# Towards a Quantum Transport Description of Particles with finite Mass Width[1]

Jörn Knoll, GSI, Darmstadt
*J.Knoll@gsi.de; http://theory.gsi.de*

Yu. B. Ivanov, Kurchatov Institute, Moscow

D. N. Voskresensky, Moscow Institute for Physics and Engineering
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The effects of the propagation of particles which have a finite life time and an according width in their mass spectrum are discussed in the context of transport descriptions. In the first part the coupling of soft photon modes to a source of charged particles is studied in a classical model which can be solved completely in analytical terms. The solution corresponds to a re-summation of certain field theory diagrams. The second part addresses the derivation of transport equations which also account for the damping width of the particles. The $\Phi$-derivable method of Baym is used to derive a self-consistent and conserving scheme. For this scheme a conserved energy-momentum tensor can be constructed. Furthermore, a kinetic entropy can be derived which besides the standard quasi-particle part also includes contributions from fluctuation.

## I. INTRODUCTION

With the aim to describe the collision of two nuclei at intermediate or even high energies one is confronted with the fact that the dynamics has to include particles like the delta or rho-meson resonances with life-times of less than 2 fm/c or equivalently with damping widths above 100 MeV. The collision rates deduced from presently used transport codes are comparable in magnitude, whereas typical mean kinetic energies as given by the temperature range between 70 to 150 MeV depending on beam energy. Thus, the damping width of most of the constituents in the system can no longer be treated as a perturbation.

As a consequence the mass spectrum of the particles in the dense matter is no longer a sharp delta function but rather acquires a width due to collisions and decays. The corresponding quantum propagators $G$ (Green's functions) are no longer the ones as in the standard text books for fixed mass, but rather have to be folded over a spectral function $A(\epsilon,\vec{p})$, which takes a Lorentz shape $A(\epsilon,\vec{p}) \sim \Gamma/((\epsilon-\epsilon(\vec{P}))^2+(\Gamma/2)^2)$ of width $\Gamma/2$ in simple approximations. One thus comes to a picture which unifies *resonances* which have already a width in vacuum due to decay modes with the "states" of particles in dense matter, which obtain a width due to collisions (collisional broadening). The theoretical concepts for a proper many body description in terms of a real time non equilibrium field theory have already been devised by Schwinger, Kadanoff, Baym and Keldysh [1] in the early sixties. First investigations of the quantum effects on the Boltzmann collision term were given Danielewicz [2], the principal conceptual problems on the level of quantum field theory were investigated by Landsmann [3], while applications which seriously include the finite width of the particles in transport descriptions were carried out only in recent times, e.g. [2,4–10]. For resonances, e.g. the delta resonance, it was natural to consider broad mass distributions and ad hoc recipes have been invented to include this in transport simulation models. However, many of these recipes are not correct as they violate some basic principle like detailed balance [4], and the description of resonances in dense matter has to be improved [9].

In this talk the consequences of the propagation of particles with short life times is readdressed and discussed. In the first part a genuine soft mode problem is studied: the coupling of a coherent classical field, the Maxwell field, to the stochastic Brownian motion of a charged particle. The rate of photons due to Bremsstrahlung, given by the classical current-current correlation function, can be obtained in closed analytical terms and discussed as a function of the macroscopic transport properties, the friction and diffusion coefficient of the Brownian particle. The result corresponds to a partial re-summation of photon self energy diagrams in the real-time formulation of field theory. In the second part of this talk a scheme is presented,

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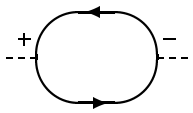
how to come to a self-consistent, conserving and thermodynamically consistent transport description of particles with finite mass width within the real-time formulation of non-equilibrium field theory. The derivation is based on and generalizes the $\Phi$-functional method of Baym [11]. The first-order gradient approximation provides a set of coupled equations of time-irreversible generalized kinetic equations for the slowly varying space-time part of the phase-space distributions and retarded equations, which provides the fast micro-scale dynamics represented by the four-momentum part of the distributions. Functional methods permit to derive a conserved energy-momentum tensor which also includes corrections arising from fluctuations besides the standard quasi-particle terms. Memory effects [12] appearing in collision term diagrams of higher order are discussed. The variational properties of $\Phi$-functional permit to derive a generalized expression for the non-equilibrium kinetic entropy flow, which includes corrections from fluctuations and memory effects. In special cases we demonstrate that the entropy can only increase with time ($H$-theorem).

## II. PRELIMINARIES

The standard text-book transition rate in terms of Fermi's golden rule, e.g. for the photon radiation from some initial state $|i\rangle$ with occupation $n_i$ to final states $|f\rangle$

$$W = \sum_{if} n_i(1-n_f) \left| \begin{array}{c} f \\ \\ i \end{array} \right|^2 \times(1+n_\omega)\ \delta(E_i - E_f - \omega_{\vec{q}}) \qquad (1)$$

with occupation $n_\omega$ for the photon, is limited to the concept of asymptotic states. It is therefore inappropriate for problems which deal with particles of finite life time. One rather has to go to the "closed" diagram picture, where the same rate emerges as

