Sigma(x,y) = \frac{\delta {\rm i}\Phi}{\delta {\rm i}G(y,x)}. \tag{2}$$

Thereby Φ, constructed from two-particle irreducible closed diagrams of the Lagrangian (1) solely depends on fully resummed, i.e. self–consistently generated propagators $G(x,y)$. In graphical terms, the variation (2) with respect to G is realized by opening a propagator line in all diagrams of Φ. Further details and the extension to include classical fields or condensates into the scheme are given in ref. [17].

Truncating Φ to a limited subset of diagrams, while preserving the variational relation (2) between $\Phi^{\rm (appr.)}$ and $\Sigma^{\rm (appr.)}(x,y)$ defines an approximation with built–in consistency. Baym[15] showed that such a Dyson resummation scheme is conserving at the expectation value level of conserved currents related to global symmetries, realised as a linear representation of the corresponding group, of the original theory, that its physical processes fulfil detailed balance and unitarity and that at the same time the scheme is thermodynamically consistent. However symmetries and conservation laws may no longer be maintained on the correlator level, a draw–back that will lead to problems for the self–consistent treatment of vector and gauge particles on the propagator level, as discussed in sect. 3.

Interested in the effects arising from the damping width of the particles we discard all changes in the real part of the self energies, keeping however the sum–rule of the spectral functions normalised. In this way we avoid renormalisation problems which require a temperature independent subtraction scheme.

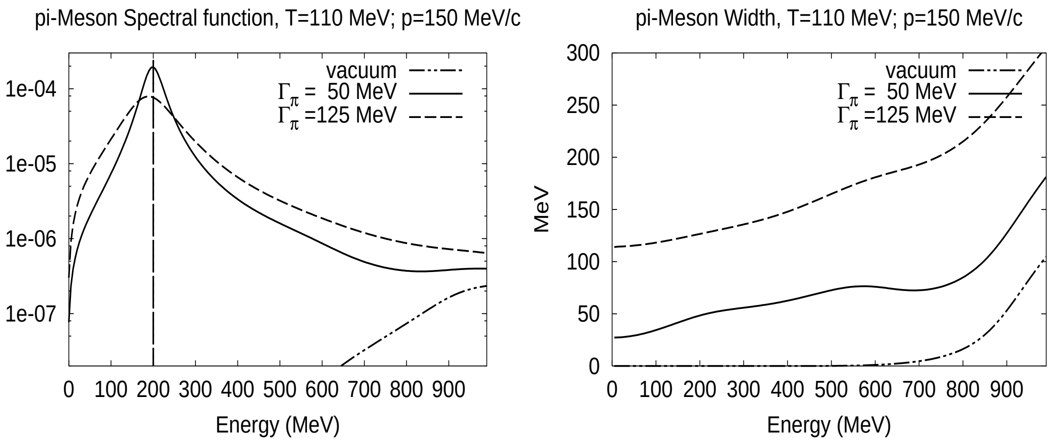
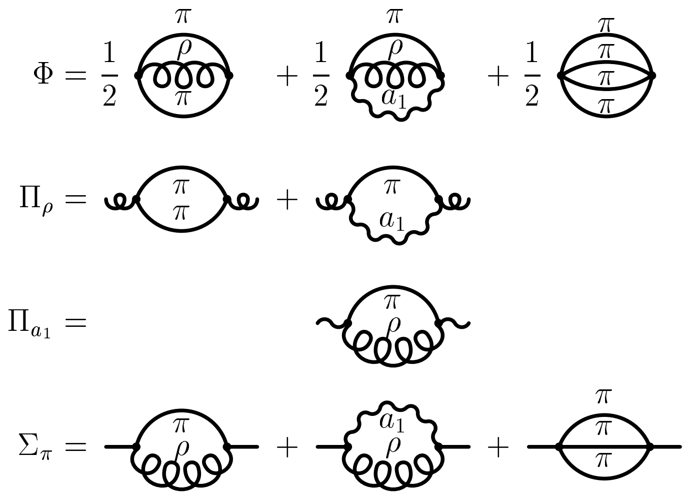


Fig. 1. *Spectral function (left) and decay width (right) of the pion as a function of the pion energy at a pion momentum of* 150 *MeV/c in the vacuum and for two self–consistent cases discussed in the text.*

The latter will be discussed in detail in a forthcoming paper [18]. Neglecting changes in real parts of the self–energies also entitles to drop tadpole contributions. The treatment of the tensor structure of the ρ– and a_1–polarisation tensors is discussed in sect. 3. Here we first discuss the results of the self–consistent calculations.

