Resonance Transport and Kinetic Entropy

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Abstract

We continue the description of the dynamics of unstable particles within the real-time formulation of nonequilibrium field theory initiated in a previous paper [1]. There we suggest to use Baym's Φ-functional method in order to achieve approximation schemes with 'built in' consistency with respect to conservation laws and thermodynamics even in the case of particles with finite damping width. Starting from Kadanoff–Baym equations we discuss a consistent first order gradient approach to transport which preserves the Φ-derivable properties. The validity conditions for the resulting quantum four-phase-space kinetic theory are discussed under the perspective to treat particles with broad damping widths. This non-equilibrium dynamics naturally includes all those quantum features already inherent in the corresponding equilibrium limit (e.g. Matsubara formalism) at the same level of Φ-derivable approximation. Various collision-term diagrams are discussed including those of higher order which lead to memory effects. As an important novel part we derive a generalized nonequilibrium expression for the *kinetic* entropy flow, which includes contributions from fluctuations and mass-width effects. In special cases an H-theorem is derived implying that the entropy can only increase with time. Memory effects in the kinetic terms provide contributions to the kinetic entropy flow that in the equilibrium limit recover the famous bosonic type $T^3 \ln T$ correction to the specific heat in the case of Fermi liquids like Helium-3.

1 Introduction

The appropriate frame for the description of nonequilibrium processes is the real-time formalism of quantum field theory, developed by Schwinger, Kadanoff, Baym and Keldysh [2–4]. This formalism finds now applications in various fields, such as quantum chromodynamics [5,6], nuclear physics [7–13], astrophysics [11,14,15], cosmology [16], spin systems [17,18], lasers [19], physics of plasma [20–22], physics of liquid ^{3}He [23], critical phenomena, quenched random systems and disordered systems [24], normal metals and

super-conductors [14,25,26], semiconductors [27,28], tunneling and secondary emission [29], etc.

The Green's function technique provides descriptions in terms of one- and two-point functions. Compared to the various equal-time operator formulations of transport theories, c.f. [30], and within the path-integral formulation [31], the Green's function approach with its non-localities in time offers probably the only convenient way for a dynamical description of spectral information of unstable particles. The corresponding mass-width effects become increasingly important especially in the realm of high-energy nuclear collisions, where one deals with resonances, like the delta resonance and rho meson, with vacuum decay widths which are even larger than the system's temperature. In spite of 40 years of work of many authors a proper dynamical scheme for such situations is still lacking. Rather ad-hoc recipes are in use that sometimes violate basic requirements as given by fundamental symmetries and conservation laws, detailed balance or thermodynamic consistency. The question of conservation laws become especially vital for the dynamics of broad resonances. In the context of Green's functions the problem of conserving approximations has first been addressed by Baym and Kadanoff [32,33] on very general grounds within the imaginary time formalism. Baym, in particular, showed [33] that so-called Φ-derivable approximations are conserving. It turned out that the Φ functional required is precisely the auxiliary functional introduced by Luttinger and Ward [34] (see also ref. [35]) in connection with the thermodynamic potential at the two-particle irreducible level.

The concept of generating functionals on the Schwinger–Keldysh contour has been explored by many authors, e.g. see refs. [16,24,28,36]. In our previous paper [1] we have made a step further in these functional methods, i.e. we reinvestigated the Φ functional on the real-time contour. The Φ functional is determined by closed vacuum skeleton diagrams in terms of full classical fields and full Green functions. All important dynamical quantities up to the two-point function level, such as the sources of classical fields, self-energies, interaction energy, etc. are derived from Φ by functional variations with respect to its arguments. The advantage of the Φ functional is that one may formulate various approximations in terms of approximate Φ (so called Φ-derivable approximations), which preserve the conservation laws related to global symmetries of the underlying theory and thermodynamic consistency. Thereby, one may restrict oneself to either a few diagrams only or to some sub-set of diagrams for Φ. The basic terms of this scheme are summarized in sect. 2.

In sect. 3 the steps towards quantum kinetic equations are summarized. They involve the Wigner transformation of the contour Dyson equation together with the first-order gradient approximation. The former formulates all quantities in terms of phase-space distribution functions in four dimensions, i.e. as a function of energy and momentum for any space-time coordinate. The gradient approximation leads to Poisson-bracket expressions which permit a classical interpretation. Thereby the standard quasiparticle approximation is not required at any step.

It should be noted though that there were many attempts in the literature to improve the

standard quasiparticle approximation and to discuss small damping width and retardation effects on different model examples, see [21,22,27,36–42] and refs. therein. However, all these derivations lose some information on finite width effects due to the inherent reduction to a 3-momentum representation of the distribution functions by some specific ansatz. With the aim to develop a self-consistent approach we defer to use such kind of reductions. Thus we'll treat the spectral information contained in the spectral functions dynamically, i.e. on the four-phase-space level. We show that this can consistently be achieved in the limit of slow space-time variations of the distribution functions thereby preserving all the invariances of the Φ-derivable approximation, employing the standard Kadanoff–Baym equations right after gradient expansion. We reinvestigate the validity condition of the resulting set of equations and in particular show that the generalization of the so called mass-shell equation loses its quasiparticle sense and becomes equivalent to the quantum four-phase-space kinetic equation within the validity of the gradient approximation. The physical meaning of the different terms in the quantum four-phase-space kinetic equation, especially the role of damping and fluctuation terms leading to a back-flow response, are discussed. The properties of Φ determine the conservation laws in terms of conserved Noether currents. For the conserved energy-momentum tensor we derive a closed expression which also includes terms arising from the the finite mass width of the particles and from fluctuations. Arguments are given that the generalized distribution functions remain positive during the time evolution thus permitting a probabilistic interpretation in four-momentum space at each space-time point.

In sect. 4 further properties are exploited with the help of the decomposition rules for the diagrams of Φ formulated in terms of full Green functions [13]. These rules lead to a multi-process decomposition of a Φ-derivable collision term, which contains space-time local and nonlocal parts, the latter representing memory effects. We demonstrate how one can systematically treat such effects in self-consistent kinetics.

The second main issue of this paper is the role of entropy. Although the entropy is a central quantity in thermodynamics and statistical mechanics, many problems connected with it, in particular, its description in terms of Green functions in the nonequilibrium case, is still open. One can find related discussions in many textbooks and reviews, e.g., in refs. [43,45–48]. The thermodynamic entropy has extensively been discussed in the literature at the end of the sixties and beginning of the seventies also within the Φ-derivable scheme for the thermodynamic potential, cf. refs. [49–51] and references therein. The fact that the quantum four-phase-space kinetic equation possesses a proper thermodynamical limit, does not yet imply that this limit will be approached during the evolution. The latter is ensured however, if one can prove an H-theorem for the equations of motion. In spite of many attempts so far, H-theorem has not yet been proven even for the classical kinetic equation including triple collision term, cf. ref. [48]. Therefore, up to now there exists no appropriate kinetic entropy expression (derived from a quantum four-phase-space kinetic description). Using the Φ-derivable properties we are able to get an expression, which takes the sense of a nonequilibrium kinetic entropy expressed in terms of Green functions and self-energies (sect. 5). For specific forms of the Φ-derivable self-energies, relevant for the majority of cases discussed in transport problems, the quantum four-phase-space

kinetic scheme provides us with an H-theorem. Our expressions for the kinetic entropy flow apply beyond the quasiparticle limit accounting for fluctuation and memory effects (the latter have to be established for each particular case). Thus the well known equilibrium expression [50] obtained in Matsubara formalism within the Φ-functional scheme is now generalized to the genuine nonequilibrium case (sect. 6). Some formal details are deferred to Appendices.

To be specific, we concentrate on systems of non-relativistic particles. Bosonic mean fields are not treated in this paper; they can however be included along the lines given in ref. [1].

2 Generating Functional Φ on Real-Time Contour

We assume the nonequilibrium system to be prepared at some initial time t_0 in terms of a given density operator $\widehat{\rho}_0 = \sum_\alpha P_\alpha |\alpha\rangle\langle\alpha|$, where the $|\alpha\rangle$ form a complete set of eigenstates of $\widehat{\rho}_0$, and formulate all expectation values on a special time contour, the *closed real-time contour* (see figure 1) with the time arguments running from t_0 to ∞ along the *time-ordered* branch and back to t_0 along the *anti-time-ordered* branch.

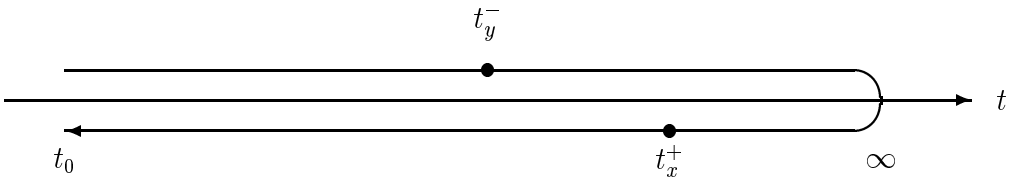


Figure 1: Closed real-time contour with two external points x, y on the contour.

Multi-point contour functions like the free (thin line) and full (thick line) propagators

$$\overleftarrow{x \quad y} = \mathrm{i}G^0(x,y) = \left\langle \mathcal{T}_\mathcal{C}\, \widehat{\varphi}_\mathrm{I}(x)\, \widehat{\varphi}_\mathrm{I}^\dagger(y) \right\rangle = \mathrm{Tr}\left[\widehat{\rho}_0 \mathcal{T}_\mathcal{C}\, \widehat{\varphi}_\mathrm{I}(x)\, \widehat{\varphi}_\mathrm{I}^\dagger(y)\right] \tag{2.1}$$

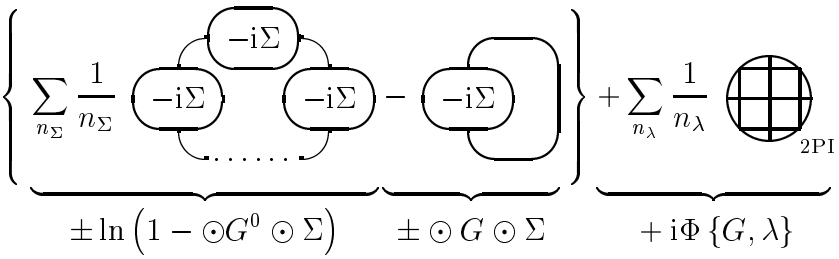
$$\overleftarrow{x \quad y} = \mathrm{i}G(x,y) = \left\langle \mathcal{T}_\mathcal{C}\, \widehat{\varphi}(x)\, \widehat{\varphi}^\dagger(y) \right\rangle = \mathrm{Tr}\left[\widehat{\rho}_0 \mathcal{T}_\mathcal{C}\, \widehat{\varphi}_\mathrm{I}(x)\, \widehat{\varphi}_\mathrm{I}^\dagger(y) \exp\left(-\mathrm{i}\int_\mathcal{C} \widehat{H}_\mathrm{I}^{\,\mathrm{int}} \mathrm{d}t\right)\right], \tag{2.2}$$

are defined as the expectation values of contour-ordered products of field operators with ρ_0, where $\widehat{\varphi}(y)$ and $\widehat{\varphi}_\mathrm{I}(y)$ denote the full and interaction picture field operators with the interaction Hamiltonian $\widehat{H}_\mathrm{I}^{\,\mathrm{int}}$, while $\mathcal{T}_\mathcal{C}$ specifies the special time-ordering, which orders the operators according to a time parameter running along the time contour $\mathcal{C}$.

With the aim to come to a self-consistent and conserving treatment on the two-point function level, we use the Φ-functional method [34,33] generalized to the real-time contour in our previous paper [1]. It is based on a decomposition of the generating functional Γ with bilocal sources into a two-particle reducible part and an auxiliary functional Φ which compiles all two-particle-irreducible (2PI) vacuum diagrams

$$\mathrm{i}\Gamma\left\{G,\lambda\right\} = \mathrm{i}\Gamma^0\left\{G^0\right\}$$

$$+\Bigg\{\underbrace{\sum_{n_\Sigma}\frac{1}{n_\Sigma}}_{\pm\ln\left(1-\odot G^0\odot\Sigma\right)}\Bigg\}\underbrace{-}_{\pm\odot G\odot\Sigma}\Bigg\}+\underbrace{\sum_{n_\lambda}\frac{1}{n_\lambda}}_{+\mathrm{i}\Phi\left\{G,\lambda\right\}}\quad . \tag{2.3}$$



Here upper signs relate to fermion quantities, whereas lower signs, to boson ones, while n_Σ and n_λ count the number of self-energy insertions in the ring diagrams and the number of vertices in the diagrams of Φ, respectively. The stationarity condition

$$\delta\Gamma\left\{G,\lambda\right\}/\delta G = 0 \tag{2.4}$$

provides the contour Dyson equation of motion for the two-point Green functions

$$S_x G(x,y) = \delta_{\mathcal{C}}(x,y) + \int\limits_{\mathcal{C}} \mathrm{d}z\Sigma(x,z)G(z,y), \tag{2.5}$$

$$S_x G^0(x,y) = \delta_{\mathcal{C}}(x,y), \qquad \text{where} \quad S_x = \mathrm{i}\partial_t + \frac{1}{2m}\partial^2_{\boldsymbol{x}}, \tag{2.6}$$

the latter in non-relativistic kinematics. Here $\delta_{\mathcal{C}}(x,y)$ is δ-function on the contour and $\int_{\mathcal{C}}$ denotes the contour integration.

The functional $\Phi\{G\}$ acts as the generating functional for the self-energy Σ via the functional variation

$$-\mathrm{i}\Sigma(x,y) = \mp\delta\mathrm{i}\Phi/\delta\mathrm{i}G(y,x). \tag{2.7}$$

The closed diagrams of Φ are expressed in terms of *full* propagators and therefore have to be of *two-particle irreducible* (2PI) topology. In graphical terms, the variation (2.7) is realized by opening a propagator line in any diagram of Φ providing proper skeleton diagrams for Σ in terms of *full propagators*, i.e. void of any self-energy insertions.

The advantage of this formulation is that Φ can be truncated at any level, thus defining approximation schemes with built in internal consistency with respect to conservation laws and thermodynamic consistency. For details we refer to the original literature [33,34] and our previous paper [1]. Thus restricting the infinite set of diagrams for Φ to either only a few of them or some sub-series of them defines a Φ-derivable approximation. Note that Φ itself is constructed in terms of "full" Green functions, where "full" now takes the sense of solving self-consistently the Dyson equation with the driving term derived from this approximate Φ through relation (2.7). It means that even restricting ourselves to a

single diagram in Φ, in fact, we deal with a whole sub-series of diagrams in terms of free propagators, and "full" takes the sense of the sum of this whole sub-series. The Wick decomposition, the truncation in Φ together with the gradient approximation, may lead to a scheme which generally is time-irreversibe [3,7,52,53,43,44], as discussed in sect. 5.