$W =$  $(1+n_\omega)\delta(\omega - \omega_{\vec{q}})$ (2)

with now two types of vertices $-$ and $+$ for the time-ordered and the anti-time ordered part of the square of the amplitude. Together with the line sense and the $-$ and $+$ marks at the vertices a unique correspondence is provided between the oriented $\overset{+\ -}{\longrightarrow}$ and $\overset{-\ +}{\longrightarrow}$ propagator lines and the initial and final states. Thus such propagator lines define the densities of occupied states or those of available states, respectively. Therefore *all standard diagrammatic rules* can be used again. One simply has to extend those rules to the two types of vertices with marks $-$ and $+$ and the corresponding 4 propagators, the usual time-ordered propagator $\overset{-\ -}{\longrightarrow}$ between two $-$ vertices, the anti-time-ordered one $\overset{+\ +}{\longrightarrow}$ between two $+$ vertices and the mixed $\overset{+\ -}{\longrightarrow}$ or $\overset{-\ +}{\longrightarrow}$ ones as densities of occupied and available states. For details I refer to the textbook of Lifshitz and Pitaevski [13]. The advantage of the formulation in terms of "correlation" diagrams, which no longer refer to amplitudes but directly relate to physical observables, like rates, is that now one is no longer restricted to the concept of asymptotic states. Rather all internal lines, also the ones which originally correspond to the "in" or "out" states are now treated on equal footing. Therefore now one can deal with "states" which have a broad mass spectrum and which therefore appropriately account for the damping of the particles. The corresponding Wigner densities $\overset{+\ -}{\longrightarrow}$ or $\overset{-\ +}{\longrightarrow}$ are then no longer on-shell $\delta$-functions in energy (on-mass shell) but rather acquire a width in terms of the spectral function, e.g. for non-relativistic particles

$$\mathrm{i}G^{-+} = \overset{-\ +}{\longleftarrow} = \mp f(p) A(p)$$

$$\mathrm{i}G^{+-} = \overset{+\ -}{\longleftarrow} = (1 \mp f(p)) A(p) \qquad (3)$$

$$A(p) = \frac{\Gamma(p)}{\left(\epsilon + \mu_F - \epsilon^0_{\vec{p}} - \mathrm{Re}\,\Sigma^R(p)\right)^2 + (\Gamma(p)/2)^2}.$$

Here $f(p)$ is the phase-space occupation at four-momentum $p = (\epsilon, \vec{p})$, $A$ is the spectral function with the damping width $\Gamma(p)$ and in-medium on-shell energy $\epsilon^0_{\vec{p}} - \mathrm{Re}\,\Sigma^R(p)$ and $\mu$ is the chemical potential. In general all quantities depend on both, energy $\epsilon$ and momentum $\vec{p}$.

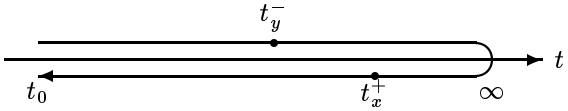



FIG. 1. Closed real-time contour with two external points $x, y$ on the contour.

The non-equilibrium theory can entirely be formulated on one special time contour, the so called closed time path [1], fig. 1, with the time argument running from some initial time $t_0$ to infinity and back with external points placed on this contour, e.g., for the four different components of Green's functions or self energies. The special

$-+$ or $+-$ components of the self energies define the gain and loss terms in transport problems, c.f. eq. (2).

## III. BREMSSTRAHLUNG FROM CLASSICAL SOURCES

For a clarification of the infra-red problem we first discuss two simple examples of soft modes in hard matter. These are examples in classical electrodynamics, which both can be solved analytically to a certain extent: there the hard matter is described either by a diffusion process or by a random walk problem, respectively [10]. As the source particles move non-relativistically these cases do not suffer from standard pathologies encountered in the hard thermal loop (HTL) problem of QCD, namely the collinear singularities, where $\vec{v}\vec{q} \approx 1$ and from diverging Bose-factors. The advantage of these examples is that damping can be fully included without violating current conserving and gauge invariants in the case of Abelian fields. The closed form results obtained correspond to a partial re-summation of certain planar diagrams, which just survive in the classical limit. The problem is related to the Landau–Pommeranchuk–Migdal effect of Bremsstrahlung in high energy scattering [14].

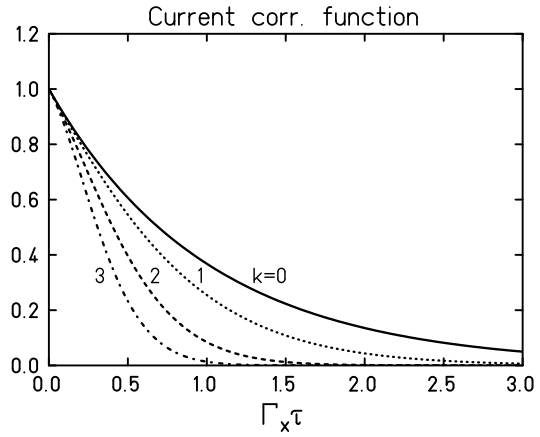



FIG. 2. Current-current correlation function in units of $e^2 < v^2 >$ as a function of time (in units of $1/\Gamma_x$) for different values of the photon momentum $q^2 = 3k^2\Gamma_x^2/< v^2 >$ with $k = 0, 1, 2, 3$.

The *diffusion process* is assumed to be described by a Fokker-Planck equation for the probability distribution $f$ of position $\vec{x}$ and velocity $\vec{v}$

$$\frac{\partial}{\partial t} f(\vec{x}, \vec{v}, t) = \left( D\Gamma_x^2 \frac{\partial^2}{\partial \vec{v}^2} + \Gamma_x \frac{\partial}{\partial \vec{v}} \vec{v} - \vec{v} \frac{\partial}{\partial \vec{x}} \right) f(\vec{x}, \vec{v}, t). \quad (4)$$

Likewise fluctuations evolve in time by this equation and this way determine the correlations. The two macroscopic parameters are the spatial diffusion coefficient $D$ and a friction constant $\Gamma_x$ which determines the relaxation rates of velocities (friction due to collisions with the medium). In the equilibrium limit ($t \to \infty$) the distribution attains a Maxwell-Boltzmann velocity distribution where $T = m \left\langle \vec{v}^2 \right\rangle /3 = mD\Gamma_x$. The correlation function can be obtained in closed form and one can discuss the resulting time correlations of the current at different fixed values of the photon momentum $\vec{q}$, fig. 2 (details are given in ref. [10]). For the transverse part of the correlation tensor this correlation decays exponentially as $\sim e^{-\Gamma_x \tau}$ at $\vec{q} = 0$, and its width further decreases with increasing momentum $q = |\vec{q}|$. Besides trivial kinematical factors, the in-medium production rate is given by the time Fourier transform $\tau \to \omega$.