For our model Lagrangian (1) and neglecting tadpole contributions one obtains the following diagrams for Φ at the two–point level which generate the subsequently given three self energies Π_ρ, Π_{a_1} and Σ_π



(3)

They are the driving terms for the corresponding three Dyson equations, which form a coupled scheme which has to be solved self–consistently. The Φ–derivable scheme pictorially illustrates the concept of Newton's principle of *actio = reactio* and detailed balance. If the self–energy of one particle is modified due to the coupling to other species, these other species also obtain a corresponding term in their self–energy. In the vacuum the ρ– and a_1–meson self–energies have the standard thresholds at $\sqrt{s} = 2m_\pi$ and at $3m_\pi$ respectively. For the pion as the only stable particle in the vacuum with a pole at m_π the threshold opens at $\sqrt{s} = 3m_\pi$ due to the first and last diagram of Σ_π. Correspondingly the vacuum spectral function of the pion shows already spectral strength for $\sqrt{s} > 3m_\pi$, c.f. fig. 1 (left).

Self–consistent equilibrium calculations are performed keeping the full dependence of all two–point functions on three momentum $\vec{p}$ and energy p_0, and treating all propagators with their dynamically determined self–energies.

The examples shown refer to a temperature of $T = 110\,\mathrm{MeV}$ appropriate for the CERES data. We discuss three different settings. First the ρ–meson polarisation tensor is calculated simply by the perturbative pion loop, i.e. with vacuum pion propagators and thermal Bose–Einstein weights (no self–consistent treatment). The two other cases refer to self–consistent solutions of the coupled Dyson scheme, where the four–π interaction is tuned such that the sun–set diagram provides a moderate pion damping width of about 50 MeV and a strong one of 125 MeV around the peak of the pion spectral function, c.f. fig. 1. Since in the thermal case any excitation energy is available, though with corresponding thermal weights, all thresholds disappear and the spectral functions show strength at *all* energies [3] ! The pion functions shown in Fig. 1 at a fixed momentum of 150MeV are plotted against energy in order to illustrate that there is significant strength also in the space–like region (below the light cone at 150 MeV) resulting from π–π scattering processes.

As an illustration we display a 3–d plot of the ρ–meson spectral function as a function of p_0 and $|\vec{p}|$ in Fig. 2, top left. The right part shows the transverse spectral function as a function of invariant mass at fixed three–momentum of 150 MeV/c in vacuum and for the two self–consistent cases. The minor changes at the low mass side of the ρ-meson spectral function become significant in the dilepton yields given in the left bottom panel. The reason lies in the statistical weights together with additional kinematical factors $\propto m^{-3}$ from the dilepton–decay mechanism described by the vector meson dominance principle[13]. For the moderate damping case ($\Gamma_\pi = 50\,\mathrm{MeV}$) we have decomposed the dilepton rate into partial contributions associated with π–π bremsstrahlung, π–π annihilation and the contribution from the a_1–meson, which can be interpreted as the a_1 Dalitz decay.

The low mass part is completely dominated by pion bremsstrahlung contributions (like–charge states in the pion loop). This contribution, which vanishes

[3] In mathematical terms: all branch–cuts in the complex energy plane reach from $-\infty$ to $+\infty$, and the physical sheets of the retarded functions are completely separated from the physical sheets of the corresponding advanced functions by these cuts.