3 Quantum Four-Phase-Space Kinetics

3.1 Gradient Expansion Scheme

For slightly inhomogeneous and slowly evolving systems, the degrees of freedom can be subdivided into rapid and slow ones. Any kinetic approximation is essentially based on this assumption. Then for any two-point function $F(x,y)$, one separates the variable $\xi = (t_1 - t_2, \boldsymbol{r_1} - \boldsymbol{r_2})$, which relates to rapid and short-ranged microscopic processes, and the variable $X = \frac{1}{2}(t_1 + t_2, \boldsymbol{r_1} + \boldsymbol{r_2})$, which refers to slow and long-ranged collective motions. The Wigner transformation, i.e. the Fourier transformation in four-space difference $\xi = x - y$ to four-momentum p leads to the corresponding Wigner densities in four-phase-space. Since the Wigner transformation is defined for physical space-time coordinates rather than for contour coordinates one has to decompose the contour integrations into its two branches, the time-ordered $\{-\}$ branch and the anti-time ordered $\{+\}$ branch, cf. Appendix A. Two-point functions then become matrices of the contour decomposed $\{-+\}$ components with physical space-time arguments. Thus

$$F^{ij}(X;p) = \int \mathrm{d}\xi e^{\mathrm{i}p\xi} F^{ij}\left(X + \xi/2, X - \xi/2\right), \qquad i,j \in \{-+\} \tag{3.1}$$

leads to a four-phase-space representation of two-point functions, which obey retarded or advanced relations, presented in Appendix A. The gradient expansion converts the Wigner transformation of any convolution of two-point functions into a product of the corresponding Wigner functions plus higher order gradient terms

$$\int \mathrm{d}\xi e^{\mathrm{i}p\xi} \left(\int \mathrm{d}z f(x,z)\varphi(z,y) \right) = \left(\exp\left[\frac{\mathrm{i}\hbar}{2} \left(\partial_p \partial_{X'} - \partial_X \partial_{p'} \right) \right] f(X,p)\varphi(X',p') \right)_{p'=p, X'=X} \tag{3.2}$$

$$\simeq f(X,p)\varphi(X,p) + \frac{\mathrm{i}\hbar}{2} \left\{ f(X,p), \varphi(X,p) \right\}, \tag{3.3}$$

where the first order terms are given by Poisson brackets

$$\left\{ f(X,p), \varphi(X,p) \right\} = \frac{\partial f}{\partial p^\mu} \frac{\partial \varphi}{\partial X_\mu} - \frac{\partial f}{\partial X^\mu} \frac{\partial \varphi}{\partial p_\mu} \tag{3.4}$$

here in covariant notation. We would like to stress that the smallness of the $\hbar\partial_X \cdot \partial_p$ comes solely from the smallness of space–time gradients ∂_X, while momentum derivatives ∂_p are not assumed to be small! This point is sometimes incorrectly treated in the literature.

The Wigner transformation of the Dyson equation (2.5) leads to the following set of equations [3]

$$\mathrm{i}v_\mu \partial_X^\mu G^{ij}(X,p) = \int \mathrm{d}\xi e^{\mathrm{i}p\xi} \int\limits_{\mathcal{C}} \mathrm{d}z \left(\Sigma(x^i,z)G(z,y^j) - G(x^i,z)\Sigma(z,y^j)\right), \tag{3.5}$$

$$\widehat{Q}_X G^{ij}(X,p) = \sigma^{ij} + \frac{1}{2}\int \mathrm{d}\xi e^{\mathrm{i}p\xi} \int\limits_{\mathcal{C}} \mathrm{d}z \left(\Sigma(x^i,z)G(z,y^j) + G(x^i,z)\Sigma(z,y^j)\right), \tag{3.6}$$

where σ^{ij} accounts for the integration sense on the two contour branches, cf. Eqs. (A.2), (A.3). For non-relativistic kinematics $v^\mu = (1, \boldsymbol{p}/m)$, and $\widehat{Q}_X = p_0 - \boldsymbol{p}^2/2m - \partial^2_{\boldsymbol{X}}/8m$. In this matrix notation, two of equations (3.5) and (3.6), involving G^{-+} and G^{+-} on the left-hand side, are known as the Kadanoff-Baym equations, here in Wigner representation [3]. Particular combinations of these equations lead to the retarded and advanced equations which completely decouple and involve only integrations over physical times rather than contour times.

It is helpful to avoid all the imaginary factors inherent in the standard Green function formulation and introduce quantities which are real and, in the quasi-homogeneous limit, positive, cf. subsect. 3.4, and therefore have a straightforward physical interpretation, much like for the Boltzmann equation. We define

$$\begin{aligned} F(X,p) &= A(X,p)f(X,p) = (\mp)\mathrm{i}G^{-+}(X,p), \\ \widetilde{F}(X,p) &= A(X,p)[1 \mp f(X,p)] = \mathrm{i}G^{+-}(X,p), \end{aligned} \tag{3.7}$$

for the generalized Wigner functions F and $\widetilde{F}$ and the corresponding *four*-phase-space distribution functions $f(X,p)$ and Fermi/Bose factors $[1 \mp f(X,p)]$. Here

$$A(X,p) \equiv -2\mathrm{Im}G^R(X,p) = \widetilde{F} \pm F = \mathrm{i}\left(G^{+-} - G^{-+}\right) \tag{3.8}$$

is the spectral function, where G^R is the retarded propagator, cf. Eq. (A.6). According to relations (A.6) and (A.7) between Green functions G^{ij}, *only two independent real functions of all the* G^{ij} *are required for a complete description.*

Likewise the reduced gain and loss rates of the collision integral are defined as

$$\Gamma_{\mathrm{in}}(X,p) = \Gamma(X,p)\gamma(X,p) = (\mp)\mathrm{i}\Sigma^{-+}(X,p), \tag{3.9}$$

$$\Gamma_{\mathrm{out}}(X,p) = \Gamma(X,p)[1 \mp \gamma(X,p)] = \mathrm{i}\Sigma^{+-}(X,p) \tag{3.10}$$

with the damping width

$$\Gamma(X,p) \equiv -2\mathrm{Im}\Sigma^R(X,p) = \Gamma_{\mathrm{out}}(X,p) \pm \Gamma_{\mathrm{in}}(X,p), \tag{3.11}$$

where Σ^R is the retarded self-energy, cf. Eq. (A.6). The opposite combination

$$I(X,p) = [2\gamma \mp 1]\Gamma = \Gamma_{\mathrm{in}}(X,p) \mp \Gamma_{\mathrm{out}}(X,p), \tag{3.12}$$

is related to fluctuations. The dimensionless quantity γ is introduced for further convenience.

In terms of the new notation (3.7)–(3.11) and within the first-order gradient approximation, the Kadanoff–Baym (KB) equations (3.5) for F and $\widetilde{F}$ take the form

$$\mathcal{D}F(X,p) - \left\{\Gamma_{\mathrm{in}}, \mathrm{Re}G^R\right\} = C(X,p), \tag{3.13}$$

$$\mathcal{D}\widetilde{F}(X,p) - \left\{\Gamma_{\mathrm{out}}, \mathrm{Re}G^R\right\} = \mp C(X,p), \tag{3.14}$$

which we denote as the *quantum four-phase-space transport equations in the KB-choice*. Here the differential drift operator is defined as

$$\mathcal{D} = \left(v_\mu - \frac{\partial \mathrm{Re}\Sigma^R}{\partial p^\mu}\right)\partial_X^\mu + \frac{\partial \mathrm{Re}\Sigma^R}{\partial X^\mu}\frac{\partial}{\partial p_\mu}, \quad \text{with} \quad v^\mu = (1, \boldsymbol{p}/m) \tag{3.15}$$

in non-relativistic kinematics,

$$C(X,p) = \Gamma_{\mathrm{in}}(X,p)\widetilde{F}(X,p) - \Gamma_{\mathrm{out}}(X,p)F(X,p) = A\Gamma[\gamma - f] \tag{3.16}$$

is the collision term with the dimensionless functions f and γ defined in (3.7) and (3.9), while $\left\{\Gamma_{\mathrm{in}}, \mathrm{Re}G^R\right\}$ and $\left\{\Gamma_{\mathrm{out}}, \mathrm{Re}G^R\right\}$ are fluctuation terms. The smallness of $|f - \gamma| \ll 1$ indeed provides the validity condition for the gradient expansion, since the gradients on the l.h.s. of (3.13) are of the order of the collision term. It implies that the macroscopic time scale τ_{macro}, characterizing kinetic processes, is much larger than the microscopic time scale τ_{micro}, relating to rapid microscopic processes. Eqs. (3.13) and (3.14) were originally derived by Kadanoff and Baym [3]. Here we have just presented them in kinetic notation useful for our further consideration.

Within the same approximation level Eq. (3.6) provides us with two alternative equations for F and $\widetilde{F}$

$$MF - \mathrm{Re}G^R\Gamma_{\mathrm{in}} = \frac{1}{4}\left(\{\Gamma, F\} - \{\Gamma_{\mathrm{in}}, A\}\right), \tag{3.17}$$

$$M\widetilde{F} - \mathrm{Re}G^R\Gamma_{\mathrm{out}} = \frac{1}{4}\left(\left\{\Gamma, \widetilde{F}\right\} - \{\Gamma_{\mathrm{out}}, A\}\right) \tag{3.18}$$

with the "mass" function

$$M(X,p) = p_0 - \frac{1}{2m}\boldsymbol{p}^2 - \mathrm{Re}\Sigma^R(X,p) \qquad \text{(for non-relativistic kinematics)}, \tag{3.19}$$

which relates to the drift operator via $\mathcal{D}f = \{M, f\}$ for any four-phase-space function f. Eqs. (3.17), (3.18) can be called the four-phase-space mass-shell equations, since in the quasiparticle limit they provide the mass-condition $M = 0$. Appropriate combinations of the two sets of equations (3.13)–(3.14) and (3.17)–(3.18) provide us with the retarded equations

$$\mathcal{D}G^R(X,p) + \frac{\mathrm{i}}{2}\left\{\Gamma, G^R\right\} = 0, \tag{3.20}$$

$$\left(M(X,p) + \frac{\mathrm{i}}{2}\Gamma(X,p)\right) G^R(X,p) = 1, \tag{3.21}$$

which are simultaneously solved by algebraic inversion of (3.21) [3,10], i.e.

$$G^R = \frac{1}{M(X,p) + \mathrm{i}\Gamma(X,p)/2} \Rightarrow \begin{cases} A(X,p) = \dfrac{\Gamma(X,p)}{M^2(X,p) + \Gamma^2(X,p)/4}, \\ \mathrm{Re}G^R(X,p) = \dfrac{M(X,p)}{M^2(X,p) + \Gamma^2(X,p)/4}. \end{cases} \tag{3.22}$$

The spectral function satisfies the sum–rule

$$\int\limits_{-\infty}^{\infty} \frac{\mathrm{d}p_0}{2\pi} A(X,p) = 1, \tag{3.23}$$

which follows from the canonical equal-time (anti) commutation relations for (fermionic) bosonic field operators.

With the solution (3.22) for G^R equations (3.13) and (3.17) become identical to (3.14) and (3.18), respectively such that one is altogether left with *three* different equations rather than *two*. These relations are known since the original text-book by Kadanoff and Baym [3], here presented in a form convenient for our further considerations. Now we like to come to some new considerations.

At first glance equations (3.13) and (3.17) are not identical, while they were identical before the gradient expansion. However, even for finite values of Γ the equivalence of these two equations can be assured within the validity range $|f(X,p) - \gamma(X,p)| \ll 1$ of the gradient approximation. Using this limit and writing all $\Gamma_{\mathrm{in}} = \gamma\Gamma$ terms as $(f + (\gamma - f))\Gamma$, the four-phase-space transport and mass-shell equations, (3.13) and (3.17), take the forms

$$\frac{A^2}{2}\left(\Gamma\left\{M,f\right\}-M\left\{\Gamma,f\right\}\right)=\Gamma A(\gamma-f)-r_{\rm kin}, \tag{3.24}$$

$$\frac{A{\rm Re}G^R}{2}\left(\Gamma\left\{M,f\right\}-M\left\{\Gamma,f\right\}\right)=MA(\gamma-f)-r_{\rm mass-eq}, \tag{3.25}$$

where the remaining terms $r_{\rm kin}$ and $r_{\rm mass-shell}$ are seen to be of order $(\gamma-f)$ times gradient terms, i.e. of second order in the gradient expansion [1] . ¿From Eq. (3.22) one has $\Gamma{\rm Re}G^R = MA$. Using this we come to the observation that the four-phase-space mass-shell equation loses its original quasiparticle-like sense, since to leading order in the gradient expansion relation (3.25) is indeed equivalent to the four-phase-space transport equation (3.24). However, the still remaining difference in the second-order terms is inconvenient from the practical point of view. Besides the retarded relations (3.20) which are to be used in any case, Kadanoff and Baym have chosen equation (3.13) as the quantum four-phase-space kinetic equation. This has the property of providing a conserved energy momentum tensor (cf. sect. 3.3) which is symmetric with respect to the appearance of real and imaginary self-energy terms and propagators. Following Botermans and Malfliet (BM) [10], who first suggested to drop the above r_{kin} term in the four-phase-space kinetic equation, we now put $r_{kin} = r_{mass} = 0$ in both Eqs. (3.24) and (3.25) with the advantage that then these equations become completely identical. Dropping the r_{kin} and r_{mass} terms in (3.24) and (3.25) amounts to replace the $\Gamma_{\rm in}$ and $\Gamma_{\rm out}$ terms by $f\Gamma$ and $(1\mp f)\Gamma$ in *all Poisson brackets*. The so obtained *quantum four-phase-space kinetic equations for F and $\widetilde{F}$ in BM-choice* then read

$$\mathcal{D}F(X,p)-\left\{\Gamma\frac{F}{A},{\rm Re}G^R\right\}=C(X,p), \tag{3.26}$$

$$\mathcal{D}\widetilde{F}(X,p)-\left\{\Gamma\frac{\widetilde{F}}{A},{\rm Re}G^R\right\}=\mp C(X,p), \tag{3.27}$$

which are identical to each other in view of the retarded relation (3.22). In terms of the four-phase-space occupation functions $f(X,p)$ both above equations reduce to

$$\frac{A^2\Gamma}{2}\left(\mathcal{D}f-\frac{M}{\Gamma}\left\{\Gamma,f\right\}\right)=C. \tag{3.28}$$

To get Eq. (3.28) we used that the additional Poisson-bracket term becomes

$$\left\{\Gamma f,{\rm Re}G^R\right\}=\frac{M^2-\Gamma^2/4}{(M^2+\Gamma^2/4)^2}\,\mathcal{D}\left(\Gamma f\right)+\frac{M\Gamma^2/2}{(M^2+\Gamma^2/4)^2}\left\{\Gamma,f\right\}. \tag{3.29}$$

[1] These terms have the explicit form

$$r_{\rm kin}=-\left\{\Gamma(\gamma-f),{\rm Re}G^R\right\},\ \ r_{\rm mass-eq}=\frac{1}{4}\left\{\Gamma(\gamma-f),A\right\}.$$

Both, the KB choice (3.13) and the BM choice (3.26) are of course equivalent within the validity range of the first order gradient approximation. Frequently, however, such equations are used beyond the limits of validity as ad-hoc equations and then the different versions may lead to different results. So far we have no physical condition which gives preference to one or the other choice. The procedure used here, where in all Poisson brackets the $\Gamma_{\rm in}$ and $\Gamma_{\rm out}$ terms have consistently been replaced by $f\Gamma$ and $(1 \mp f)\Gamma$, respectively is therefore optional. However, in doing so we gained few advantages. First, four-phase-space kinetic and mass-shell equation are from now *exactly* equivalent to each other, as they were before the gradient expansion. Second, the so obtained quantum kinetic equations (3.26)-(3.27) has particular feature with respect to the definition of a nonequilibrium entropy flow and the formulation of an *exact* H-theorem in certain cases (sect. 5). If we omit these substitutions, both these features would become approximate up to the second-order gradient terms.