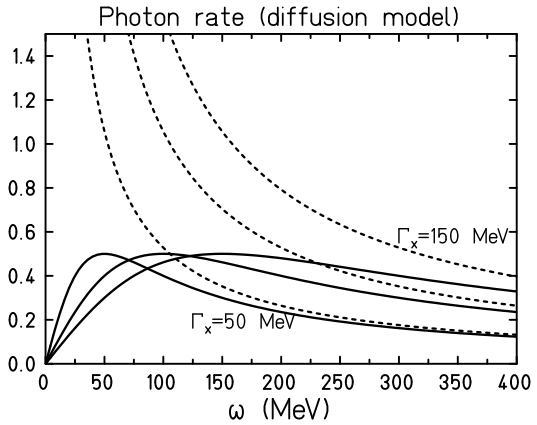



FIG. 3. Rate of real photons $\mathrm{d}^2N/(\mathrm{d}\omega \mathrm{d}t)$ in units of $4\pi e^2 \left\langle \vec{v}^2 \right\rangle /3$ for a non-relativistic source for $\Gamma_x$ =50,100,150 MeV; for comparison the IQF results (dashed lines) are also shown.

Fig. 3 displays the corresponding total production rates $\mathrm{d}^2N/(\mathrm{d}\omega \mathrm{d}t)$ of on-shell photons (number per time and energy; which is dimensionless) in units of $4\pi e^2 \left\langle \vec{v}^2 \right\rangle /3$. One sees that the hard part of the spectrum behaves as expected, namely, like in the IQF approximation the rate grows proportional to $\Gamma_x$ and this way proportional to the microscopic collision rate $\Gamma$ (c.f. below). However independent of $\Gamma_x$ the rate saturates at a value of $\sim 1/2$ in these units around $\omega \sim \Gamma_x$, and the soft part shows the inverse behavior. That is, with increasing collision rate the production rate is more and more suppressed! This is in line with the picture that such photons cannot resolve the individual collisions any more. Since the soft part of the spectrum behaves like $\omega/\Gamma_x$, it shows a genuine non-perturbative feature which cannot be obtained

by any power series in $\Gamma_x$. For comparison: the dashed lines show the corresponding IQF yields, which agree with the correct rate for the hard part while they completely fail and diverge towards the soft end of the spectrum. For non-relativistic sources $\langle \vec{v}^2 \rangle \ll 1$ one can ignore the additional $q$-dependence (dipole approximation; c.f. fig. 2) and the entire spectrum is determined by one macroscopic scale, the relaxation rate $\Gamma_x$. This scale provides a quenching factor

$$C_0(\omega) = \frac{\omega^2}{\omega^2 + \Gamma_x^2} \,. \tag{5}$$

by which the IQF results have to be corrected in order to account for the finite collision time effects in dense matter.

In the *microscopic Langevin picture* one considers a classical process, where hard scatterings occur at random with a constant *mean collision rate* $\Gamma$. These scatterings consecutively change the velocity of a point charge from $\vec{v}_m$ to $\vec{v}_{m+1}$ to $\vec{v}_{m+2}$, ... (in the following subscripts $m$ and $n$ refer to the collision sequence). In between scatterings the charge moves freely. For such a multiple collision process some explicit results can be given, since the correlated probability to find the charge at time $t_1$ and $t_2$ at two different segments with $n$ scatterings in between follows from the iterative folding of the exponential decay law with decay time $1/\Gamma$. Therefore the space integrated current-current correlation function takes a simple Poisson form

$$\begin{aligned} \mathrm{i}\Pi^{\mu\nu}_{-+} &\propto \int \mathrm{d}^3x_1 \mathrm{d}^3x_2 \langle j^\mu(\vec{x}_1, t - \tfrac{\tau}{2}) j^\nu(\vec{x}_2, t + \tfrac{\tau}{2}) \rangle \\ &= e^2 \langle v^\mu(0) v^\nu(\tau) \rangle \\ &= e^2 e^{-|\Gamma\tau|} \sum_{n=0}^{\infty} \frac{|\Gamma\tau|^n}{n!} \left\langle v^\mu_m v^\nu_{m+n} \right\rangle_m \end{aligned} \tag{6}$$

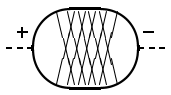
with $v = (1, \vec{v})$. This result represents a genuine multiple collision description of the correlation function. Here $\langle \ldots \rangle_m$ denotes the average over the discrete collision sequence $\{m\}$. This form, which one writes down intuitively, directly includes what one calls *damping* in the corresponding quantum case. Fourier transformed it determines the spectrum in completely regular terms (void of any infra-red singularities) where each term describes the interference of the photon being emitted at a certain time or $n$ collisions later.

In special cases where velocity fluctuations are degraded by a constant fraction $\alpha$ in each collision, such that $\langle \vec{v}_m \cdot \vec{v}_{m+n} \rangle_m = \alpha^n \langle \vec{v}_m \cdot \vec{v}_m \rangle_m$, one can re-sum the whole series in (6) and thus recover the relaxation result with $2\Gamma_x \langle \vec{v}^2 \rangle = \Gamma \langle (\vec{v}_m - \vec{v}_{m+1})^2 \rangle$ at least for $\vec{q} = 0$ and the corresponding quenching factor (5).

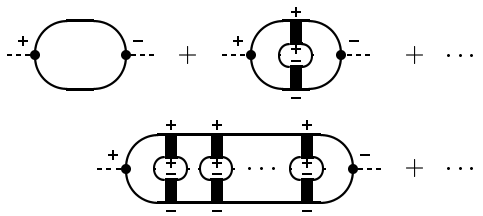
This clarifies that the diffusion result represents a re-summation of the Langevin multiple collision picture and altogether only macroscopic scales are relevant for the form of the spectrum and not the details of the microscopic collisions. Note also that the classical results, both for the diffusion equation (c.f. fig. 2) and for the Langevin process fulfill the classical version ($\hbar \to 0$) of the sum rules discussed in refs. [15,10].