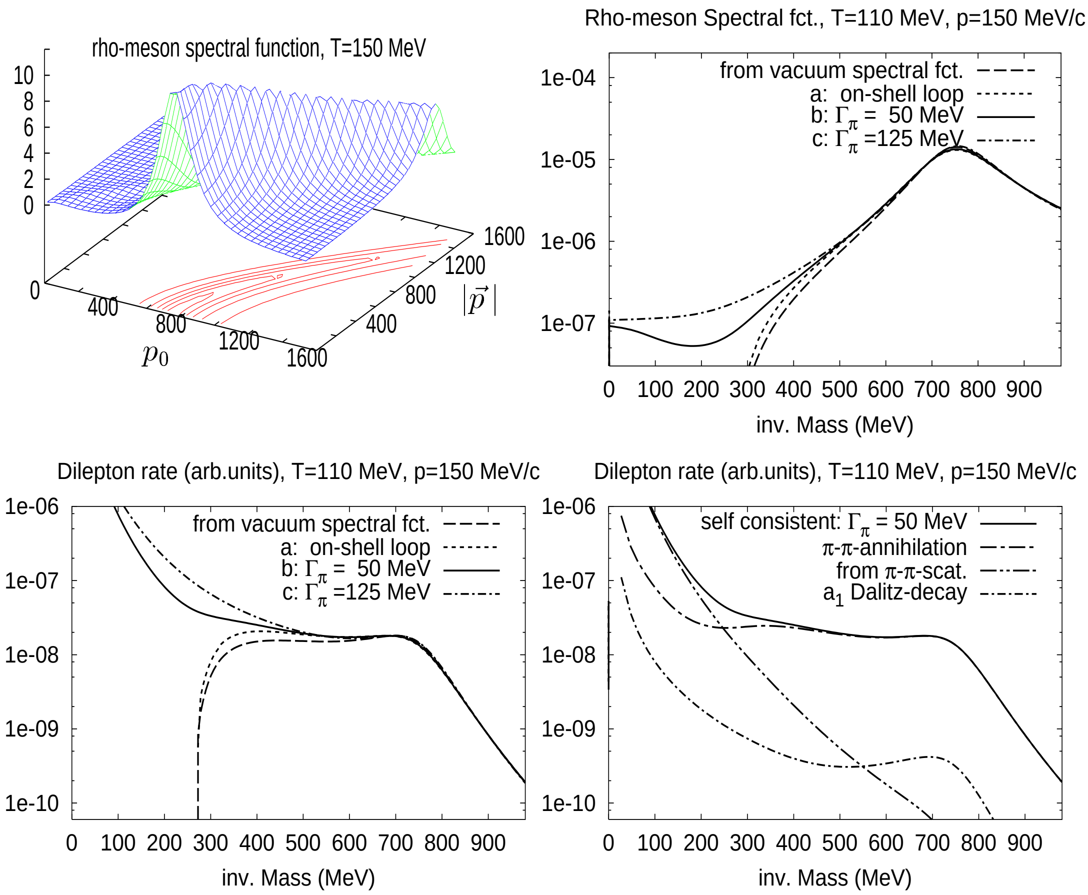


Fig. 2. *top: ρ–meson spectral function, bottom: thermal dilepton rate.*

in lowest order perturbation theory, is *finite* for pions with finite width. It has to be interpreted as bremsstrahlung, since the finite width results from collisions with other particles present in the heat bath. Compared to the standard treatment, where the bremsstrahlung is calculated independently of the π–π annihilation process, this self–consistent treatment has a few advantages. The bremsstrahlung contribution is calculated consistently with the annihilation process, it appropriately accounts for the Landau–Pomeranchuk suppression at low invariant masses [19] and at the same time includes the in–medium pion electromagnetic form–factor for the bremsstrahlung part. As a result the finite pion width adds significant strength to the mass region below 500 MeV compared to the trivial treatment with the vacuum spectral function. Therefore the resulting dilepton spectrum essentially shows no drop any more in this low mass region already for a moderate pion width of 50 MeV. The a_1 Dalitz decay contribution can be read off from the partial ρ–meson width due to the π–a_1 loop in Π_ρ. This component is seen to be unimportant at all energies in the present calculations where medium modifications of the masses of the mesons are discarded. The latter can be included through renormalised dispersion relations within such a consistent scheme.