So far the gradient approximation has been applied to the space-time foldings occurring between the self-energies and the propagators appearing in the collision term. They give rise to the gradient terms on the l.h.s. of the kinetic equations. This is sufficient as long as the self-energies are calculated without further approximation. Commonly one also likes to obtain $\Gamma_{\rm in}$ and $\Gamma_{\rm out}$ in a kind of local approximation evaluated with all Green functions taken at the same space-time point X. Then for self-energy diagrams with more than two points, also nonlocal gradient corrections arise for such diagrams, which have to be treated in a consistent gradient approximation scheme. The latter fact gives rise to memory effects, which will be discussed in detail in subsect. 4.2, below.

3.2 Physical interpretation and quasiparticle limit

We now provide a physical interpretation of various terms in the quantum four-phase-space kinetic equation (3.26) or equivalently (3.28). This physical interpretation relies on the similarity of most of the terms to conventional kinetic equations, for example, such as the Landau kinetic equation for Fermi liquids (see, e.g., refs. [51,54]) and ref. [55] for the relativistic version), proposed by Landau on the basis of an intuitive quasiparticle picture [56], although the complete spectral dependence on energy is treated dynamically.

For this purpose it is advantage to convert the drift operator (3.15) into a space and time separated form

$$\mathcal{D} = \frac{1}{Z}\left(\partial_t + \boldsymbol{v}_g \partial_{\mathbf{X}}\right) + \partial_t \mathrm{Re}\Sigma^R \cdot \partial_{p_0} - \partial_{\mathbf{X}} \mathrm{Re}\Sigma^R \cdot \partial_{\boldsymbol{p}} \ , \tag{3.30}$$

where

$$\boldsymbol{v}_g(X,p) = Z\left(\boldsymbol{v} + \frac{\partial \mathrm{Re}\Sigma^R}{\partial \boldsymbol{p}}\right) \quad \text{with} \quad Z = \left(v_0 - \frac{\partial \mathrm{Re}\Sigma^R}{\partial p_0}\right)^{-1} \tag{3.31}$$

takes the meaning of the group 3-velocity in the quasiparticle approximation, while Z is the standard renormalization factor. In the quantum four-phase-space kinetics p_0 and $\boldsymbol{p}$ are independent in (3.31).

Thus, the drift term $\mathcal{D}F$ on the l.h.s. of Eqs. (3.13), (3.26) is the usual kinetic drift term including the corrections from the self-consistent field $\mathrm{Re}\Sigma^R$ into the convective transfer in four-phase-space. In the collision-less case $\mathcal{D}F = 0$ (Vlasov equation), the characteristic curves of quasi-linear first-order differential operator $\mathcal{D}$ define classical paths in four-phase-space with conserved probability $F(X,p)$ in this case. The formulation in terms of a Poisson bracket in four dimensions implies a generalized Liouville theorem. In the collisional case, both the collision, C, and fluctuation terms (3.29) change the phase-space probabilities of the "generalized" particles during their propagation along the the four-phase-space paths given by $\mathcal{D}$. Thereby particles are no longer bound to their mass-shell, $M = 0$, during propagation due to damping processes governed by the collision term. The Poisson-bracket term (3.29) is special. It contains genuine contributions from the finite damping width of the particles and describes the response of the surrounding matter due to fluctuations. This can be seen from the conservation laws discussed below. In particular the first term in (3.29) gives rise to a back-flow component of the surrounding matter. It restores the Noether currents as the conserved ones rather than the intuitively expected sum of convective currents arising from the convective $\mathcal{D}F$ term in (3.26). Furthermore, it is seen from Eq. (3.28) that the term $M\{\Gamma, f\}/\Gamma$ gives no contribution in the quasiparticle limit due to the factor M and thus represents a specific off-mass-shell response.

The r.h.s. of Eqs. (3.13), (3.26), (3.28) specifies the collision term C in terms of gain and loss terms, which also can account for multi-particle processes. Since F in Eqs. (3.13), (3.26) includes a factor A, C further deviates from the standard Boltzmann-type form in as much that it is multiplied by the spectral function A, which accounts for the finite width of the particles.

The equations so far presented, mostly with the KB choice (3.13), were the starting point for many derivations of extended Boltzmann and generalized kinetic equations, ever since these equations have been formulated in 1962. Most of those derivations use the equal-time reduction by integrating the four-phase-space equations over energy p_0, thus reducing the description to three-phase-space information, cf. refs. [21,22,27,36–42] and refs. therein. This can only consistently be done in the limit of small width Γ employing some kind of quasi-particle ansatz for the spectral function $A(X,p)$. It is important to realize that the function $\frac{1}{2}A^2\Gamma$ in front of the drift term in Eq. (3.28) is more sharply peaked than the original spectral function A. Both $\frac{1}{2}A^2\Gamma$ and A are reduced to the same δ-function in the quasiparticle limit, however corrections to the δ-function in powers of Γ start linear for A whereas they start of order Γ^3 for $\frac{1}{2}A^2\Gamma$ [50]. Particular attention has been payed to the treatment of the time-derivative parts in the Poisson brackets, which in the four-phase-space formulation still appear time-local, i.e. Markovian, while they lead to retardation effects in the equal-time reduction. Generalized quasiparticle ansätze were proposed, which essentially improve the quality and consistency of the approximation, providing those extra terms to the naive Boltzmann equation (some times called additional

collision term) which are responsible for the correct second-order virial corrections and the appropriate conservation of total energy, c.f. [27,38] and refs. therein.

At the simplest level the quasiparticle distribution takes the form [57]

$$F^{\rm qp}(X,p) = 2\pi\ Z(X,\varepsilon,\boldsymbol{p})\,\delta\left(p_0 - \varepsilon(X,\boldsymbol{p})\right) f^{\rm qp}(X,\boldsymbol{p}) \tag{3.32}$$

which is a function of only three-momentum through the quasiparticle dispersion relation for the energy $\varepsilon(X,\boldsymbol{p})$

$$\varepsilon(X,\boldsymbol{p}) = \frac{1}{2m}\boldsymbol{p}^2 + {\rm Re}\ \Sigma^R\left(X,\varepsilon(X,\boldsymbol{p}),\boldsymbol{p}\right). \tag{3.33}$$

However, all these quasi-particle schemes rely on the smallness of the damping widths. Yet, in order to describe the transport of particles with large mass widths, in particular, broad resonances, one has to stay at the level of the *quantum four-phase-space kinetic equations* (3.28) together with the retarded Eq. (3.22), which preserve all the spectral information. It is the main objective of this paper to study the properties of this four-phase-space kinetics at the most general level.

3.3 Conservations of Charge and Energy–Momentum

The quantum four-phase-space kinetic equation (3.26) weighted either with the charge e or with 4-momentum p^ν, integrated over momentum and summed over internal degrees of freedom like spin (Tr) gives rise to the charge or energy–momentum conservation laws, respectively, with the Noether 4-current and Noether energy–momentum tensor defined by the following expressions

$$j^\mu(X) = e{\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}v^\mu F(X,p), \tag{3.34}$$

$$\Theta^{\mu\nu}(X) = {\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}v^\mu p^\nu F(X,p) + g^{\mu\nu}\left(\mathcal{E}^{\rm int}(X) - \mathcal{E}^{\rm pot}(X)\right). \tag{3.35}$$

Here

$$\mathcal{E}^{\rm int}(X) = \left\langle -\widehat{\mathcal{L}}^{\rm int}(X)\right\rangle = \left.\frac{\delta\Phi}{\delta\lambda(x)}\right|_{\lambda=1} \tag{3.36}$$

is the total interaction energy density, which in terms of Φ is given by a functional variation with respect to a space-time dependent coupling strength of the interaction part of the Lagrangian density $\widehat{\mathcal{L}}^{\rm int} \to \lambda(x)\widehat{\mathcal{L}}^{\rm int}$, cf. ref. [1]. The potential energy density $\mathcal{E}^{\rm pot}$ as introduced in ref. [1] determines that potential energy which a probe particle with Wigner

density $F(X,p)$ would have due to the interaction with all other particles in the system. For the BM choice [2] it takes the form

$$\mathcal{E}^{\rm pot}(X) = {\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}\left[{\rm Re}\Sigma^R F + {\rm Re}G^R\frac{\Gamma}{A}F\right] = {\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}\left(p_0-\epsilon^0(\boldsymbol{p})\right)F(X,p)$$
$$\text{with}\ \ \epsilon^0(\boldsymbol{p}) = \frac{\boldsymbol{p}^2}{2m} \tag{3.37}$$

This auxiliary quantity is directly related to the self-energy Σ. Whereas the first term in the square bracket complies with quasiparticle expectations, namely mean potential times density, the second term displays the role of the width in the potential energy density. For the BM choice the entire expression can be reduced to a simple trace over $F(X,p)$.

In general, $\mathcal{E}^{\rm int}(X)$ has to be determined according to its diagrams given by Φ, cf. [1]. Only for specific interactions there are direct relations to $\mathcal{E}^{\rm pot}(X)$. This is the case, if to all interaction vertices of $\widehat{\mathcal{L}}^{\rm int}$ the same number α of field operators is attached. Then one simply deduces

$$\mathcal{E}^{\rm int}(X) = \frac{2}{\alpha}\mathcal{E}^{\rm pot}(X). \tag{3.38}$$

In particular, for two-body interactions one has $\alpha = 4$ and thus $2\mathcal{E}^{\rm int} = \mathcal{E}^{\rm pot}$ which provides the energy momentum tensor as a simple trace over $F(X,p)$

$$\Theta^{\mu\nu}_{\rm (two-body)}(X) = {\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}\left[v^\mu p^\nu - \frac{1}{2}g^{\mu\nu}\left(p_0-\epsilon^0(\boldsymbol{p})\right)\right]A(X,p)f(X,p), \tag{3.39}$$

with the free energy $\epsilon^0(\boldsymbol{p})$ from (3.37). a form commonly used in extended quasiparticle approximations, e.g. [38]. Please, also notice from (3.35) that the special combination

$$\Theta^{00}(X) + \frac{1}{3}\sum_{i=1}^{3}\Theta^{ii}(X) = {\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}A(X,p)f(X,p)\left(p_0+\frac{2}{3}\epsilon^0(\boldsymbol{p})\right) \tag{3.40}$$

depends on the specific form of the interaction only via the spectral function. As we will see below, in local thermodynamic equilibrium this combination simply relates to the entropy density.

[2] For the KB choice it takes the form

$$\mathcal{E}^{\rm pot}(X) = {\rm Tr}\int\frac{{\rm d}^4p}{(2\pi)^4}\left[{\rm Re}\Sigma^R F + {\rm Re}G^R\Gamma_{\rm in}\right].$$

The conservation laws only hold, if all the self-energies are Φ-derivable. In ref. [1], it has been shown that this implies consistency relations, Eqs. (6.5) and (6.9) in [1], which after Wigner transformation and first-order gradient expansion lead to

$$\mathrm{iTr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\left[\left\{\mathrm{Re}\Sigma^R,F\right\}-\left\{\mathrm{Re}G^R,\frac{\Gamma}{A}F\right\}+C\right]=0, \tag{3.41}$$

for the conserved current and

$$\partial^\nu\left(\mathcal{E}^{\mathrm{int}}-\mathcal{E}^{\mathrm{pot}}\right)=\mathrm{Tr}\int\frac{p^\nu\mathrm{d}^4p}{(2\pi)^4}\left[\left\{\mathrm{Re}\Sigma^R,F\right\}-\left\{\mathrm{Re}G^R,\frac{\Gamma}{A}F\right\}+C\right]. \tag{3.42}$$

for the energy-momentum tensor. The contributions from the Markovian collision term C drop out in both cases, cf. Eq. (4.8) below. The first term in each of the two relations refers to the change from the free velocity $\boldsymbol{v}$ to the group velocity $\boldsymbol{v}_g$, cf. Eq. (3.31), in the medium. It can therefore be associated with a corresponding *drag–flow* contribution of the surrounding matter to the current or energy–momentum flow. The second (fluctuation) term compensates the former contribution and can therefore be associated with a *back–flow* contribution, which restores the Noether expressions (3.34) and (3.35) to be indeed the conserved quantities. In this compensation we see the essential role of the fluctuation term (3.29) in the quantum four-phase-space kinetic equation. Dropping this term would spoil the conservation laws.

Expressions (3.34) and (3.35) for the conserved current and energy–momentum tensor, as well as self-consistency relations (3.41) and (3.42) are written explicitly for the case of non-relativistic particles with fixed particle number. This follows from the conventional way of non-relativistic renormalization for such particles based on normal ordering. When the number of particles is not conserved (e.g., for phonons or a system of relativistic particles), one should replace $F(X,p)\to\frac{1}{2}\left(F(X,p)\mp\tilde{F}(X,p)\right)$ in all above formulas in order to take proper account of zero point vibrations (e.g., of phonons) or of the vacuum polarization in the relativistic case. These symmetrized equations, derived from special $(\mp)$ combinations of the transport equations (3.26) and (3.27), are generally ultra-violet divergent, and hence, have to be properly renormalized at the vacuum level.