## IV. RADIATION ON THE QUANTUM LEVEL

We have seen that on the classical level the problem of radiation from dense matter can be solved quite naturally and completely at least for simple examples, and figs. 2 and 3 display the main physics. They show, that the *damping* of the particles due to scattering is an important feature, which in particular has to be included right from the onset. This does not only assure results which no longer diverge, but also provides a systematic and convergent scheme. On the *quantum level* such problems requires techniques beyond the standard repertoire of perturbation theory or the quasi-particle approximation.




The production or absorption rates are given by photon self energy diagrams of the type to the right with an in– and outgoing photon line (dashed). The hatched loop area denotes all strong interactions of the source. The latter give rise to a whole series of diagrams. As mentioned, for the particles of the source, e.g. the nucleons, one has to re-sum Dyson's equation with the corresponding full complex self energy in order to determine the full Green's functions in dense matter. Once one has these Green's functions together with the interaction vertices at hand one could in principle calculate the required diagrams. However both, the computational effort to calculate a single diagram and the number of diagrams, are increasing dramatically with the loop order of the diagrams, such that in practice only lowest order loop diagrams can be considered in the full quantum case. In certain limits some diagrams drop out. We could show that in the *classical limit* of the quantum description, which in this case implies the hierarchy $\omega, |\vec{q}|, \Gamma \ll T \ll m$ together with low phase-space occupations for the source, i.e. $f(x,p) \ll 1$, only the following set of diagrams survive



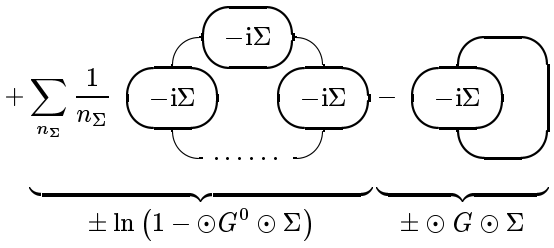
(7)

In these "Langevin" diagrams the bold lines denote the full nucleon Green's functions which also include the damping width, the black blocks represent the effective nucleon-nucleon interaction in matter, and the full dots the coupling vertex to the photon. Each of these diagrams with $n$ interaction loop insertions just corresponds to the $n^{th}$ term in the classical Langevin result (6). Thus the classical multiple collision example provides a quite intuitive picture about such diagrams. Thereby the diagram of order $n$ describes the interference of the amplitude where the photon is "emitted" at some time and that where it is "emitted" $n$ collisions later. Further details are given in [10].

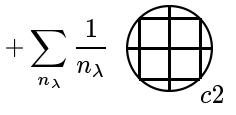
## V. Φ-DERIVABLE APPROXIMATIONS

Following Luttinger, Ward [16], and the reformulation by Cornwall, Jackiw and Tomboulis [17] using path-integral methods for equilibrium case, the generating functional $\Gamma\{\phi, G\}$ for the equations of motions, both, for the classical fields $\phi = <\widehat{\phi}>$ and Dyson's equation for the propagators $G$, can be expressed in terms of an auxiliary functional $\Phi$, where $\Phi$ is solely given in terms of full, i.e. re-summed, propagators $G$ and full classical fields $\phi$. Following [18] it can be generalized to the real time case with the diagrammatic representation

$$\mathrm{i}\Gamma\{\phi, G\} = \mathrm{i}\Gamma^0\{G^0\} + \oint \mathrm{d}x \mathcal{L}^0\{\phi, \partial_\mu \phi\}$$




$$\pm \ln\left(1 - \odot G^0 \odot \Sigma\right) \qquad \pm \odot G \odot \Sigma$$



$$+ \mathrm{i}\Phi\{\phi, G\} \qquad (8)$$

Here upper signs relate to fermion quantities, whereas lower signs, to boson quantities. Thereby $n_\Sigma$ counts the number of self-energy $\Sigma$-insertions in the ring diagrams, while for the closed diagram of $\Phi$ the value $n_\lambda$ counts the number of vertices building up the functional $\Phi$. Due to this factor such a set of diagrams is not resumable in the standard diagrammatic sense. The $\Gamma^0$ solely depends on the unperturbed propagator $G^0$ (thin line) and, hence, is treated as a constant with respect to the functional variations in $G(x, y)$ and $\phi(x)$. The diagrams contributing to $\Phi$ are given in terms of full propagators $G$ (thick lines) and classical fields $\phi$. As a consequence, these $\Phi$-diagrams have to be *two-particle irreducible* (label $c2$), i.e. they cannot be decomposed into two pieces by cutting two propagator lines. The latter property matches diagrammatic rules for the re-summed self-energy $\Sigma(x, y)$ and the source current $J(x)$ of the classical field equations, which results from functional variation of $\Phi$ with respect to any propagator $G(y, x)$, i.e.

$$-\mathrm{i}\Sigma = \mp \delta \mathrm{i}\Phi / \delta \mathrm{i}G, \quad \mathrm{i}J = \delta \mathrm{i}\Phi / \delta\phi. \qquad (9)$$

It directly follows from the stationarity condition of $\Gamma$ (8) with respect to variations of $G$ and $\phi$ on the contour

$$\delta\Gamma\{\phi, G\}/\delta G = 0, \quad \delta\Gamma\{\phi, G\}/\delta\phi = 0, \qquad (10)$$

which indeed provides the Dyson equation with self-energy consistent with respect to the $\Phi$-functional and the classical field equation. In graphical terms, the variation (9) with respect to $G$ is realized by opening a propagator line in all diagrams of $\Phi$. The resulting set of thus opened diagrams must then be that of proper skeleton diagrams of $\Sigma$ in terms of *full propagators*, i.e. void of any self-energy insertions.