3 Longitudinal and transverse components

While scalar particles and couplings can be treated self–consistently with no principle problems at any truncation level, considerable difficulties and undesired features arise in the case of vector particles. The origin lies in the fact that, though in Φ–derivable Dyson resummations symmetries and conservation laws are fulfilled at the expectation value level, they are generally no longer guaranteed at the correlator level. Considering the ρ-meson as a gauge particle one has to care about local gauge symmetries, where the situation is even worse, because the symmetry of the quantised theory is not the original one but the non–linear BRST symmetry [20,21]. Contrary to perturbation theory, where the loop expansion corresponds to a strict power expansion in $\hbar$ and symmetries are maintained order by order, partial resummations mix different orders and thus are violating the corresponding symmetries. It is obvious that the scheme discussed above indeed violates the Ward identities on the correlator level and thus the vector–meson polarisation tensor is no longer 4–dimensionally transverse. This means that unphysical states are propagated within the internal lines of the Φ–derivable approximation scheme which leads to a number of conceptual difficulties and to explicit difficulties in the numerical treatment of the problem. In the above calculations we have worked around this problem in the following way.

For the exact polarisation tensor we know that for $\vec{p} = 0$ the temporal components exactly vanish $\Pi^{00}(q) = \Pi^{0i}(q) = \Pi^{i0}(q) = 0$ for $q_0 \neq 0$, while this is not the case for the self–consistently constructed tensor [19]. Indeed these components are tied to the conservation of charge and therefore involve a relaxation time for a conserved quantity which is of course infinite while the self–consistent result always reflects the damping time of the propagators in the loop. This behaviour is studied in detail in ref. [19], both on the classical and quantum many body Green's function level within the real time formalism. There it has been shown that current conservation can only be restored through a resummation of all the scattering processes in a transport picture which amounts to a Bethe–Salpeter ladder resummation in the corresponding quantum field theory description. This will be discussed to some extent in sect. 4. At this level it is important to realise that the spatial components generally suffer less corrections from this resummation in case that the relaxation time for the transverse current–current correlator is comparable to the damping time of the propagators in the loop. The time components, however, suffer significant corrections. Thus our strategy for the self–consistent loop calculation is the following: from the loop calculation of the polarisation tensors Π (of the ρ–meson) we evaluate only the information obtained for the spatial components Π^{ik}. Taking the following two spatial traces (details are given in the appendix A)