3.4 Positive Definiteness of Kinetic Quantities

For a semi-classical interpretation one likes to have the Wigner distributions $F(X,p)$ and $\tilde{F}(X,p)$ to be positive semi-definite, hereto after just called "positive". Using the operator definition for the Green functions (2.1) and integrating it over a large space-time volume, one arrives at, e.g.,

$$\int\mathrm{d}XF(X,p)=\left\langle\left(\int\mathrm{d}y e^{\mathrm{i}py}\widehat{\varphi}^\dagger(y)\right)\left(\int\mathrm{d}x e^{-\mathrm{i}px}\widehat{\varphi}(x)\right)\right\rangle, \tag{3.43}$$

and similarly for self-energies Σ expressed through the current–current correlator. It indicates that the r.h.s. of such equality is real and non-negative. Thus, we get the following set of constraints

$$\int \mathrm{d}X\widetilde{F}(X,p) \geq 0, \quad \int \mathrm{d}XF(X,p) \geq 0, \quad \int \mathrm{d}X\Gamma_{\mathrm{out}}(X,p) \geq 0, \quad \int \mathrm{d}X\Gamma_{\mathrm{in}}(X,p) \geq 0. \tag{3.44}$$

Similar relations are obtained for the integration over four-momentum space rather than space and time. As a result, in stationary and spatially homogeneous systems, in particular in equilibrium systems, the quantities F, $\widetilde{F}$, Γ_{in} and Γ_{out} are real and non-negative[3], i.e.

$$F(p) \geq 0, \quad \widetilde{F}(p) \geq 0; \quad \Gamma_{\mathrm{in}}(p) \geq 0, \quad \Gamma_{\mathrm{out}}(p) \geq 0. \tag{3.45}$$

In deriving constraints (3.44) and (3.45), we did not use the fact that the Green functions are solutions of the Dyson's equation. However, we used the operator picture. Any approximation, in particular, if formulated in the space of Green functions, may spoil such rigorous statements like (3.43). Nevertheless, both the Φ-derivable scheme and the gradient approximation preserve the retarded relations (A.6) among the different contour components and the retarded and advanced functions of any contour function, with definite values for the imaginary parts of the corresponding retarded Wigner functions

$$-2\mathrm{Im}G^R(X,p) = A(X,p) \geq 0, \quad -2\mathrm{Im}\Sigma^R(X,p) = \Gamma(X,p) \geq 0, \tag{3.46}$$

which even hold locally. In particular, solution (3.21) for the retarded Green function shows that all retarded relations hold locally: the momentum part is the same as that in the homogeneous case with the space-time coordinate X as a parameter. Under the condition $|f-\gamma| \ll 1$, cf. the discussion around Eq. (3.24), one finds that

$$\Gamma_{\mathrm{out}}(X,p) \approx f\Gamma(X,p) > 0, \qquad \Gamma_{\mathrm{in}}(X,p) \approx (1 \mp f)\Gamma(X,p) > 0 \tag{3.47}$$

as long as the Wigner densities f and $1 \mp f$ are positive. As the gradient approximation is a quasi-homogeneous approximation, one may therefore expect the positivity of Γ_{in} and Γ_{out} to be preserved even in the self-consistent treatment discussed here. Diagrammatic rules may also corroborate this, since diagrams for Γ_{in} and Γ_{out} are calculated like in the homogeneous case.

We now like to show that, if Γ_{in} and Γ_{out} are positive, also under minor restrictions the kinetic equation (3.28) preserves the positivity of f, once initially started from a positive f. This equation is of integro-quasi-linear first-order partial differential type in 8 dimensions with all derivatives placed on the l.h.s. and the collision term

$$C = A(\Gamma_{\mathrm{in}} - \Gamma f) \tag{3.48}$$

[3] In relativistic descriptions one has appropriately to separate particles from anti-particles.

provides a term linear in f with a negative coefficient. Therefore overall stability requires the coefficient in front of the time derivative term in Eq. (3.28) to be positive [4]. Given the solution of (3.28), let us then discuss properties of f along the characteristic curves determined by the quasi-linear drift operator on the l.h.s. of (3.28) with a curve parameter s growing monotonically with time. Assuming f positive initially, the occurrence of a negative value at some later time requires the first zero value to occur at one of the characteristic curves. Be s_0 such a place, one finds

$$\frac{1}{2}A\Gamma\frac{\mathrm{d}}{\mathrm{d}s}\,f(s)\Big|_{s\,=\,s_0} = [A\Gamma_{\mathrm{in}}]_{s\,=\,s_0}\;>0. \tag{3.49}$$

If the r.h.s. is positive, one comes to a contradiction, as the approach from some positive value to a zero value which then becomes negative would require a non-positive derivative at s_0. Thus, the overall stability requirement of the equations is sufficient to preserve positive distribution functions.

4 Collision Term

To further discuss the transport treatment we need an explicit form of the collision term (3.16), which is provided from the Φ functional in the $-+$ matrix notation via the variation rules (A.8) as

$$C(X,p) = \frac{\delta \mathrm{i}\Phi}{\delta\widetilde{F}(X,p)}\widetilde{F}(X,p) - \frac{\delta \mathrm{i}\Phi}{\delta F(X,p)}F(X,p). \tag{4.1}$$

Here we assumed Φ be transformed into the Wigner representation and all $\mp\mathrm{i}G^{-+}$ and $\mathrm{i}G^{+-}$ to be replaced by the Wigner-densities F and $\widetilde{F}$. Thus, the structure of the collision term can be inferred from the structure of the diagrams contributing to the functional Φ. To this end, in close analogy to the consideration of ref. [13], we discuss various decompositions of the Φ-functional, from which the in- and out-rates are derived. This consideration is based on the standard real-time diagrammatic rules, where the contour integrations are decomposed into two branches with $-$ and $+$ vertices for the time- and anti-time-ordered branches, cf. Appendix A. For the sake of physical transparency, we confine our treatment to the *local* case, where in Wigner representation all the Green functions are taken at the same space-time coordinate X and all non-localities, i.e. derivative corrections, are disregarded. Derivative corrections give rise to memory effects in the collision term, which will be analyzed separately for the specific case of triangle diagram (see subsect. 4.4).

[4] If the physical situation allows for instabilities, the dynamics will carry the system over into new regime of stability, like for phase transitions. In the latter case the kinetic equation has to be supplemented by an equation describing the time evolution of the condensate (one-point function) which couples to the propagator, cf. the general scheme discussed in [1].

Consider a given closed diagram of Φ, at this level specified by a certain number n_λ of vertices and a certain contraction pattern which links all vertices with lines of certain arrow sense for complex fields. This fixes the topology of such a contour diagram. It leads to 2^{n_λ} different diagrams in the $-+$ notation from the summation over all $-+$ signs attached to each vertex. Thus for any $-+$ type diagram of Φ also the diagram, where all $+$ and $-$ vertex signs are interchanged, contributes to Φ. Furthermore, for any diagram with given line senses the diagram with all line senses reversed also is a valid diagram of Φ. We shall exploit these two discrete symmetry operations to further determine the properties of Φ. The simultaneous application of the interchange of all vertex signs and the reversion of the line sense leads to the adjoint expressions, since in the underlying operator picture it adjungates all operators and inverts the operator ordering. The corresponding values are then complex conjugate to one another, cf. (A.7).

Using these symmetries one can convert the functional Φ into the following general form

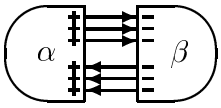
$$\mathrm{i}\Phi^{\mathrm{local}} = \frac{1}{2}\int \mathrm{d}X \sum_{m,m'} \left[\mathrm{i}\Phi_{m,m'}(X) + \mathrm{i}\Phi_{m',m}(X)\right], \tag{4.2}$$

$$\begin{aligned}\mathrm{i}\Phi_{m,m'} = \int \frac{\mathrm{d}^4p_1}{(2\pi)^4}\cdots\frac{\mathrm{d}^4p_m}{(2\pi)^4}\frac{\mathrm{d}^4p'_1}{(2\pi)^4}\cdots\frac{\mathrm{d}^4p'_{m'}}{(2\pi)^4}\ (2\pi)^4\delta^4\left(\sum_{i=1}^{m} p_i - \sum_{i=1}^{m'} p'_i\right)\\ \times R_{m,m'}(X;p_1,\ldots,p_m;p'_1,\ldots,p'_{m'})\ F_1\cdots F_m\widetilde{F}'_1\cdots\widetilde{F}'_{m'}\ ,\end{aligned} \tag{4.3}$$

where in view of the local approximation the four momentum conservation has been extracted. While Φ_{00} compiles with terms void of any Wigner densities, i.e. from diagrams where all vertices have the same sign and which do not contribute to the collision term, the nontrivial $\Phi_{m,m'}$ terms sum the sub-class of diagrams of Φ with precisely $m+m'$ Wigner densities $F_i = F(X,p_i)$ and $\widetilde{F}'_i = \widetilde{F}(X,p'_i)$, respectively. According to Eq. (4.1) each $\Phi_{m,m'}$ in (4.3) generates multi-particle gain or loss contributions expressed in terms of integrals over products of generalized distribution functions F, and Fermi/Bose factors $\widetilde{F}$. Every term in the sum (4.2) has been duplicated, repeating each term in its line reverse form. The corresponding transition rates $R_{m,m'}(X;p_1,\ldots,p_m;p'_1,\ldots,p'_{m'})$ are real due to the adjungation symmetry. As explained below, they result from the product of chronological Feynman amplitudes, given by the sub-diagram compiled from all $-$ vertices linked by $\mathrm{i}G^{--}$ Green functions times that of the anti-chronological part containing the $\mathrm{i}G^{++}$ functions.

The generic form (4.3) of $\Phi_{m,m'}$ can be illustrated diagrammatically. For this purpose consider a $-+$ notation diagram contributing to $\Phi_{m,m'}$, which contains vertices of either sign. It can be decomposed into two pieces, denoted by a compact bra-ket notation, say $(\alpha|$ and $|\beta)$, in such a way that each of the two sub-pieces contains vertices of only one

type of sign [5]

$$\mathrm{i}\Phi^{+-}_{\alpha\beta} = \frac{1}{n_{\alpha\beta}}$$  $$= \frac{1}{n_{\alpha\beta}} \left(\alpha \left| F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{m'} \right| \beta\right)$$

$$= \frac{1}{n_{\alpha\beta}} \int \frac{\mathrm{d}^{4(m+m')} p_{mm'}}{(2\pi)^{4(m+m')}} \, (2\pi)^4 \delta^4 \left(\sum_{i=1}^{m} p_i - \sum_{i=1}^{m'} p'_i \right)$$
$$\times \, V^*_\alpha(X; p_{mm'}) \, F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{m'} \, V_\beta(X; p_{mm'}), \tag{4.4}$$

where $n_{\alpha\beta}$ counts the number of vertices in the closed diagram $(\alpha|\beta)$. The short-hand notation $p_{mm'} = \{p_1, \ldots, p_m; p'_1, \ldots, p'_{m'}\}$ summarizes the momenta, type and internal quantum numbers of the set of ordered valences, to be joint to the product of $m + m'$ Wigner densities [6] $F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{\tilde{m}}$ linking the two amplitudes. The "end-caps" $(\alpha|$ and $|\beta)$ represent multi-point vertex functions, in simple cases of tree type, of only one sign for the vertices, i.e. they are either entirely time ordered ($-$ vertices) or entirely anti-time ordered ($+$ vertices). Each such vertex function

$$|\beta) = V_\beta(X; p_{mm'}), \qquad (\alpha| = V^*_\alpha(X; p_{mm'}), \tag{4.5}$$

to be determined by normal Feynman diagram rules, has m F-valences and m' $\widetilde{F}$-valences, respectively. In (4.5) we used the fact that adjoint expressions are complex conjugate to each other, cf. (A.7). Accumulating all diagrams of Φ that lead to the same set of Wigner densities $F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{\tilde{m}}$ provides us with the generic form (4.3) with the partial process rates

$$R_{m,m'}(X; p_{mm'}) = \sum_{(\alpha\beta)\in\Phi_{m,m'}} \frac{1}{n_{\alpha\beta}} \mathrm{Re}\left\{V^*_\alpha(X; p_{mm'}) V_\beta(X; p_{mm'})\right\}. \tag{4.6}$$

The restriction to the real part arises, since with $(\alpha|\beta)$ also the adjoint $(\beta|\alpha)$ diagram contributes to the subclass $\Phi_{m,m'}$. However these rates are not necessarily positive as in perturbation theory [7]. In this point, the Φ-derivable scheme differs from the conventional

[5] To construct the decomposition, just deform a given mixed-vertex diagram of Φ in such a way that all $+$ and $-$ vertices are placed left and respectively right from a vertical division line and then cut along this line.

[6] This product of Wigner-densities originates from the $\mp\mathrm{i}G^{-+}$ and $\mathrm{i}G^{+-}$ Green functions, (cf. Eq. (3.7)). In closing the diagram by these Wigner densities extra fermion loops may appear besides the ones accounted for in the amplitudes α and β. Using the product $F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{m'}$ just take care of the additional (-1) factors arising from this closing of the diagram.

[7] In perturbation theory they are positive, since there the sum in (4.6) leads to absolute squares. In the general case with resummed propagators this positivity may be lost due to the restriction to diagrams of Φ which are globally two-particle irreducible. The latter excludes certain combinations of amplitude diagrams, implying that the rates of genuine multi-particle processes are

Boltzmann kinetics. Even the two rates, $R_{mm'}$ and its line sense reverse $R_{m'm}$, have not to be necessarily identical to each other. Still Eq. (4.3) represents the most general form of Φ expressed through the Wigner densities $F(X,p)$ and $\widetilde{F}(X,p)$ in the local approximation. It is however important to realize that in many physically relevant cases, e.g. those discussed below, one indeed finds that

$$R_{m,m'} = R_{m',m}. \tag{4.7}$$

This property will be used as a sufficient condition for the derivation of the H-theorem. In the following we will restrict the discussion to cases where (4.7) is assumed. The treatment of more general cases, in particular in connection with the H-theorem, will be deferred to another publication.

Since Φ is two-line irreducible, there are at least three lines connecting $(\alpha|$ and $|\beta)$ and in many cases $(\alpha|$ and $|\beta)$ are connected by, at least, four lines, like in the case of two-body interactions. In such diagrams each of the amplitudes α or β necessarily form a connected diagram for the complex field case for binary ($m = m' = 2$) and for triple scattering ($m = m' = 3$) case, while in the general case of multi-particle (more than triple) processes such amplitudes may be disconnected.

No further symmetry can be specified at this level without additional knowledge on possible topological and other symmetries of Φ and the different particle species involved. The decomposition discussed here solely relies on a straightforward application of the contour rules for multi-point functions. They differ from other cutting rules like those derived by Danielewicz [8], which represent the result in terms of generalized retarded functions.

4.2 Local Collision Term and Memory Corrections

The gradient corrections to the folding of the self-energies with the propagators in the collision term essentially give rise to in-medium corrections of the convective part (l.h.s) of the quantum four-phase-space kinetic equation (3.26). Within the spirit of the gradient approximation one also likes to express the self-energies themselves by space-time local quantities. Thus, for a consistent gradient approximation further gradient corrections are admissible once the self-energy diagrams and thereby the diagrams of Φ consist of more than 2 vertices. We call the collision term, evaluated with all Green functions in the Wigner representation taken at the same space-time point X, the *local* collision term.