In order to arrive at a closed and consistent scheme we consider the so-called $\Phi$-derivable approximation, first introduced by Baym [11] based on ref. [19] within linear response to external perturbation of equilibrated systems. They used the corresponding imaginary time formulation. A $\Phi$-derivable approximation is constructed by confining the infinite set of diagrams for $\Phi$ to either only a few of them or some sub-series of them. Note that $\Phi$ itself is constructed in terms of "full" Green's functions and classical fields, where "full" now takes the sense of solving self-consistently the Dyson and Classical field equation with the driving terms $\Sigma$ and $J$ derived from this $\Phi$ through relation (9). It means that even restricting ourselves to a single diagram in $\Phi$, in fact, we deal with a whole sub-series of perturbation theory diagrams, and "full" takes the sense of the sum of this whole sub-series. Thus, a $\Phi$-derivable approximation offers a natural way of

introducing closed, i.e. consistent approximation schemes based on summation of diagrammatic sub-series. In order to preserve the original symmetry of the exact $\Phi$ we postulate that the set of diagrams defining the $\Phi$-derivable approximation complies with all such symmetries. As a consequence, approximate forms of $\Phi^{(\mathrm{appr.})}$ define *effective* theories, where $\Phi^{(\mathrm{appr.})}$ serves as a generating functional for approximate self-energies $\Sigma^{(\mathrm{appr.})}(x,y)$ and source currents $J(x)$ through relation (9), which then enter as driving terms for the Dyson equations. The propagators solving this set of Dyson equations are still called "full" in the sense of the $\Phi^{(\mathrm{appr.})}$-derivable scheme. Below, we omit the superscript "appr.".

## VI. GENERALIZED KINETIC EQUATION

### A. Gradient Expansion Scheme

For slightly inhomogeneous and slowly evolving systems, the degrees of freedom can be subdivided into rapid and slow ones. Any kinetic approximation is essentially based on this assumption. Then for any two-point function $F(x,y)$, one separates the variable $\xi = (t_1 - t_2, \vec{r_1} - \vec{r_2})$, which relates to rapid and short-ranged microscopic processes, and the variable $X = \frac{1}{2}(t_1 + t_2, \vec{r_1} + \vec{r_2})$, which refers to slow and long-ranged collective motions. The Wigner transformation, i.e. the Fourier transformation in four-space difference $\xi = x - y$ to four-momentum $p$ of the contour decomposed components of $F^{ij}, i,j \in \{-+\}$

$$F^{ij}(X;p) = \int \mathrm{d}\xi e^{\mathrm{i}p\xi} F^{ij}\left(X + \xi/2, X - \xi/2\right) \tag{11}$$

leads to a (co-variant) four phase-space formulation of two-point functions. The Wigner transformation of Dyson's equation (10) in $\{-+\}$ notation is straight forward. For details and the extensions to include the coupling to classical field equations we refer to ref. [18].

Standard transport descriptions usually involve two approximation steps: (i) the gradient expansion for the slow degrees of freedom, as well as (ii) the quasi-particle approximation for rapid ones. We intend to avoid the latter approximation and will solely deal with the gradient approximation for slow collective motions by performing the gradient expansion of the coupled Dyson equations. This step indeed preserves all the invariances of the $\Phi$ functional in a $\Phi$-derivable approximation.

### B. Generalized Kinetic Equation in Physical Notation

It is helpful to avoid all the imaginary factors inherent in the standard Green's function formulation and change to quantities which are real and in the homogeneous limit positive and therefore have a straight physical interpretation much like for the Boltzmann equation. We define

$$\begin{aligned} F(X,p) &= A(X,p) f(X,p) = \mathrm{i}(\mp) G^{-+}(X,p), \\ \widetilde{F}(X,p) &= A(X,p)[1 \mp f(X,p)] = \mathrm{i} G^{+-}(X,p) \end{aligned} \tag{12}$$

for the generalized Wigner functions $F$ and $\widetilde{F}$ and the corresponding *four* phase space distribution functions $f(X,p)$ and Fermi/Bose factors $[1 \mp f(X,p)]$. Here

$$A(X,p) \equiv -2\mathrm{Im}\, G^R(X,p) = \widetilde{F} \pm F \tag{13}$$

is the spectral function. According to retarded relations between Green's functions $G^{ij}$, *only two of these real functions are required for a complete description of the system's evolution.*

The reduced gain and loss rates and total width of the collision integral are

$$\begin{aligned} \Gamma_{\mathrm{in}}(X,p) &= \mathrm{i}(\mp)\Sigma^{-+}(X,p), \\ \Gamma_{\mathrm{out}}(X,p) &= \mathrm{i}\Sigma^{+-}(X,p). \end{aligned} \tag{14}$$

They determine the damping width

$$\begin{aligned} \Gamma(X,p) &\equiv -2\mathrm{Im}\, \Sigma^R(X,p) \\ &= \Gamma_{\mathrm{out}}(X,p) \pm \Gamma_{\mathrm{in}}(X,p), \end{aligned} \tag{15}$$

where $G^R$ and $\Sigma^R$ are the retarded propagator and self-energy, respectively. The opposite combinations

$$I(X,p) = \Gamma_{\mathrm{in}}(X,p) \mp \Gamma_{\mathrm{out}}(X,p), \tag{16}$$

determines the fluctuations.