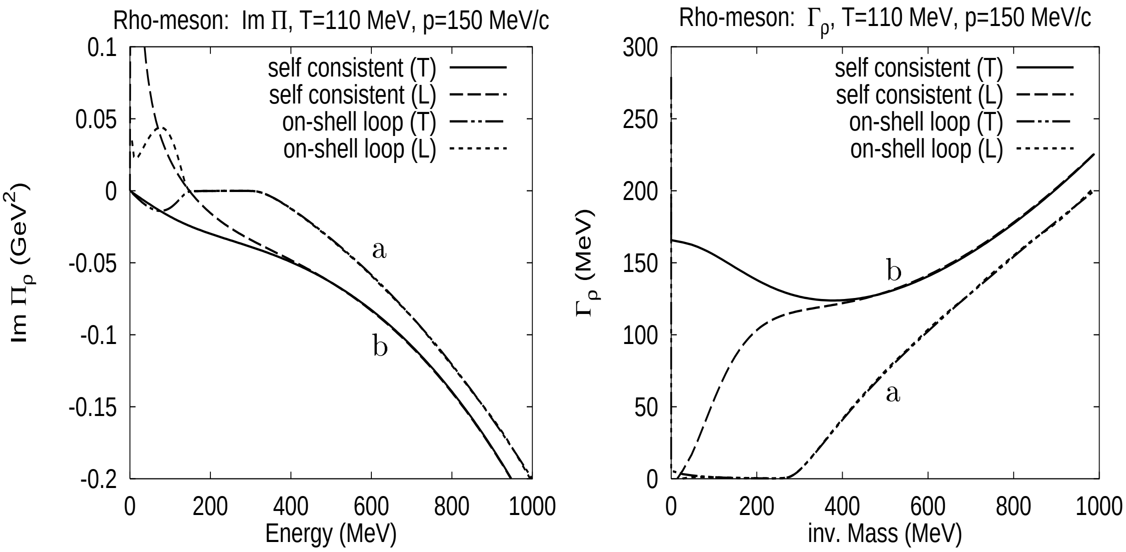


Fig. 3. *Left: Imaginary part of the Longitudinal (L) and transverse (T) ρ–meson polarisation tensor at $T = 110\,MeV$. case (a): finite temperature on-shell–loop calculation; case (b): self–consistent calculation for the case where $\Gamma_\pi = 125\,MeV$. Right: the same for the corresponding widths $\Gamma_\rho = -\mathrm{Im}\,\Pi_\rho/p^0$.*

$$\left.\begin{aligned} 3\Pi_3 &= -g_{ik}\Pi^{ik} = 2\Pi_T + \tfrac{(q^0)^2}{q^2}\Pi_L \\ \Pi_1 &= \frac{p_i p_k}{\vec{q}^2}\Pi^{ik} = \frac{(q^0)^2}{q^2}\Pi_L \end{aligned}\right\} \quad \Leftarrow \quad \partial_\mu \Pi^{\mu\nu} = 0 \tag{4}$$

permits to deduce the 3–dim. longitudinal and transverse tensor components Π_L and Π_T under the condition that the polarisation tensor is exactly 4–dim. transversal. This construction thus fulfils current conservation on the correlator level.

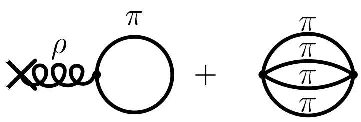
The result of this procedure is shown in fig. 3 for the components of the ρ–meson polarisation tensor for a finite spatial momentum of $\vec{p} = 150\,\mathrm{MeV}$. The plots show $\mathrm{Im}\,\Pi_L$ and $\mathrm{Im}\,\Pi_T$ first for the on–shell loop result, i.e. with vacuum pion spectral functions and thermal occupations. For this on–shell loop case to very good approximation one finds $\Pi_L = \Pi_T$ for time like momenta, while as expected they deviate in sign for space–like momenta. The longitudinal component exactly vanishes on the light cone and changes sign there. Thus the tensor is entirely transverse on the light cone as it should! Switching to the self–consistent results the threshold gap between $\sqrt{s} \in [0, 2m_\pi]$ is completely filled. At non–zero momenta $\vec{p}$ the longitudinal and tranverse component deviate from one another towards low invariant masses, i.e. $\sqrt{s} < 400\,\mathrm{MeV}$ in this case, while they are identical for large $\sqrt{s}$ as they should. As both components Π_L and Π_T are constructed from different moments of the numerically given Π^{ik}, the agreement of the two components at large $\sqrt{s}$ shows the numerical precision of the employed loop integration method. The resulting behaviour is further clarified in the right part of fig. 3, which shows the resulting damping width of the ρ–meson $\Gamma_\rho(p) = -\mathrm{Im}\,\Pi_\rho(p)/p^0$. One sees that the typical threshold behaviour of the on–shell loop is completely changed in the self–consistent result. The transverse width is with $\Gamma_T \approx 150$MeV almost constant over the displayed invariant mass range! For the longitudinal component one has to consider the kinematical factor entering in specific tensor components, e.g. $\Pi^{00} = \Pi_L\, \vec{q}^2/q^2$, such that also Γ^{00} is about constant.

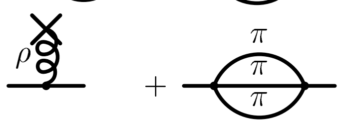
While current conservation has been restored on the correlator level by the procedure above, the Ward–Takahashi identities are certainly not fulfilled. Thus the whole procedure will not be gauge covariant. Still from the experience discussed in ref. [19] we expect that this method provides a good approximation to the in–medium polarisation tensor at finite temperature. In particular for the light pions which at $T = 150$ MeV have already quite relativistic energies we expect that the mutual scattering leads to fairly isotropic distributions after each scattering such that the memory on some initial fluctuation of the pion current is already lost after the first collision, c.f. the discussions in ref. [19] and in the next section.