In terms of the representation (4.3) of Φ and implying line-sense reversal symmetry for the rates (4.7) the matrix variation rules (4.1) determine the following local part of the collision term (3.16) for a particle of flavor "a" as

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not necessarily positive.

$$C_a^{\rm loc}(X,p) = \frac{1}{2}\sum_{m,m'}\int \frac{{\rm d}^4p_1}{(2\pi)^4}\cdots\frac{{\rm d}^4p_m}{(2\pi)^4}\frac{{\rm d}^4p'_1}{(2\pi)^4}\cdots\frac{{\rm d}^4p'_{m'}}{(2\pi)^4}$$
$$\times R_{m,m'}(X;p_1,\cdots,p_m;p'_1,\cdots,p'_{m'})\left\{\widetilde{F}_1\cdots\widetilde{F}_m F'_1\cdots F'_{m'} - F_1\cdots F_m\widetilde{F}'_1\cdots\widetilde{F}'_{m'}\right\}$$
$$\times\left[\sum_{i=1}^{m}\delta_{aa_i}(2\pi)^4\delta^4(p_i-p) - \sum_{i=1}^{m'}\delta_{aa_i}(2\pi)^4\delta^4(p'_i-p)\right](2\pi)^4\delta^4\left(\sum_{i=1}^{m}p_i - \sum_{i=1}^{m'}p'_i\right). \quad (4.8)$$

The functional variations of Φ with respect to F_a and $\widetilde{F}_a$ are expressed in terms of the four-momentum projectors $\delta_{aa_i}(2\pi)^4\delta^4(p_i-p)$, cf. eq. (A.9), which in case of multi-component systems of particles with different flavors and internal quantum numbers "a" also include the proper projections onto the different flavors $a_i = a$. This expression nicely visualizes the detailed balance property namely that the same multi-particle rate determines both the forward and the backward processes.

The first-order gradient corrections to the local collision term (4.8) are called *memory* corrections. Nonlocal collision terms have been already studied by many authors, in particular, within the nonequilibrium Green's function technique,e.g., cf. refs [37,38,58–63]. Only recently it was realized [63] that within the first-order gradient expansion we must keep the linear gradient corrections to the collision term. Therefore, memory effects are, in general, unavoidable within kinetic treatment. However, memory effects are quite commonly neglected in transport models. Indeed, *only self-energy diagrams of third and higher order in the number of vertices give rise to memory effects*, as it is discussed below (sect. 4.4) within a simple model example. In particular, it means that the popular Born (or self-consistent Born) approximation does not involve any memory corrections. Until recently this property of the Born approximation was not that obvious within the equal-time formalism because of complications resulting from reduction schemes. Spicka *et al.* [62] showed that a proper equal-time reduction really maintains this property.

Memory corrections are linear in the time-spatial gradients of Wigner functions and the displacement factors in front of these Taylor expansions give momentum gradients of associated other multi-point functions. Since our prime goal here is only to demonstrate how memory corrections arise within our method of decomposing the Φ functional, we refrain from specifying them in general terms. To this end only a simple model example is considered below (sect. 4.4).

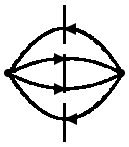
4.3 Φ-Derivable Collision Terms

4.3.1 Two-body Potential Interaction

To be specific we consider a system of fermions interacting via a zero-range two-body potential $V = V_0\delta(x-y)$, and, for the sake of simplicity, disregard its spin structure, by relating spin and anti-symmetrization effects to a degeneracy factor d. To derive the decomposition of a Φ-derivable collision term, we employ the rules as described in subsect.

4.1.

In the first example, we consider the generating functional Φ to be approximated by the following two diagrams

$$\mathrm{i}\Phi = \frac{1}{2}$$  $$+ \frac{1}{4}$$  (4.9)

with the dashed line illustrating the decomposition according to (4.4). Here the $1/n_\lambda$ factors start with $1/2, 1/4, \ldots$ according to the non-relativistic diagram rules for two-body interaction, cf. Appendix C, i.e. the vertex dots are considered as the zero range limit of a finite range interaction. In the $\{-+\}$ matrix notation of the Green functions, one can easily see that the one-point diagram does not contribute to the collision term, while decomposing the second diagram along the dashed line leads to a purely local result

$$C^{(2)} = d^2 \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \frac{\mathrm{d}^4 p_3}{(2\pi)^4} \Big|$$  $$\Big|^2$$
$$\times (2\pi)^4 \delta^4 \left(p + p_1 - p_2 - p_3\right) \left(F_2 F_3 \widetilde{F} \widetilde{F}_1 - \widetilde{F}_2 \widetilde{F}_3 F F_1\right), \tag{4.10}$$

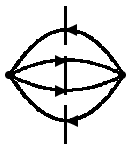
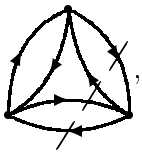
where the brief notation of the previous subsect. is used for F_i and $\widetilde{F}_i$. This collision integral has precisely the form of the binary collision term of Boltzmann–Uehling–Uhlenbeck (BUU), except for the fact that distribution functions are not constrained to the mass shell. The binary transition rate

$$R_2^{(2)} = V_0^2 = \Big|$$  $$\Big|^2 \tag{4.11}$$

is non-negative in this case. Here and below, the superscripts in brackets $^{(2)}$ (or $^{(3)}$) point out the origin of the quantity (C, R, etc.) from the second (or third) diagram of Φ. The subscript $_2$ in the transition rate of Eq. (4.11) indicates the binary-collision nature of this transition rate. Note that external 4-momenta (in-going and out-going) of the scattering amplitude  are not confined to the mass shell. For the trivial case under consideration, this fact does not give rise to any important consequences. However, for more complicated examples below, it means that the collision term is determined by off-shell scattering amplitudes.

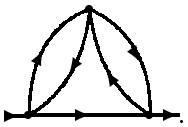
The picture becomes more complicated, if Φ involves diagrams of higher orders. For instance, let us add the three point diagram to Φ, i.e.

$$\mathrm{i}\Phi = \mathrm{i}\left(\Phi^{(1)} + \Phi^{(2)} + \Phi^{(3)}\right)$$

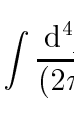
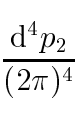
$$= \frac{1}{2}$$  $$+ \frac{1}{4}$$  $$+ \frac{1}{6}$$  , (4.12)

where one possible decomposition is illustrated by dashed lines. The corresponding self-energy becomes

$$-\mathrm{i}\Sigma = -\mathrm{i}\left(\Sigma^{(1)} + \Sigma^{(2)} + \Sigma^{(3)}\right) =$$

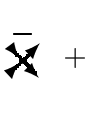
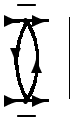
$$=$$  $$+$$  $$+$$  . (4.13)

Now the collision term contains a *nonlocal* part due to the last diagram. This nonlocal contribution is discussed in the next subsect. in detail. The local part can easily be derived in the form

$$C^{(2)} + C^{(3)}_{\rm loc} = d^2 \int \frac{\mathrm{d}^4p_1}{(2\pi)^4}\frac{\mathrm{d}^4p_2}{(2\pi)^4}\frac{\mathrm{d}^4p_3}{(2\pi)^4} \left(\left| \right.$$  $$\left. \right|^2 - \left| \right.$$  $$\left. \right|^2 \Big)$$

$$\times (2\pi)^4 \delta^4\left(p + p_1 - p_2 - p_3\right)\left(F_2F_3\tilde{F}\tilde{F}_1 - \tilde{F}_2\tilde{F}_3FF_1\right), \qquad (4.14)$$

where all the vertices in the off-shell scattering amplitudes are of the same sign, say " $-$ " for definiteness, i.e. there are no " $+-$" and " $-+$" Green functions left. The quantity $C^{(2)} + C^{(3)}_{\rm loc}$ is again of the Boltzmann form with the transition rate

$$R^{(2)}_2 + R^{(3)}_2 = \left| \right.$$  $$\left. \right|^2 - \left| \right.$$  $$\left. \right|^2 .$$ (4.15)

At the first glance, one may argue that this rate is not necessarily positive in the limit of strong coupling. Indeed, the first term in Eq. (4.15), i.e. $-\mathrm{i}V_0$, is purely imaginary, whereas the second one—the loop—given by

$$\mathrm{loop} = -d \int \frac{\mathrm{d}^4p_1}{(2\pi)^4} \mid V_0 \mid^2 \mathrm{i}G^{--}(x, p + p_1)\mathrm{i}G^{--}(x, p_1),$$

has both real and imaginary parts. Hence, the real part of this loop is canceled out in Eq. (4.15). If $\mid \mathrm{Im(loop)} \mid > \mid V_0 \mid$, the rate $(R^{(2)}_2 + R^{(3)}_2)$ may become negative, depending on signs of V_0 and Im(loop). However, one has to keep in mind that the Green functions in the loop cannot be chosen arbitrarily. Rather in a consistent treatment, as shown in Eq. (80) of ref. [13], the loop reveals a factor $\propto |1/\Gamma| \propto |1/V_0^2|$ resulting from the imaginary part

of the retarded self-energy in the propagators, which balances the total value of the loop. Indeed, in equilibrium the gain part of collision term and thus $(R_2^{(2)} + R_2^{(3)})$ is positive. This illustrates that the question of positive definiteness is quite subtle.

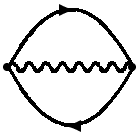
4.3.2 Bosonization of the Interaction

It is obvious that the situation becomes complicated once one extends the picture to ring diagrams with more than three vertices. Yet, there is a simple (however, not general) strategy to proceed. We may avoid the two-particle potential interaction from the very beginning and rather introduce an interaction mediated by an artificially introduced neutral heavy scalar boson (h.b.) of mass $m_{\text{h.b.}}$ much larger than any characteristic momentum transfers in the system. Then the free retarded Green function of the boson approximately equals

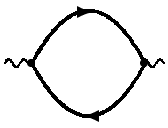
$$\Delta^{0R}_{\text{h.b.}} \simeq \frac{-1}{m^2_{\text{h.b.}} - \mathrm{i}0}, \tag{4.16}$$

and the vertex of fermion–heavy-boson interaction being $g = \sqrt{|V_0|}m_{\text{h.b.}}$. Moreover, by the same reason the heavy-boson occupation numbers may be put to zero, $\Delta^{0-+}_{\text{h.b.}} = 0$, since typical excitation energies are much less than the boson mass. The fact that this boson is very heavy makes the fermion–fermion interaction almost point-like. To be specific, we assume that $V_0 < 0$, i.e. attractive, which can be mediated by a scalar boson. In case of repulsive interactions, a vector boson would be an appropriate choice.

Thus, *from now on, we deal with a system of interacting fermions and heavy bosons.* Let us take the following approximation for the corresponding Φ functional (we call it $\Phi_{\text{h.b.}}$)

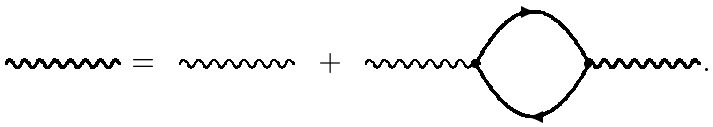
$\mathrm{i}\Phi_{\text{h.b.}} = \frac{1}{2}$  (4.17)

in terms of full Green functions of fermions (the bold solid lines) and bosons (the bold wavy line). In this approximation, the boson self-energy is given by (cf. Eq. (2.7)) [8]

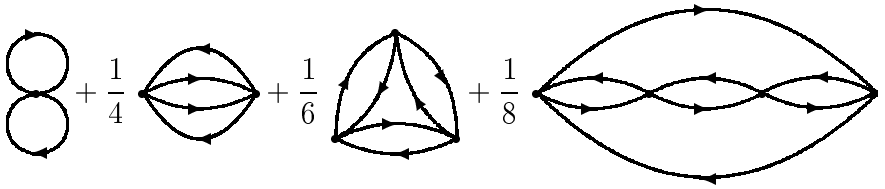
$-\mathrm{i}\frac{1}{2}\Sigma_{\text{h.b.}} = \frac{\delta \mathrm{i}\Phi_{\text{h.b.}}}{\delta \mathrm{i}G_{\text{h.b.}}} =$  (4.18)

[8] Note that for neutral bosons, which number is not conserved, the additional factor 1/2 appears in Eq. (2.7), cf. ref. [1].

and the heavy-boson Green function is defined by the standard Dyson's equation

$$\mathrm{i}G_{\mathrm{h.b.}} =$$  (4.19)

We would like to compare this model with the model described in subsection 4.3.1. Eliminating the artificial heavy boson one effectively sums up all ring diagrams of the type

$$\mathrm{i}\Phi_{\mathrm{ring}} = \frac{1}{2}$$  $$+ \ldots$$ (4.20)

Note that $\Phi_{\mathrm{ring}} \neq \Phi_{\mathrm{h.b.}}$, since the summations of loops in Eq. (4.17) and Eq. (4.20) have different sense. In Eq. (4.19) we have the conventional diagrammatic summation, while in Eq. (4.20) the summation is logarithmic, i.e. with the factors $1/n$, where n is the number of vertices in the diagram. However, this difference in summations is compensated for by the heavy-boson contribution to the generating functional (2.3). The equivalence of approximations (4.20) and (4.17) can be actually seen, e.g., from the fact that they result in the same approximation for the fermion self-energy $\mathrm{i}\Sigma_{\mathrm{f}} = \delta\Phi_{\mathrm{ring}}/\delta G_{\mathrm{f}} = \delta\Phi_{\mathrm{h.b.}}/\delta G_{\mathrm{f}}$ (cf. Eq. (2.7)) after substituting Eq. (4.19) for the heavy-boson propagator. Here and below, the sub-label "f" denotes fermion quantities.

In the heavy-boson picture we have to deal with two coupled transport equations—for non-relativistic fermions and for heavy bosons—with the following collision terms

$$\begin{aligned} C_{\mathrm{f}}(X,p) = & \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4}\frac{\mathrm{d}^4 p_2}{(2\pi)^4} g^2 (2\pi)^4 \delta^4\left(p - p_1 - p_2\right) \\ & \times \left[\widetilde{F}_{\mathrm{f}}(X,p) F_{\mathrm{f}}(X,p_1) F_{\mathrm{h.b.}}(x,p_2) - F_{\mathrm{f}}(X,p)\widetilde{F}_{\mathrm{f}}(X,p_1)\widetilde{F}_{\mathrm{h.b.}}(x,p_2)\right], \end{aligned} \tag{4.21}$$

for fermions and

$$\begin{aligned} C_{\mathrm{h.b.}}(X.p) = & d \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4}\frac{\mathrm{d}^4 p_2}{(2\pi)^4} g^2 (2\pi)^4 \delta^4\left(p + p_1 - p_2\right) \\ & \times \left[\widetilde{F}_{\mathrm{h.b.}}(X,p)\widetilde{F}_{\mathrm{f}}(X,p_1) F_{\mathrm{f}}(X,p_2) - F_{\mathrm{h.b.}}(X,p) F_{\mathrm{f}}(X,p_1)\widetilde{F}_{\mathrm{f}}(X,p_2)\right], \end{aligned} \tag{4.22}$$

for heavy bosons, where $g^2 = -V_0 m_{\mathrm{h.b.}}^2 > 0$ is defined in terms of two-particle interaction strength $V_0 < 0$ and the heavy boson mass $m_{\mathrm{h.b.}}$.