In terms of the new notation (12) - (15) and in the first gradient approximation the *generalized kinetic* equation for $F$ takes the form

$$\mathcal{D}F(X,p) - B = C(X,p) \tag{17}$$

with the differential drift operator (for simplicity in non-relativistic kinematics)

$$\mathcal{D} = \left(v_\mu - \frac{\partial \mathrm{Re}\,\Sigma^R}{\partial p^\mu}\right)\partial_X^\mu + \frac{\partial \mathrm{Re}\,\Sigma^R}{\partial X^\mu}\frac{\partial}{\partial p_\mu} \tag{18}$$

with $v^\mu = (1, \vec{p}/m)$. Further $C(X,p)$ and $B(X,p)$ are the collision and a fluctuation term, respectively

$$\begin{aligned} C(X,p) &= \Gamma_{\mathrm{in}}(X,p)\widetilde{F}(X,p) - \Gamma_{\mathrm{out}}(X,p)F(X,p) \\ B &= \left\{\Gamma_{\mathrm{in}}, \mathrm{Re}\, G^R\right\}. \end{aligned} \tag{19}$$

We need still one more equation, which can be provided by the retarded Dyson equation. In terms of the new notation it takes the simple form

$$\mathcal{D}G^R(X,p) + \frac{\mathrm{i}}{2}\left\{\Gamma, G^R\right\} = 0, \qquad (20)$$

$$\left(M(X,p) + \frac{\mathrm{i}}{2}\Gamma(X,p)\right) G^R(X,p) = 1, \qquad (21)$$

with the "mass" function

$$M(X,p) = p_0 - \frac{1}{2m}\vec{p}^2 - \mathrm{Re}\,\Sigma^R(X,p), \qquad (22)$$

which relates to the drift operator via $Df = \{M, f\}$ for any four phase-space function $f$. Subset (20) - (21) is solved by [20]

$$G^R = \frac{1}{M(X,p) + \mathrm{i}\Gamma(X,p)/2} \qquad (23)$$

$$\Rightarrow \begin{cases} A(X,p) &= \dfrac{\Gamma(X,p)}{M^2(X,p) + \Gamma^2(X,p)/4}, \\ \mathrm{Re}\,G^R(X,p) &= \dfrac{M(X,p)}{M^2(X,p) + \Gamma^2(X,p)/4}. \end{cases}$$

The spectral function satisfies the sum–rule

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}p_0}{2\pi} A(X,p) = 1, \qquad (24)$$

which follows from the canonical equal-time (anti) commutation relations for (fermionic) bosonic field operators.

We now provide a physical interpretation of various terms in the generalized kinetic equation (17). The drift term $\mathcal{D}F$ on the l.h.s. of eq. (17) is the usual kinetic drift term including the corrections from the self-consistent field Re $\Sigma^R$ into the convective transfer of real and also virtual particles. In the collision-less case $C = B = 0$, i.e. $\mathcal{D}F = 0$ (Vlasov equation), the quasi-linear first order differential operator $\mathcal{D}$ defines characteristic curves. They are the standard classical paths in the Vlasov case. Thereby the four-phase-space probability $F(X,p)$ is conserved along these paths. The formulation in terms of a Poisson bracket in four dimensions implies a generalized Liouville theorem. In the collisional case both, the collision term $C$ and the fluctuation term $B$ change the phase-space probabilities of the "generalized" particles during their propagation along the "generalized" classical paths given by $\mathcal{D}$. We use the term "generalized" in order to emphasize that particles are no longer bound to their mass-shell, $M = 0$, during propagation due to the collision term, i.e. due decay, creation or scattering processes.

The r.h.s. of eq. (17) specifies the collision term $C$ in terms of gain and loss terms, which also can account for multi-particle processes. Since $F$ includes a factor $A$, $C$ further deviates from the standard Boltzmann-type form in as much that it is multiplied by the spectral function $A$, which accounts for the finite width of the particles.

The additional Poisson-bracket term

$$B = \left\{\Gamma_{\mathrm{in}}, \mathrm{Re}\,G^R\right\} = \frac{M^2 - \Gamma^2/4}{(M^2 + \Gamma^2/4)^2}\,\mathcal{D}\,\Gamma_{\mathrm{in}} + \frac{M\Gamma}{(M^2 + \Gamma^2/4)^2}\left\{\Gamma_{\mathrm{in}}, \Gamma\right\} \qquad (25)$$

is special. It contains genuine contributions from the finite mass width of the particles and describes the response of the surrounding matter due to fluctuations. This can be seen from the conservation laws discussed below. In particular the first term in (25) gives rise to a back-flow component of the surrounding matter. It restores the Noether currents as the conserved ones from the intuitively expected sum of convective currents arising from the convective $\mathcal{D}\widetilde{F}$ terms in (17). The second term of (25) gives no contribution in the quasi-particle limit of small damping width limit and represents a specific off mass-shell response, c.f. [21,22].

### C. Conservations of the Current and Energy–Momentum

Special combinations of the transport equations (17) and the corresponding one for $\widetilde{F}$ weighted with $e$ and $p^\nu$, and integrated over momentum give rise to the charge and energy–momentum conservation laws, respectively, with the Noether charge current and Noether energy–momentum tensor defined by the following expressions

$$j^\mu(X) = \frac{e}{2}\mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4} v^\mu \left(F(X,p) \mp \widetilde{F}(X,p)\right),$$

$$\Theta^{\mu\nu}(X) = \frac{1}{2}\mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4} v^\mu p^\nu \left(F(X,p) \mp \widetilde{F}(X,p)\right) + g^{\mu\nu}\left(\mathcal{E}^{\mathrm{int}}(X) - \mathcal{E}^{\mathrm{pot}}(X)\right). \qquad (26)$$

Here

$$\mathcal{E}^{\mathrm{int}}(X) = \left\langle -\widehat{\mathcal{L}}^{\,\mathrm{int}}(X)\right\rangle = \left.\frac{\delta\Phi}{\delta\lambda(x)}\right|_{\lambda=1} \qquad (27)$$

is the interaction energy density, which in terms of $\Phi$ is given by a functional variation with respect to a space-time dependent coupling strength of $\widehat{\mathcal{L}}^{\,\mathrm{int}} \to \lambda(x)\,\widehat{\mathcal{L}}^{\,\mathrm{int}}$, c.f. ref. [18]. The potential energy density $\mathcal{E}^{\mathrm{pot}}$ takes the form

$$\mathcal{E}^{\mathrm{pot}} = \frac{1}{2}\mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\left[\mathrm{Re}\,\Sigma^R\left(F \mp \widetilde{F}\right) + \mathrm{Re}\,G^R I\right] \qquad (28)$$