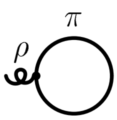
4 Symmetries and gauge invariance

In view of the difficulties to provide a gauge–invariant scheme one may raise the question: *is there a self–consistent truncation scheme beyond the mean field level for the gauge fields, which preserves gauge invariance?* In particular we are interested that the internal dynamics, i.e. the dynamical quantities like classical fields and propagators which enter the self–consistent set of equations remain gauge covariant.

At the mean field level the gauge fields couple to the expectation values of the vector currents and gauge covariance is fully maintained. This level is explored in all hard thermal loop (HTL) approaches [22–25]. For the π–ρ–meson system the mean field approximation is given by the following Φ–derivable scheme (again omitting the tadpole term for the pion self–energy)

$\Phi\{G_\pi, \rho\} =$  (5)

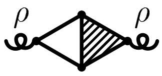
$\Sigma_\pi =$  (6)

$\left(\partial^\nu \partial_\nu - m^2\right) \rho^\mu = j^\mu =$  (7)

Here full lines represent the self–consistent pion propagators and curly lines with a cross represent the classical ρ–meson field, governed by the classical field equations of motion (7). Since Φ is invariant with respect to gauge transformations of the classical vector field ρ^μ, the resulting equations of motion are gauge covariant.

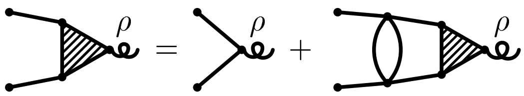
The step to construct symmetry preserving correlation functions is provided by considering the linear response of the system on fluctuations in the background field [26,15], see also [27] in the context of gauge and Goldstone bosons. Thereby gauge covariance also holds for fluctuations $\rho^\mu + \delta\rho^\mu$ around mean field solution of (6 - 7). Thus, one can then define a gauge covariant *external* polarisation tensor via variations with respect to the background field $\delta\rho^\mu$

$$\Pi^{\text{ext}}_{\mu\nu}(x_1, x_2) = \frac{\delta}{\delta\rho^\mu(x_2)} \left. \frac{\delta\Phi[G_\pi, \rho]}{\delta\rho^\nu(x_1)} \right|_{G_\pi[\rho]} = \tag{8}$$



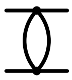
as a linear response to fluctuations around the mean field. This tensor can be accessed through a corresponding three–point–vertex equation

$$\frac{\delta G_\pi}{\delta\rho^\mu} = \tag{9}$$



In order to maintain all symmetries and invariances, the four-point Bethe–Salpeter Kernel in this equation has to be chosen consistently with the Φ–functional (5) [15,26], i.e. as a second functional variation of Φ with respect to the propagators

$$K_{1234} = \frac{\delta^2\Phi}{\delta G_{12}\delta G_{34}}. = \tag{10}$$



Thereby the pion propagator entering the ladder resummation (9) is determined by the self–consistent solution of Eq. (6) at vanishing classical ρ-field. In particular this ladder resummation accounts for real physical scattering processes, a phenomenon already discussed in [19] for the description of Bremsstrahlung within a classical transport scheme (Landau–Pomeranchuk–Migdal effect). From this point of view one clearly sees that the pure Φ–functional formalism without the vertex corrections (9) just describes the "decay of states" due to collision broadening. Thus in the Φ–Dyson scheme (3) all components of the *internal* ρ–meson polarisation tensor have a *time–decaying* behaviour with a decay constant given by the pion damping rate Γ_π. However, the exact tensor has at least two decay times, one for the transverse components and a second one which involves the conserved charge and which naturally is infinite. The Dyson resummation fails to cope with this, since there also the 00–component approximately behaves like

$$\Pi^{00}_\rho(\tau, \vec{p} = 0) \propto e^{-\Gamma_\pi \tau} \tag{11}$$

in a mixed time–momentum representation. This clearly violates charge conservation, since $\partial_0 \Pi_\rho^{00}(\tau,\vec{p})|_{\vec{p}=0}$ does not vanish! Yet, accounting coherently for the multiple scattering of the particles through the vertex resummation (9) keeps track of the "charge flow" into other states and thus restores charge conservation. Within classical considerations the ladder resummation (9) indeed yields