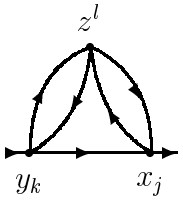
The collision terms (4.21) and (4.22) are very simple in spite of the fact that they involve the whole series of ring diagrams. The corresponding gain and loss terms are positive and

contain no memory effects, as they are hidden in the boson, while in the pure fermionic case already the triangle diagram gives rise to memory effects. Indeed, there is no contradiction here. If one wants to eliminate the bosonic degree of freedom, one has to resolve the bosonic transport equation with respect to the bosonic generalized distribution function $F_{\text{h.b.}}$ for the entire past and substitute this into the fermionic collision term. In this way, the resulting collision term becomes highly complicated and nonlocal and thus contains memory effects.

Hence, we have demonstrated that sometimes it is useful to introduce new degrees of freedom in order to achieve a reasonable collision term. Of course, all the considerations above remain valid also for particle–particle interaction mediated by a real boson rather than only by the artificially introduced one.

4.4 Memory Effects in Collision Term

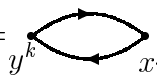
A general treatment of memory effects in the collision term is a cumbersome task. In this subsection we continue to consider a system of non-relativistic fermions interacting via the contact two-body potential and concentrate on the third diagram in Eq. (4.12), which has already been considered in a Φ-derivable scheme for the thermodynamic potential and entropy in refs. [49,50]. This is the first ring diagram to contribute to memory effects. The corresponding self-energy diagram reads

$-\mathrm{i}\Sigma^{(3)}_{jk}(x,y) =$  , $\quad j,k \in \{+,-\}.$ (4.23)

Standard diagrammatic rules in the matrix representation present $\Sigma^{(3)}_{ij}$ in analytic form as

$$-\mathrm{i}\Sigma^{(3)}_{jk}(x,y) = \int \mathrm{d}z \cdot \mathrm{i}V_0 \sigma_{jj'}\sigma_{kk'}\mathrm{i}G^{j'k'}(x,y)L^{k'l}(y,z)\sigma_{ll'}L^{l'j'}(z,x), \tag{4.24}$$

where we have introduced the loop function

$L^{jk}(x,y) =$  $= d\mathrm{i}V_0\mathrm{i}G^{jk}(x,y)\mathrm{i}G^{kj}(y,x),$ (4.25)

and σ_{ij} are given by Eq. (A.2). As above, the factor d results from the trace over spin. In the Wigner representation, L^{jk} takes the form

$$L^{jk}(X,p') = \int \frac{\mathrm{d}^4 p''}{(2\pi)^4} \widetilde{L}^{jk}(X; p''+p', p''), \tag{4.26}$$

where

$$\widetilde{L}^{jk}(X; p''+p', p'') = d\mathrm{i}V_0 \mathrm{i}G^{jk}(X, p''+p')\mathrm{i}G^{kj}(X, p''). \tag{4.27}$$

The loop functions L possess notable properties

$$L^{++} + L^{--} = L^{-+} + L^{+-}, \quad L^{jk}(x,y) = L^{kj}(y,x), \quad L^{jk}(X,p) = L^{kj}(X,-p). \tag{4.28}$$

The former property follows from the general property (A.6) of the two-point functions and holds in both the coordinate and Wigner representations. Proceeding from Eq. (4.24) and with the help of relation (3.3), we can immediately evaluate the Wigner transform of $\Sigma^{(3)}_{ij}$ (cf. Eq. (3.1)) and perform its gradient expansion

$$\Sigma^{(3)}_{jk} \simeq \left(\Sigma^{(3)}_{jk}\right)_{\mathrm{loc}} + \left(\Sigma^{(3)}_{jk}\right)_{\mathrm{mem}}, \tag{4.29}$$

where

$$-\mathrm{i}\left(\Sigma^{(3)}_{jk}\right)_{\mathrm{loc}}(X,p) = \int \frac{\mathrm{d}^4 p'}{(2\pi)^4} \mathrm{i}V_0 \sigma_{jj'}\sigma_{kk'}\sigma_{ll'} \mathrm{i}G^{j'k'}(X, p'+p) L^{k'l}(X,p') L^{l'j'}(p', X) \tag{4.30}$$

is the local contribution to $\Sigma^{(3)}_{ij}$, and

$$\begin{aligned} -\mathrm{i}\left(\Sigma^{(3)}_{jk}\right)_{\mathrm{mem}}(X,p) = \frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^4 p'}{(2\pi)^4} \mathrm{i}V_0 \sigma_{jj'}\sigma_{kk'}\sigma_{ll'} \mathrm{i}G^{j'k'}(X, p'+p) \\ \times \left\{ L^{k'l}(X,p'), L^{l'j'}(X,p') \right\}_{p',X} \end{aligned} \tag{4.31}$$

is the first-order gradient (memory) correction, which is of central interest in this subsection. Here, the Poisson bracket is taken with respect to the (p', X) variables indicated in the subscript.

To evaluate the memory correction for the collision term (cf. Eqs. (3.9) and (3.16)), we need to consider the following combinations of self-energies with Green functions

$$\left(\Sigma^{(3)}_{+-}\right)_{\mathrm{mem}}(X,p) G^{-+}(X,p) = \frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^4 p'}{(2\pi)^4} \frac{1}{d} \widetilde{L}^{+-}(X; p'+p, p) \sigma_{ll'} \left\{ L^{-l}, L^{l'+} \right\}_{p',X}, \tag{4.32}$$

$$G^{+-}(x,p) \left(\Sigma^{(3)}_{-+}\right)_{\mathrm{mem}}(X,p) = \frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^4 p'}{(2\pi)^4} \frac{1}{d} \widetilde{L}^{-+}(X; p'+p, p) \sigma_{ll'} \left\{ L^{+l}, L^{l'-} \right\}_{p',X}. \tag{4.33}$$

With the symmetry relations

$$\sigma_{ll'}\left\{L^{-l},L^{l'+}\right\}_{p',X}=\sigma_{ll'}\left\{L^{+l},L^{l'-}\right\}_{p',X}=\left\{L^{+-},L^{-+}\right\}_{p',X} \tag{4.34}$$

deduced from (4.28) one determines the first-order gradient correction to the collision term induced by graph (4.23) as

$$\begin{aligned}C^{(3)}_{\rm mem}(X,p)&=\left[\left(\Sigma^{(3)}_{+-}\right)_{\rm mem}(X,p)G^{-+}(X,p)-G^{+-}(X,p)\left(\Sigma^{(3)}_{-+}\right)_{\rm mem}(X,p)\right]\\&=\frac{\rm i}{2}\int\frac{{\rm d}^4p'}{(2\pi)^4}\frac{1}{d}\left[\widetilde{L}^{+-}(X;p'+p,p)-\widetilde{L}^{-+}(X;p'+p,p)\right]\left\{L^{+-},L^{-+}\right\}_{p',X}\,.\end{aligned} \tag{4.35}$$

The corresponding local collision term is given by Eq. (4.14).

5 H–Theorem

5.1 Time-Irreversibility of the Quantum Four-Phase-Space Kinetic Description

Compared to exact description, which is time-reversible, reduced description schemes in terms of relevant degrees of freedom have access only to some limited information and thus normally lead to irreversibility through the implied coarse graining. Various reduction schemes in particular at the equal time operator level have been discussed and developed over the years. In the Green's function formalism presented here, we have done only three kind of approximations. First, we assumed that the Wick theorem holds in our case. This assumption implies that either the initial state is uncorrelated or initial correlations are quickly dying according to Bogolyubov's condition of weakening of initial correlations [3,7,64]. We further suggested to truncate the series of skeleton diagrams in the Φ functional in order to arrive at tractable Φ-derivable approximations. This truncation, though not equivalent to the truncation in the Martin-Schwinger hierarchy [9] for multi-point Green's functions [52,53], leads to a loss of higher-order correlations beyond a certain level. Finally, the gradient approximation brought us to space time local expression and gradient corrections implying a further loss of information, cf. [44] and refs. therein. All these three steps lead to a scheme, which generally is time-irreversible. Retaining diagrams up to two vertices in Φ together with the gradient approximation leads to Markovian transport equations, void of any memory effects which are time irreversible. However in the special case, when only one-point functions are retained in Φ, which corresponds to the self-consistent Hatree approximation, transport processes are dropped and this scheme is explicitly time-reversible.

[9] The appropriate classical limit leads to the Bogolyubov–Born–Green–Kirkwood–Yvon chain of equations.

At the operator level the nonequilibrium entropy can be straightforwardly formulated in terms of von Neumann's entropy, which is an entropy in the information theory sense [65]

$$S = -\mathrm{Tr}\,\widehat{\rho}\ln\widehat{\rho}. \tag{5.1}$$

It is given by the expectation value of the logarithm of the density operator $\widehat{\rho}$ itself. The problem of the Green function formalism is that it does not give a direct access to the density operator itself but rather describes the space-time dependence of the expectation values of well defined operators, say $\left\langle \widehat{A(t)} \right\rangle = \mathrm{Tr}\,\widehat{A}\,\widehat{\rho}(t)$. To this extend, there is no immediate formulation of the entropy (5.1) in terms of Green functions for the nonequilibrium case. The situation is different at equilibrium, where with $\widehat{\rho} = \exp(-\beta(\widehat{H} - \mu\widehat{N}))/Z$ a well defined density operator exists, which leads to the well known Matsubara or real-time formulation of the equilibrium entropy, cf. sect. 6 below. As known since long time [50], even in equilibrium, the entropy expression is not priori given but rather depends on the choice of Φ in a Φ-derivable scheme.

In order to access a nonequilibrium expression relevant for our quantum four-phase-space transport equation (3.26), we shall start from this transport equation and derive a flow expression s^μ with the property that its divergence grows in time, i.e.

$$\partial_\mu s^\mu(X) \geq 0, \tag{5.2}$$

and which in the equilibrium limit merges the corresponding equilibrium form of the entropy flow. That is, we explicitly show the existence of an H-theorem for our quantum four-phase-space kinetic description and $s^\mu(X)$ is thus identified with the kinetic entropy flow. Thereby, the validity conditions for the derivation of this kinetic entropy flow coincide with those of the kinetic equations themselves.

5.2 Markovian Entropy Flow

We start with general manipulations which lead us to definition of the kinetic entropy flow. We multiply Eq. (3.27) by $(\mp)\ln(\widetilde{F}/A)$, Eq. (3.26) by $-\ln(F/A)$, take their sum, integrate it over $\mathrm{d}^4p/(2\pi)^4$ and finally sum the result over internal degrees of freedom like spin (Tr). Using the identity for the Poisson brackets

$$\{B, Af\}\ln f \pm \{B, A(1\mp f)\}\ln(1\mp f) = \{B, Af\ln f \pm A(1\mp f)\ln(1\mp f)\} \tag{5.3}$$

for any functions A, B and positive f and $1\mp f$, one then arrives at the following relation

$$\partial_\mu s^\mu_{\mathrm{loc}}(X) = \mathrm{Tr}\sum_a \int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}_a}{F_a}C_a(X,p), \tag{5.4}$$

where the quantity

$$s^{\mu}_{\rm loc} = \sum_a s^{\mu}_{{\rm loc},a} = {\rm Tr} \sum_a \int \frac{{\rm d}^4 p}{(2\pi)^4} \left[\left(v^{\mu} - \frac{\partial {\rm Re}\Sigma^R_a}{\partial p_{\mu}} \right) \left(\mp \widetilde{F}_a \ln \frac{\widetilde{F}_a}{A_a} - F_a \ln \frac{F_a}{A_a} \right) \right.$$
$$\left. - {\rm Re} G^R_a \left(\mp \ln \frac{\widetilde{F}_a}{A_a} \frac{\partial}{\partial p_{\mu}} \left(\Gamma_a \frac{\widetilde{F}_a}{A_a} \right) - \ln \frac{F_a}{A_a} \frac{\partial}{\partial p_{\mu}} \left(\Gamma_a \frac{F_a}{A_a} \right) \right) \right] \qquad (5.5)$$

obtained from the l.h.s. of the kinetic equation is interpreted as the local (Markovian) part of the entropy flow. Here we have restored the summation over "a" denoting the different particle species and intrinsic quantum numbers for a multi-component system. It illustrates that, although this entropy expression accounts for interactions among all particles, it can be expressed as a sum of the individual contributions, each of which is solely determined by the particle self-energy ${\rm Re}\Sigma^R_a$ and its width Γ_a. Gradient corrections to the collision term (i.e. $C_{\rm mem}$) give rise to extra memory contributions to the entropy flow.

Partial integrations in Eq. (5.5) lead us to a more transparent expression for the entropy flow in terms of four-phase-space distribution functions $f_a(X,p)$ (cf. Eq.(3.7))

$$s^{\mu}_{\rm loc} = {\rm Tr} \sum_a \int \frac{{\rm d}^4 p}{(2\pi)^4} A^{\mu}_{sa}(X,p)\, \sigma\left(f_a(X,p) \right), \quad \sigma(f) = \mp(1 \mp f) \ln(1 \mp f) - f \ln f, \qquad (5.6)$$

where

$$A^{\mu}_{sa}(X,p) = \frac{A_a \Gamma_a}{2} B^{\mu}_a, \qquad (5.7)$$

the zero component of which A^0_{sa} has the meaning of an entropy-flow spectral function, while the zero component of

$$B^{\mu}_a = A_a \left[\left(v^{\mu} - \frac{\partial {\rm Re}\Sigma^R}{\partial p_{\mu}} \right) - M_a \Gamma_a^{-1} \frac{\partial \Gamma_a}{\partial p_{\mu}} \right], \qquad (5.8)$$

is the flow spectral function, cf. the corresponding drift term (proportional to $\partial_{\mu} f$ in Eq. (3.28)). These entropy-flow and flow spectral functions coincide with the corresponding two functions introduced in refs. [66,67] for the case of equilibrium systems. Moreover, they satisfy the sum rules

$$\int_{-\infty}^{\infty} \frac{{\rm d}p_0}{2\pi} A^0_{sa} = \int_{-\infty}^{\infty} \frac{{\rm d}p_0}{2\pi} B^0_a = \int_{-\infty}^{\infty} \frac{{\rm d}p_0}{2\pi} A_a = 1, \qquad (5.9)$$

which can directly be obtained from the sum rule (3.23) for the spectral function A. For the case of a resonance, like the Δ or ρ-meson resonances in hadron physics, the B^0

function relates to the energy variations of scattering phase shift of the scattering channel coupling to the resonance in the virial limit, for details see e.g. refs. [66,67,1].