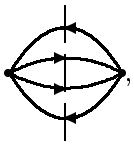
where $I = \Gamma_{\rm in} \mp \Gamma_{\rm out}$. Whereas the first term complies with quasi-particle expectations, namely mean potential times density, the second term displays the role of fluctuations $I = \Gamma_{\rm in} \mp \Gamma_{\rm out}$ in the potential energy density. Since in many cases interaction and potential energy are proportional to each other, the same statement applies to the interaction energy, too. This fluctuation term precisely arises form the $B$-term in the kinetic eq. (17), discussed around eq. (25). It restores that the Noether expressions (26) are indeed the conserved quantities. In this compensation we see the essential role of the fluctuation term in the generalized kinetic equation. Dropping or approximating this term would spoil the conservation laws. Indeed, both expressions in (26) comply exactly with the generalized kinetic equation (17), i.e. they are exact integrals of the generalized kinetic equations of motion. As usual the existence of such conservation laws require certain invariances which lead to certain consistency relations. In ref. [18,22] it has been shown that these are met if all the self-energies are $\Phi$-derivable.

In the field theoretical case there are contributions in (26), describing modifications of the vacuum-polarization in matter. These terms are generally ultra-violet divergent, and hence, have to be properly renormalized on the vacuum level. Alongside the spectral sum-rule (24) gets modified by wave-function renormalization.

### D. Multiprocess Decomposition of Φ-Derivable Collision Term

To be specific we consider a system of fermions interacting via a two-body potential $V = V_0\delta(x-y)$, and, for the sake of simplicity, disregard its spin structure, by reducing spin and antisymmetrization effects to a degeneracy factor $d$. To derive the decomposition of a $\Phi$-derivable collision term, we employ the same rules as described in ref. [22].

In the first example we consider the generating functional $\Phi$ to be approximated by the following two diagrams

$$\mathrm{i}\Phi = \frac{1}{2}$$  $$+ \frac{1}{4}$$  , (29)

the dashed line illustrating the decomposition. In the $\{-+\}$ matrix notation of the Green's functions one can easily see that one-point diagrams do not contribute to the collision term, while decomposing the second one along the dashed line leads to a purely local result

$$C_{(2)} = d^2 \int \frac{\mathrm{d}^4p_1}{(2\pi)^4}\frac{\mathrm{d}^4p_2}{(2\pi)^4}\frac{\mathrm{d}^4p_3}{(2\pi)^4}$$  (30)

$$\times\delta^4\left(p+p_1-p_2-p_3\right)\left(F_2F_3\widetilde{F}\widetilde{F}_1 - \widetilde{F}_2\widetilde{F}_3FF_1\right),$$

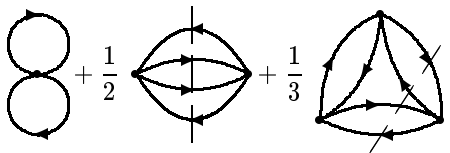
where the brief notation $F_i = F(X,p_i)$ etc. is used. This collision integral has precisely the form of the binary collision term of Boltzmann–Uehling–Uhlenbeck (BUU), except for the fact that the distribution functions are not constrained by the mass shell. The binary transition rate

$$R_2^{(2)} = V_0^2 =$$  (31)

is non-negative in this case.

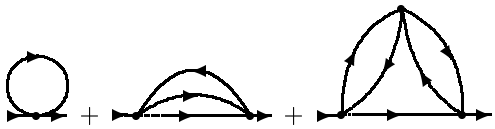
The picture becomes more complicated, if $\Phi$ involves diagrams of higher orders. For instance, let us add the following three point diagram to $\Phi$, which is next in a series of ring diagrams, i.e.

$$\mathrm{i}\Phi = \mathrm{i}\left(\Phi_{(1)} + \Phi_{(2)} + \Phi_{(3)}\right)$$

$$=$$  (32)

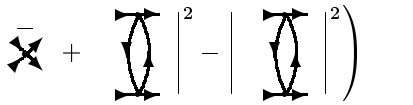
where one possible decomposition is illustrated by the dashed line. The corresponding self-energy becomes

$$-\mathrm{i}\Sigma = -\mathrm{i}\left(\Sigma_{(1)} + \Sigma_{(2)} + \Sigma_{(3)}\right) =$$

 (33)

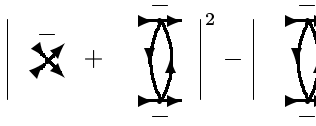
Now the collision term contains a *non-local* part due to the last diagram. The local part can easily be derived in the form

$$C_{(2)}^{\rm loc} + C_{(3)}^{\rm loc} = d^2 \int \frac{\mathrm{d}^4p_1}{(2\pi)^4}\frac{\mathrm{d}^4p_2}{(2\pi)^4}\frac{\mathrm{d}^4p_3}{(2\pi)^4}$$

$$\times$$  (34)

$$\times\delta^4\left(p+p_1-p_2-p_3\right)\left(F_2F_3\widetilde{F}\widetilde{F}_1 - \widetilde{F}_2\widetilde{F}_3FF_1\right),$$

where all the vertices in the off-shell scattering amplitudes are of the same sign, say " $-$ " for definiteness, i.e., there are no " $+-$" and " $-+$" Green's functions left. The quantity $C_{(2)}^{\rm loc} + C_{(3)}^{\rm loc}$ is again of the Boltzmann form

$$R_2^{(2)} + R_2^{(3)} =$$  ,

the sub-label 2 denoting that 2 pairs of particle-hole lines are affected by the decomposition cut. It can be shown that under normal circumstances also this rate coefficient is positive.