$$\Pi_\rho^{00}(\tau,\vec{p}=0) \propto \sum_n \frac{(\Gamma\tau)^n}{n!}\, e^{-\Gamma\tau} = 1 \tag{12}$$

confirming charge conservation. For further details c.f. ref. [19]. From the physics discussions above it is clear that these conclusions hold also for constant self–energies with a constant imaginary part. This is opposed to the vacuum case where a constant self–energy would not require any vertex correction! The formal origin of this difference lies in the fact that in the real time formulation of the field theory all relations become matrix relations from the contour time ordering. In particular the three point functions then have three independent retarded components [4] (c.f. [28,29]) and the corresponding Ward-Takahashi identities involve both retarded and advanced self energy terms which differ in the sign of their imaginary parts. Thus even for constant self–energies the terms related to the width do no longer cancel out in these identities in the true real time case at finite T.

5 Comments and prospects

We presented self–consistent calculations of the vector meson production in a pion gas environment, where the width of the pion was generated by a pion four-vertex interaction. The novel part is that through the damping width standard thresholds known from vacuum calculations disappear and that contributions arising from pion bremsstrahlung and from π–π–annihilation are treated within the same scheme, and are therefore consistent with one another. With reasonable damping width for the pions the calculations show significant contributions to the dilepton spectrum in the mass range below 400 MeV. In the second part we discussed the particular features related to the self–consistent treatment of vector or gauge bosons.

Vertex corrections of various types were already considered in the literature in the context of the ρ–meson. They dealt with the case, where the pion couples to the nuclear sector, and vertex corrections related to the p–wave coupling of the π-N-Δ–vertex where included on phenomenological grounds using

[4] In the Matsubara formalism this corresponds to the fact that the different energy arguments of three point functions can be placed on different half planes.

Landau–Migdal parameterisations for the Δ-N interaction [30,31]. There the employed quasi-particle loops are straightforward and the "bubble" resummation (algebraic) also. More involved vertex corrections were considered in refs. [6] and by many others later, e.g. [12]. However in none of those papers the vertex corrections were done self–consistently, nor were they proven to restore gauge invariance, nor has there the question been addressed which vertex corrections are required, once the propagators in the self–energy loop of the gauge particles have a significant damping width. The latter question was addressed in ref. [19] in the context of photon production and put on formal grounds here. Using the background field scheme we explained above the steps to come to a consistent vertex equation (9-10). It is neither some vertex equation nor the exact vertex equation: it is precisely *the* vertex equation which pertains to the self–consistent pion self–energy given by (6) at vanishing background field. The consistency comes about by the fact that both, the pion self energy (6) and the Bethe-Salpeter kernel (10), are generated from the same Φ-functional (5). The method is general and applies to any kind of Φ-functional supplemented by terms coupling to background gauge fields.

Two problems are to be mentioned in this context, a practical and a principle one. The practical problem concerns the fact that there is no feasible algorithm to calculate the external self–energy (8). Already for our most simple example one has to solve the ladder re–summation in the Bethe–Salpeter equation with *full off–shell momentum dependence* for the three–point vertex given by (9). Even if one restricts oneself to the simplifying case of vanishing ρ–meson three–momentum for this vertex the numerical effort increases by about two orders of magnitude (for the full momentum dependence a factor 10^5) compared to the presented numerical solution of self–consistent self–energies and is thus out of reach in practice.

However there is yet a principle problem. What one constructs by the vertex equation is the *external* polarisation tensor of the vector meson. It has all desired features. However, the corresponding vector–meson propagator does not take part in the self–consistent scheme. This so constructed external propagator is fine in all cases, where the vector meson couples perturbatively to a source, e.g. like the photon to the electromagnetic current of a source system. For the case of the ρ–meson recoupling effects may already be of some importance, and for sure for gluons in an interacting quark-gluon plasma such recoupling effects are important, and the vector meson is a sensible component in the self–consistent scheme. In such cases, however, one sees that in self–consistent truncation schemes there is generally a difference between the self–consistent internal propagator and the external propagator constructed from the Bethe–Salpeter ladder resummation. While the first violates Ward–Takahashi identities, the latter fulfils them.