In the non-interacting limit, the entropy (the zero component of $s^\mu_{\rm loc}$) directly transforms into the proper ideal gas expression, cf. ref. [68]. In the quasiparticle approximation, the $\Gamma_{\rm in}$ and $\Gamma_{\rm out}$ terms have an additional smallness, which allows to neglect these terms. Thus, expression (5.5) for the entropy flow takes the form

$$(s^\mu_{\rm loc})^{\rm qp} = \left(\begin{array}{c} s_0 \\ \boldsymbol{s} \end{array} \right) = {\rm Tr} \sum_a \int \frac{{\rm d}^3 p}{(2\pi)^3} \left(\begin{array}{c} 1 \\ \dfrac{\partial \varepsilon_a}{\partial \boldsymbol{p}} \end{array} \right) \Big[\mp (1 \mp f_a^{\rm qp}) \ln (1 \mp f_a^{\rm qp}) - f_a^{\rm qp} \ln f_a^{\rm qp} \Big] \quad (5.10)$$

in the quasiparticle limit, which follows from the substitution of Eqs. (3.32) into Eq. (5.5). From (5.6) Eq. (5.10) is also easily recovered, since $A^2\Gamma/2$ transforms to the corresponding δ-function in the limit $\Gamma \to 0$.

To prove explicitly the H-theorem we have to show that the r.h.s. of Eq. (5.4) is non-negative. To this end, we should consider the convolution of the collision term with $\ln(\widetilde{F}/F)$. First, we do this for collision terms in local approximation. For the special case of the Hatree approximation (Φ-diagrams with only one vertex) the collsion term vanishes and the corresponding entropy is exactly conserved.

5.3 Local Collision Term and H-Theorem

Using the multi-particle process decomposition (4.8) we arrive at the relation

$$\begin{aligned} {\rm Tr} \int \frac{{\rm d}^4 p}{(2\pi)^4} \ln \frac{\widetilde{F}}{F} C_{\rm loc}(X,p) = {\rm Tr} \sum_{m,m'} \frac{1}{2} \int \frac{{\rm d}^4 p_1}{(2\pi)^4} \cdots \frac{{\rm d}^4 p_m}{(2\pi)^4} \frac{{\rm d}^4 p'_1}{(2\pi)^4} \cdots \frac{{\rm d}^4 p'_{m'}}{(2\pi)^4} \\ \times \Big\{ F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{m'} - \widetilde{F}_1 \cdots \widetilde{F}_m F'_1 \cdots F'_{m'} \Big\} \ln \frac{F_1 \cdots F_m \widetilde{F}'_1 \cdots \widetilde{F}'_{m'}}{\widetilde{F}_1 \cdots \widetilde{F}_m F'_1 \cdots F'_{m'}} \\ \times R_{m,m'} \; (2\pi)^4 \delta^4 \left(\sum_{i=1}^m p_i - \sum_{i=1}^{m'} p'_i \right) . \end{aligned} \quad (5.11)$$

Here we assumed different flavors and intrinsic quantum numbers to be absorbed in the momenta p_1 and p'_i. In the case when all rates $R_{m,m'}$ are non-negative, i.e. $R_{m,m'} \geq 0$, this expression is non-negative, since $(x-y)\ln(x/y) \geq 0$ for any positive x and y. In particular, $R_{m,m'} \geq 0$ takes place for all Φ-functionals up to two vertices. Then the divergence of $s^\mu_{\rm loc}$ is non-negative which proves the H-theorem in this case with (5.5) as the nonequilibrium entropy flow.

We explicitly discuss the two examples introduced already in sect. 4.3 with Markovian collision terms, i.e. with the Φ functional consisting of one- and two-point diagrams only. In the pure fermionic case with collision term (4.9), one can state an exact H-theorem

$$\mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}}{F}C^{(2)}=d^3\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4p_1}{(2\pi)^4}\frac{\mathrm{d}^4p_2}{(2\pi)^4}\frac{\mathrm{d}^4p_3}{(2\pi)^4}\Big|\quad\Big|^2$$
$$\times(2\pi)^4\delta^4\left(p+p_1-p_2-p_3\right)\ln\frac{\widetilde{F}\widetilde{F}_1F_2F_3}{FF_1\widetilde{F}_2\widetilde{F}_3}\left(\widetilde{F}\widetilde{F}_1F_2F_3-FF_1\widetilde{F}_2\widetilde{F}_3\right)\geq 0. \tag{5.12}$$

Furthermore, it is instructive to consider the Φ approximation (4.17), where a heavy scalar boson has been introduced in order to sum up the series of ring diagrams of Eq. (4.20), cf. subsect. 4.3.2. In this case, there are two coupled kinetic equations with collision terms (4.21) for fermions and (4.22) for heavy bosons, leading to

$$\partial_\mu s^\mu_{\mathrm{loc}}=\partial_\mu s^\mu_{\mathrm{f}}+\partial_\mu s^\mu_{\mathrm{h.b.}}=\int\frac{\mathrm{d}^4p}{(2\pi)^4}\left(d\ln\frac{\widetilde{F}_{\mathrm{f}}}{F_{\mathrm{f}}}C_{\mathrm{f}}+\frac{1}{2}\ln\frac{\widetilde{F}_{\mathrm{h.b.}}}{F_{\mathrm{h.b.}}}C_{\mathrm{h.b.}}\right), \tag{5.13}$$

where s^μ_{loc} is given by the sum of the proper fermion (s^μ_{f}) and heavy-boson ($s^\mu_{\mathrm{h.b.}}$) contributions. The thermodynamic entropy for this system, i.e. for Φ given by diagram (4.17), has recently been obtained in ref. [6]. Here we present the corresponding nonequilibrium entropy flow together with an affirmation of the H-theorem also for this case. The collision term, indeed, becomes

$$\partial_\mu s^\mu_{\mathrm{loc}}=d\int\frac{\mathrm{d}^4p_1}{(2\pi)^4}\frac{\mathrm{d}^4p_2}{(2\pi)^4}\frac{\mathrm{d}^4p_b}{(2\pi)^4}g^2(2\pi)^4\delta^4\left(p_1-p_2-p_b\right)$$
$$\times\ln\frac{\widetilde{F}_1F_2F_{\mathrm{h.b.}}}{F_{\mathrm{f}1}\widetilde{F}_{\mathrm{f}2}\widetilde{F}_{\mathrm{h.b.}}}\left(\widetilde{F}_{\mathrm{f}1}F_{\mathrm{f}2}F_{\mathrm{h.b.}}-F_{\mathrm{f}1}\widetilde{F}_{\mathrm{f}2}\widetilde{F}_{\mathrm{h.b.}}\right)\geq 0, \tag{5.14}$$

which is non-negative.

Our representation of the entropy of a system interacting via two-body potential (i.e. as a sum of a purely fermionic part and that of the artificially introduced heavy boson) is also very similar to that derived by Riedel [49] within the ring-diagram model of Φ-derivable thermodynamics. In both cases, the bosonic part of the entropy $s^\mu_{\mathrm{h.b.}}$ takes account of the fermion–fermion interaction calculated within the ring-diagram approximation (4.20). In thermodynamics, this interaction part of the entropy gives rise to the famous correction to the specific heat of liquid ^{3}He [49–51]: $\sim T^3\ln T$, where T is the temperature. As has been found by Carneiro and Pethick [50], this correction to the specific heat emerges already solely from the third diagram of the whole ring series (4.20). To demonstrate the same within our kinetic approach, we should consider the Φ-derivable model (4.12)

involving only the first three ring diagrams. Moreover, since the local entropy expression (cf. Eq. (5.5)) derived above does not contain such kind of corrections, one has to explicitly consider memory effects in the collision term (4.35).

5.5 Memory Effects in Entropy Flow and H-Theorem

We assume that the fermion–fermion potential interaction is such that the corresponding transition rate (4.15) is always non-negative, so that the H-theorem takes place in the local approximation, i.e. when we keep only $C^{(2)} + C^{(3)}_{\rm loc}$. Our aim now is to derive the entropy, which takes into account memory effects in the collision term ($C^{(3)}_{\rm mem}$).

Proceeding similarly to that in subsect. 5.2, we multiply Eq. (3.27) by $-\ln(\widetilde{F}/A)$, Eq. (3.26) by $-\ln(F/A)$, sum and integrate it over $\mathrm{d}^4p/(2\pi)^4$. Thus, we arrive at the equation

$$\partial_\mu s^\mu_{\rm loc}(X) = \mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}}{F}(C^{(2)} + C^{(3)}_{\rm loc}) + \mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}}{F}C^{(3)}_{\rm mem}, \tag{5.15}$$

where $s^\mu_{\rm loc}$ is still the Markovian entropy flow defined by Eq. (5.5). Our aim here is to present the last term on the r.h.s. of Eq. (5.15) in the form of full x-derivative

$$\mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}}{F}C^{(3)}_{\rm mem} = -\partial_\mu s^\mu_{\rm mem}(X) + \delta c_{\rm mem}(X) \tag{5.16}$$

of some function $s^\mu_{\rm mem}(X)$, which we then interpret as a non-Markovian correction to the entropy flow of Eq. (5.5), plus a correction ($\delta c_{\rm mem}$) which is small in some sense. Indeed, this term on the r.h.s. of Eq. (5.16) is linear in X- and p-derivatives. Hence, it cannot be transformed into sign-definite form. The only possibility which is left is to construct a full derivative out of it. If we succeed to find a proper $s^\mu_{\rm mem}(X)$, then relying on smallness of $\delta c_{\rm mem}$ we obtain

$$\partial_\mu\,(s^\mu_{\rm loc} + s^\mu_{\rm mem}) \simeq \mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}}{F}(C^{(2)} + C^{(3)}_{\rm loc}) \geq 0, \tag{5.17}$$

which is the H-theorem for the non-Markovian kinetic equation under consideration with $s^\mu_{\rm loc} + s^\mu_{\rm mem}$ as the proper entropy flow. The r.h.s. of Eq. (5.17) is non-negative due to our assumption that the corresponding transition rate (4.15) is always non-negative.

Hence, considering the last term on the r.h.s. of Eq. (5.15) we substitute expression (4.35) for $C^{(3)}_{\rm mem}$, shift the integration variable $p \to p - p'/2$ and arrive at

$$\mathrm{Tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\ln\frac{\widetilde{F}}{F}C^{(3)}_{\rm mem} = \frac{\mathrm{i}}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}^4p'}{(2\pi)^4}\left[\widetilde{L}^{+-}(X;p+\frac{p'}{2},p-\frac{p'}{2}) - \widetilde{L}^{-+}(X;p+\frac{p'}{2},p-\frac{p'}{2})\right]$$

$$\times \left\{ L^{+-}(X,p'), L^{-+}(X,p') \right\}_{p',X} \ln \frac{\widetilde{F}(X,p-p'/2)}{F(X,p-p'/2)}. \tag{5.18}$$

By taking the average of Eq. (5.18) with that with p' and $-p'$ interchanged we arrive at the symmetric form of this equation

$$\begin{aligned} \mathrm{Tr} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln \frac{\widetilde{F}}{F} C^{(3)}_{\mathrm{mem}} = \frac{\mathrm{i}}{4} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{d}^4 p'}{(2\pi)^4} \left[\widetilde{L}^{-+}(X; p + \frac{p'}{2}, p - \frac{p'}{2}) - \widetilde{L}^{+-}(X; p + \frac{p'}{2}, p - \frac{p'}{2}) \right] \\ \times \{ L^{-+}(X,p'), L^{+-}(X,p') \}_{p',X} \ln \frac{\widetilde{L}^{+-}(X; p + p'/2, p - p'/2)}{\widetilde{L}^{-+}(X; p + p'/2, p - p'/2)}, \end{aligned} \tag{5.19}$$

which after simple algebraic transformations can be decomposed into two terms

$$\mathrm{Tr} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln \frac{\widetilde{F}}{F} C^{(3)}_{\mathrm{mem}} = c_{\mathrm{mem}} + \delta c_{\mathrm{mem}}, \tag{5.20}$$

with

$$\begin{aligned} c_{\mathrm{mem}}(X) = \frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{d}^4 p'}{(2\pi)^4} \left\{ L^{-+}(X,p'), L^{+-}(X,p') \widetilde{L}^{-+} \left(X; p + \frac{p'}{2}, p - \frac{p'}{2} \right) \right. \\ \left. \times \left[\ln \frac{\widetilde{L}^{+-}(X; p + p'/2, p - p'/2)}{\widetilde{L}^{-+}(X; p + p'/2, p - p'/2)} - 1 \right] \right\}_{p',X}, \end{aligned} \tag{5.21}$$

$$\begin{aligned} \delta c_{\mathrm{mem}}(X) = \frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{d}^4 p'}{(2\pi)^4} \left[\left\{ L^{-+}(X,p'), L^{+-}(X,p') \right\}_{p',X} \widetilde{L}^{-+} \left(X; p + \frac{p'}{2}, p - \frac{p'}{2} \right) \right. \\ - \left\{ L^{-+}(X,p'), \widetilde{L}^{-+} \left(X; p + \frac{p'}{2}, p - \frac{p'}{2} \right) \ln \frac{\widetilde{L}^{+-}(X; p + p'/2, p - p'/2)}{\widetilde{L}^{-+}(X; p + p'/2, p - p'/2)} \right\}_{p',X} \\ \left. \times L^{+-}(X,p') \right]. \end{aligned} \tag{5.22}$$

Using partial integration, we have subdivided the quantities in Eq. (5.20) in such a way that the first term c_{mem} takes the form of the full divergence, and thus defines the non-Markovian contribution to the entropy flow (cf. Eq. (5.16))

$$\begin{aligned} s^{\nu}_{\mathrm{mem}}(X) = -\frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{d}^4 p'}{(2\pi)^4} \widetilde{L}^{-+} \left(X; p + \frac{p'}{2}, p - \frac{p'}{2} \right) L^{+-}(X,p') \\ \times \left[\ln \frac{\widetilde{L}^{+-}(X; p + p'/2, p - p'/2)}{\widetilde{L}^{-+}(X; p + p'/2, p - p'/2)} - 1 \right] \frac{\partial L^{-+}(X,p')}{\partial p'_{\nu}}. \end{aligned} \tag{5.23}$$

This $c_{\rm mem}$-term remains non-zero even in local thermal equilibrium, which is defined by the former equilibrium relations (B.1)–(B.4) but with temperature $T(X)$, 4-velocity $U^\nu(X)$ and chemical potential $\mu(X)$ depending on the coordinate X. On the other hand, as we show below, the second term $\delta c_{\rm mem}$ vanishes in the limit of local thermal equilibrium.