### E. Kinetic Entropy

Ignoring higher order gradients the generalized kinetic equation (17) provides us with the following relation

$$\partial_\mu s^\mu(x) = \sum_a \int \frac{\mathrm{d}^4p}{(2\pi)^4} \ln \frac{\widetilde{F}_a}{F_a} C_a(x,p), \qquad (35)$$

where the quantity

$$s^\mu = \sum_a s_a^\mu = \sum_a \int \frac{\mathrm{d}^4p}{(2\pi)^4}$$
$$\left[ \left( v^\mu - \frac{\partial \mathrm{Re}\,\Sigma_a^R}{\partial p_\mu} \right) \left( \mp \widetilde{F}_a \ln \frac{\widetilde{F}_a}{A_a} - F_a \ln \frac{F_a}{A_a} \right) \right.$$
$$\left. - \mathrm{Re}\, G_a^R \left( \mp \frac{\partial \Gamma^a_{\mathrm{out}}}{\partial p_\mu} \ln \frac{\widetilde{F}_a}{A_a} - \frac{\partial \Gamma^a_{\mathrm{in}}}{\partial p_\mu} \ln \frac{F_a}{A_a} \right) \right], \qquad (36)$$

obtained from the l.h.s. of the kinetic equation is interpreted as the Markovian part of the entropy flow. Here we have introduced a summation over $a$ denoting the different particle species and intrinsic quantum numbers for a multi-component system. The interesting aspect is that for special local collision terms $C_a$ as the ones discussed above the r.h.s. of (35) can be shown to be non-negative and therefore gives rise to an H-theorem. Again the functional properties of $\Phi$ have been used. The positivity of r.h.s. of (35 is exactly given for $\Phi$-functionals with two internal points for which in the equilibrium limit the zero component of the non-equilibrium entropy flow (36) agrees with the corresponding equilibrium entropy. Memory corrections as contained in $\Phi$-functionals with more than two points give rise to extra gradient terms which contribute to the entropy flow. For details we refer to our forthcoming paper [22].

## VII. CONCLUSION

In the first part of this talk the problem of soft modes in hard dense matter is discussed under circumstances which can be treated completely in analytical terms. The hard modes are described by a Fokker-Planck equation. They couple to a classical Maxwell field for the soft modes, c.f. Fig. 4. For this Abelian case the result is conserving and completely gauge invariant even though the damping of the source particles is fully included. The friction coefficient (closely related to the damping) determines the scale that separates soft from hard modes. This classical scheme is seen to re-sum a certain set of planar diagrams in the quantum case, which do survive in the classical limit. Such concepts are quite a general. In recent times they have been applied to the hard thermal loop (HTL) re-summation [23] in terms of classical transport [24,25]. In the non-abelian QCD case, however, in order to preserve gauge invariance, the transport part is limited to the collision-less Vlasov equation, i.e. neglecting the damping of the source particles. A historical hard-loop re-summation scheme is the Fermi-liquid problem, where soft RPA-modes are treated by the coupling to the fermions in the Fermi-sea, the latter representing the hard modes, c.f. Fig. 4.

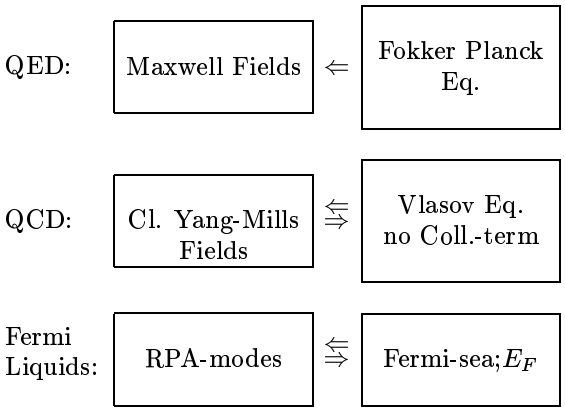



FIG. 4. Hard Loop Re-Summation

In practical terms we have seen that the spectrum of soft particles resulting from collisions in dense matter can no longer appropriately be described by the *quasi-particle* approximation, since it leads to divergent results in the soft limit. Rather the finite time between successive collisions and the ensuing relaxation rates $\Gamma_x$ in dense matter lead to a considerable quenching of the production rate, e.g. at small photon energies. This can be compiled in the simple quenching factor (5). Fig. 3 summarizes the main behavior, which also is relevant in a quantum treatment of the source.

In the second part a scheme is presented that leads to self-consistent conserving transport equations. There we essentially followed ideas suggested by Kadanoff and Baym in particular. The central quantity is a functional $\Phi$ which generates the driving terms for the classical field and transport equations. It can be truncated at any

desired loop order and still provides equations which fulfill conservation laws. We explicitly constructed the energy momentum tensor for this $\Phi$-derivable method. The gradient approximation provided equations of classical type for the phase-space distribution functions in four dimensions. At no place the quasi-particle approximation was necessary. Alongside from the $\Phi$-derivable properties a kinetic entropy could be derived, which in some cases leads to an H-theorem.

In summary the method has the following advantages:

- ♣ provides a self-consistent & conserving transport scheme;
- ♣ allows to include classical fields (soft modes);
- ♣ includes all QM effects that are accounted for in the corresponding equilibrium treatment;
- ♣ has no limitation to small widths;
- ♣ includes delay-time, drag & back flow, and memory effects.

There are two limitations: first, the derivation is limited to slow space-time variations of the macroscopic quantities; secondly, local symmetries, like gauge invariance, may be violated by such re-summation schemes. The latter problem is inherent to all approaches, based on truncated self-consistent dynamical equations.

Our considerations are of particular importance for the theoretical description of nucleus-nucleus collisions at intermediate to relativistic energies. The kinematical feature are such that damping effects play an essential role, i.e. the energy uncertainty of the particles is comparable with their mean kinetic energy! In particular the bulk production and absorption rates of all particles with masses less than $T$, if calculated in standard quasi-particle approximation, are seriously subjected to the here discussed effects.

In summary, the combined effort from many sides to include the finite width of the particles in dense matter, may give hope for a unified transport theory which appropriately describes both, the propagation of resonances and of off-shell particles in the dense matter environment.

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