Therefore presently we see no obvious self–consistent scheme where vector particles are treated dynamically beyond mean field, i.e. with dynamical propagators, and which at the same time complies with gauge invariance also for the *internal* propagation, unless one solves the exact theory. The workaround presented in sect. 3 at least guarantees that the polarisation tensor remains four–dimensional transverse and thus no unphysical modes are appearing in the scheme. The problem of renormalisation omitted here is investigated separately using subtracted dispersion relations [18]. Thus for vector particles a fully self–consistent scheme with all the features of the Φ–functional, especially to ensure the consistency of dynamical and thermodynamical properties of the calculated propagators together with the conservation laws on both the expectation value and the correlator level remains an open problem.

The self–consistent equilibrium calculations presented here also serve the goal to gain experience about particles with broad damping width aiming towards a transport scheme for particles beyond the quasi–particle limit [32], see also [33–35].

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A Decomposition of the polarisation tensor

In spherically symmetric systems the polarisation tensor Π can be decomposed into three components (4–longitudinal (l) and the two 4-transverse components, the longitudinal (L) and transverse (T) one)

$$\Pi^{\mu\nu} = \Pi_l^{\mu\nu} + \Pi_L^{\mu\nu} + \Pi_T^{\mu\nu} \tag{A.1}$$

$$\Pi_l^{\mu\nu} = -\frac{q^\mu q^\nu}{q^2}\Pi_l \tag{A.2}$$

$$\Pi_L^{\mu\nu} = \left(-g^{\mu\nu} - \delta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} + \frac{\vec{q}^\mu \vec{q}^\nu}{\vec{q}^2}\right)\Pi_L \tag{A.3}$$

$$\Pi_T^{\mu\nu} = \left(\delta^{\mu\nu} - \frac{\vec{q}^\mu \vec{q}^\nu}{\vec{q}^2}\right)\Pi_T. \tag{A.4}$$

Here $\delta^{\mu\nu}$ and $\vec{q}^{\mu}$ are defined such that the time–components vanish. The spatial part and the 00–component become

$$\Pi^{ik} = \left(\delta^{ik} - \frac{\vec{q}^i\vec{q}^k}{\vec{q}^2}\right)\Pi_T + \frac{(q^0)^2}{q^2}\frac{q^iq^k}{\vec{q}^2}\Pi_L - \frac{q^iq^k}{q^2}\Pi_l \tag{A.5}$$

$$\Pi^{00} = \left(-g^{00} + \frac{(q^0)^2}{q^2}\right)\Pi_L - \frac{(q^0)^2}{q^2}\Pi_l. \tag{A.6}$$

In terms of the 4– and 3–traces we define

$$\begin{aligned} 4\Pi_4 = -\,\mathrm{Tr}_4\{\Pi^{\mu\nu}\} &= -g_{\mu\nu}\Pi^{\mu\nu} = -\Pi^{00} + \mathrm{Tr}_3\{\Pi^{ik}\} \\ &= \Pi_l + \Pi_L + 2\Pi_T \end{aligned} \tag{A.7}$$

$$3\Pi_3 \;= \mathrm{Tr}_3\{\Pi^{ik}\} \;= -g_{ik}\Pi^{ik} = 2\Pi_T + \frac{(q^0)^2}{q^2}\Pi_L - \frac{\vec{q}^2}{q^2}\Pi_l. \tag{A.8}$$

One further can use

$$\Pi_1 = \frac{p_ip_k}{\vec{q}^2}\Pi^{ik} = \frac{(q^0)^2}{q^2}\Pi_L - \frac{\vec{q}^2}{q^2}\Pi_l. \tag{A.9}$$

Taking into account the information contained in the two traces Π_1 and Π_3, and the condition for 4–dim. transversality, $\Pi_l = 0$, a conserved polarisation tensor can be constructed.

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