In local thermal equilibrium the Kubo-Martin-Schwinger condition (B.1) provides the following relations

$$\left(\frac{\widetilde{L}^{+-}(X;p+p'/2,p-p'/2)}{\widetilde{L}^{-+}(X;p+p'/2,p-p'/2)}\right)_{\rm loc.Eq.} = \left(\frac{L^{+-}(X;p')}{L^{-+}(X,p')}\right)_{\rm loc.Eq.} = \exp\left(\frac{p'_\nu U^\nu(X)}{T(X)}\right), \quad (5.24)$$

which can also be derived from (B.2)–(B.5) proceeding from definitions of L^{ij} (4.26) and $\widetilde{L}^{ij}$ (4.27). Guided by (5.24) we write the $\widetilde{L}^{ij}$ ratio in the ln-term of (5.22) as

$$\frac{\widetilde{L}^{+-}(X;p+p'/2,p-p'/2)}{\widetilde{L}^{-+}(X;p+p'/2,p-p'/2)} = \frac{L^{+-}(X;p')}{L^{-+}(X,p')}(1+\xi), \quad (5.25)$$

where ξ is expected to be small within the validity range of the quantum four-phase-space kinetic equation (3.26), i.e. $|\xi| \approx |f-\gamma|$. Substituting this into expression (5.22) for $\delta c_{\rm mem}$, the $\ln(L^{-+}/L^{-+})$-term in the second Poisson bracket cancels against the first term, since the p-integration converts the linear $\widetilde{L}^{ij}$ factor into L^{ij}. Thus one obtains

$$\begin{aligned}\delta c_{\rm mem} &= -\frac{\rm i}{2}\int \frac{{\rm d}^4p}{(2\pi)^4}\frac{{\rm d}^4p'}{(2\pi)^4}\left\{L^{-+},\widetilde{L}^{-+}\ln(1+\xi)\right\}_{p',X} L^{+-} \\ &\simeq \frac{\rm i}{2}\int \frac{{\rm d}^4p}{(2\pi)^4}\frac{{\rm d}^4p'}{(2\pi)^4}\left\{L^{-+},\widetilde{L}^{-+}\frac{1}{2}\xi^2\right\}_{p',X} L^{+-}, \end{aligned} \quad (5.26)$$

where also the term linear in ξ exactly cancels out, since

$$\begin{aligned}&-\frac{\rm i}{2}\int \frac{{\rm d}^4p}{(2\pi)^4}\frac{{\rm d}^4p'}{(2\pi)^4}\left\{L^{-+},\widetilde{L}^{-+}\xi\right\}_{p',X} L^{+-} \\ &= -\frac{\rm i}{2}\int \frac{{\rm d}^4p}{(2\pi)^4}\frac{{\rm d}^4p'}{(2\pi)^4}\left\{L^{-+},\left(\frac{L^{-+}}{L^{+-}}\widetilde{L}^{+-}-\widetilde{L}^{-+}\right)\right\}_{p',X} L^{+-} = 0. \end{aligned} \quad (5.27)$$

As above, the $\widetilde{L}^{ij}$ can be readily integrated over p to produce L^{ij}, revealing a cancelation of the terms in the Poisson bracket in (5.27). Thus, $\delta c_{\rm mem}$ is not only zero in local equilibrium ($\xi=0$). It is of the second-order in the small parameter $|\xi| \approx |f-\gamma|$ times gradients and therefore negligible compared to c_{mem} in (5.21) which is of linear order in $|\xi| \approx |f-\gamma|$ times gradients. Thus within the validity of the quantum four-phase-space kinetic equation (3.26) $s^\nu_{\rm mem}(X)$ as given in (5.23) represents the appropriate non-Markovian memory

correction to the entropy flow. In local equilibrium this term can be further simplified to

$$(s^{\nu}_{\rm mem})_{\rm eq} = \frac{\rm i}{2}\int \frac{{\rm d}^4p'}{(2\pi)^4} L^{-+}(X;p')L^{+-}(X,p')\left[\ln\frac{L^{+-}(X,p')}{L^{-+}(X,p')} - 1\right]\frac{\partial L^{-+}(X,p')}{\partial p'_\nu} \quad (5.28)$$

by means of relation (5.24).

6 Thermodynamic Limit of Entropy

6.1 Thermodynamic Entropy

In the Matsubara technique the thermodynamic potential Ω (see, e.g., ref. [35]) is a functional of Matsubara Green functions $G({\rm i}\varepsilon_n, \boldsymbol{p})$. Its important property is that in the form analogous to (2.3) it is stationary under variations of $G({\rm i}\varepsilon_n, \boldsymbol{p})$ at fixed free Matsubara Green function $G^0({\rm i}\varepsilon_n, \boldsymbol{p})$

$$\left(\frac{\delta\Omega}{\delta G({\rm i}\varepsilon_n, \boldsymbol{p})}\right)_{G^0} = 0. \quad (6.1)$$

Since $G({\rm i}\varepsilon_n, \boldsymbol{p})$ has the spectral representation

$$G({\rm i}\varepsilon_n, \boldsymbol{p}) = \int\limits_{-\infty}^{\infty} \frac{{\rm d}\varepsilon'}{2\pi}\frac{A(\varepsilon', \boldsymbol{p})}{{\rm i}\varepsilon_n - \varepsilon'}, \quad (6.2)$$

property (6.1) implies that Ω is stationary under variations of the spectral density $A(\varepsilon, \boldsymbol{p})$ keeping the Matsubara frequencies ε_n and, thus, the free Matsubara Green function $G^0({\rm i}\varepsilon_n, \boldsymbol{p})$ unaltered.

In the standard way, cf. (B.8)–(B.10), the Matsubara sum over ε_n is converted into an energy integral over the distribution functions $n(\varepsilon - \mu)$, cf. (B.4). The thermodynamic potential expressed in terms of the real-time Green functions and self-energies (cf. Eqs. (B.2) and (B.3)) then becomes [50,1]

$$\Omega = {\rm Tr}\int {\rm d}^3x \frac{{\rm d}^4p}{(2\pi)^4} n(\varepsilon - \mu)\left[-2{\rm Im}\ \ln\left(-G^R\right) - {\rm Re}G^R\Gamma - A{\rm Re}\Sigma^R\right] + \Phi_T \quad (6.3)$$

where $\varepsilon = p_0$ in the rest frame of the equilibrated system and Φ_T is represented by the same set of closed diagrams as Φ. Due to the stationarity property of Ω, only the explicit

T-dependences of the Matsubara frequencies or in the occupations $n(\varepsilon-\mu)$ in the integral formulation (6.3) are to be taken into account in calculating the entropy from

$$S = -\left(\partial\Omega/\partial T\right)_{\mu}/V, \tag{6.4}$$

where V is the volume of the system. Thus one finds

$$S = -\mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\partial n(\varepsilon-\mu)}{\partial T}\left[-2\mathrm{Im}\ \ln\left(-G^R\right) - \mathrm{Re}G^R\Gamma - A\mathrm{Re}\Sigma^R\right] - \frac{\partial\Phi_T}{\partial T} \tag{6.5}$$

for the entropy density. With the help of the identity (B.6) we replace $\partial n(\varepsilon-\mu)/\partial T$ in Eq. (6.5) and then perform partial integration over p_0 and obtain

$$S = S_{\mathrm{loc}} + S_{\mathrm{mem}}, \qquad \text{where} \tag{6.6}$$

$$S_{\mathrm{loc}} = -\mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\,\sigma\left(n(\varepsilon-\mu)\right)\frac{\partial}{\partial p_0}\left[-2\mathrm{Im}\ \ln(-G^R) - \mathrm{Re}G^R\Gamma\right], \tag{6.7}$$

$$S_{\mathrm{mem}} = -\frac{\partial\Phi}{\partial T} + \mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\,\sigma\left(n(\varepsilon-\mu)\right)\frac{\partial\left(A\mathrm{Re}\Sigma^R\right)}{\partial p_0}, \tag{6.8}$$

$$\sigma(n) = \mp(1\mp n)\ln(1\mp n) - n\ln n. \tag{6.9}$$

We have used subscripts "local" and "memory" to denote these two different contributions, because below we demonstrate that they are indeed associated with the local (Markovian) s^0_{loc}, cf. Eq. (5.5), and the memory (non-Markovian) s^0_{mem}, cf. Eq. (5.23), parts of the kinetic entropy. Taking derivatives in Eq. (6.7), we readily get

$$S_{\mathrm{loc}} = \mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\,\sigma\left(n(\varepsilon-\mu)\right)A^0_s(p) \tag{6.10}$$

with A^0_s defined in Eq. (5.7). Thus, S_{loc}, indeed, coincides with 0-component of the kinetic entropy flow (5.6).

In order to clarify the meaning of the values S_{loc} and S_{mem} we first inspect the quasiparticle limit, in which the spectral function reduces to a delta-function. In this limit, the value (6.10) for S_{loc} is given by

$$S^{\mathrm{qp}}_{\mathrm{loc}} = \mathrm{Tr}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\,\sigma\left(n\left(\varepsilon(\boldsymbol{p})-\mu\right)\right), \tag{6.11}$$

where the quasiparticle energy $\varepsilon(\boldsymbol{p})$ is determined by solution of the dispersion equation (3.33). The full $S^{\mathrm{qp}}_{\mathrm{loc}}$ is just the sum of single-particle contributions, as if one deals with a non-interacting ideal gas of quasiparticles. This is the standard picture in the quasiparticle approximation. Corrections to $S^{\mathrm{qp}}_{\mathrm{loc}}$, resulting from S_{loc}, are of higher order in the width

Γ. At the same time, $S_{\rm mem}$ provides corrections to $S^{\rm qp}_{\rm loc}$ even at the zero-order level in Γ, which are associated with real rescatterings of on-mass-shell quasiparticles. This fact was demonstrated by Carneiro and Pethick in ref. [50]. For Fermi liquids, they showed that the first Φ diagram contributing to $S_{\rm mem}$ is the triangle $\Phi^{(3)}$ of (4.12). Contributions from $\Phi^{(1)}$ and $\Phi^{(2)}$ are zero. In the quasiparticle approximation, the contribution from $\Phi^{(3)}$ is (cf. Eqs. (45) and (72) of ref. [50])

$$S^{\rm qp}_{\rm mem} = \frac{1}{12}\int \frac{{\rm d}^4p}{(2\pi)^4}\left(\Gamma^{\rm qp}_{\rm b}(p)\right)^3 \frac{\partial n_{\rm b}(p_0)}{\partial T}, \tag{6.12}$$

where $n_{\rm b}$ is the thermal occupation (B.4) number of the artificial boson. The width $\Gamma_{\rm b} = -2{\rm Im}L^R$ of the bosonic loop (4.25) is given by

$$\Gamma_{\rm b}(p) = V_0 d\int \frac{{\rm d}^4p'}{(2\pi)^4}\left[n_{\rm f}\left(\varepsilon' - \frac{1}{2}p_0\right) n_{\rm f}\left(\varepsilon' + \frac{1}{2}p_0\right)\right] A_{\rm f}\left(p' - \frac{1}{2}p\right) A_{\rm f}\left(p' + \frac{1}{2}p\right) \tag{6.13}$$

with $\varepsilon' = p'_0 - \mu$ and the $n_{\rm f}$ are the thermal fermion occupations (B.4). As above, V_0 stands for the strength of the two-body potential. To get the quasiparticle approximation to this width ($\Gamma^{\rm qp}_{\rm b}$), one should replace the exact spectral function $A_{\rm f}$ in (6.13) by its quasiparticle approximation $A^{\rm qp}_{\rm f}$. Note that now $S^{\rm qp}_{\rm mem}$ is expressed in terms of bosonic quantities $n_{\rm b}$ and $\Gamma_{\rm b}$, although initially we have started with a purely fermionic system with two-body interaction. This fact provides the link to the thermodynamic calculation of Riedel [49], where the correction to the standard quasiparticle entropy of Fermi liquids is presented in the form of an effective boson contribution re-summing the entire series of ring diagrams rather than considering only the first three of them as in Eq. (4.12). At low temperatures, $S^{\rm qp}_{\rm mem} \sim T^3 \ln T$ [50] gives the leading correction to the standard quasiparticle entropy. This is the famous correction to the specific heat of liquid ^{3}He [51,49,50]. Since this correction is quite comparable (numerically) to the leading term in the specific heat ($\sim T$), one may claim that liquid ^{3}He is a liquid with quite strong memory effects from the point of view of kinetics.

Note that using the thermodynamic relation $E + PV - \mu N = TSV$ for homogenous systems a different but even simpler form of the thermodynamic entropy including memory corrections follows from Eq. (3.40)

$$TS = {\rm Tr}\int \frac{{\rm d}^4p}{(2\pi)^4} A(p)\, n(p_0 - \mu)\left(p_0 + \frac{2}{3}\epsilon^0(\boldsymbol{p}) - \mu\right) \tag{6.14}$$

with the free single-particle energy $\epsilon^0(\boldsymbol{p}) = \boldsymbol{p}^2/(2m)$ in non-relativistic kinematics.

We now evaluate the memory correction $(s^\nu_{\rm mem})_{\rm eq}$ to the kinetic entropy flow (see Eq. (5.28)) in thermal equilibrium. Proceeding from the definition of L^{ij} (4.26), as well as from equilibrium relations (B.2)–(B.4), and identity (B.5), we present L^{ij} in the form

$$L^{-+}(p) = \mathrm{i} n_{\rm b}(\omega)\Gamma_{\rm b}(p), \quad L^{+-}(p) = \mathrm{i}\left[1+n_{\rm b}(\omega)\right]\Gamma_{\rm b}(p), \tag{6.15}$$

where $\omega = p_\nu U^\nu$, $\Gamma_{\rm b}$ is defined by Eq. (6.13), and $n_{\rm b}$ is the bosonic occupation number. Now, $(s^\nu_{\rm mem})_{\rm eq}$ of Eq. (5.28) takes the form

$$\begin{aligned}(s^\nu_{\rm mem})_{\rm eq} &= -\frac{1}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4} n_{\rm b}(\omega)\left[1+n_{\rm b}(\omega)\right]\Gamma^2_{\rm b}(p)\left(\frac{\omega}{T}-1\right)\frac{\partial}{\partial p_\nu}\left[n_{\rm b}(\omega)\Gamma_{\rm b}(p)\right] &(6.16)\\ &= -\frac{1}{2}\int \frac{\mathrm{d}^4p}{(2\pi)^4} n_{\rm b}\left(1+n_{\rm b}\right)\left(\frac{\omega}{T}-1\right)\left(\Gamma^3_{\rm b}\frac{\partial n_{\rm b}}{\partial p_\nu}+\frac{1}{3}\frac{\partial \Gamma^3_{\rm b}}{\partial p_\nu}n_{\rm b}\right)\\ &= \frac{1}{6}U^\nu\int \frac{\mathrm{d}^4p}{(2\pi)^4}\Gamma^3_{\rm b}\left[n^2_{\rm b}\left(1+n_{\rm b}\right)\frac{\omega}{T^2}\right]. &(6.17)\end{aligned}$$

The last line is obtained through partial integration, explicitly taking the derivatives of the bosonic occupations and using the equilibrium property $\mathrm{d}n_{\rm b}/\mathrm{d}\omega = -n_{\rm b}(1+n_{\rm b})/T$. Now, we change the integration variable $p \to -p$ in the last line of Eq. (6.17) and use the parity properties valid for bosonic loops, phonons or relativistic bosons

$$\Gamma_{\rm b}(-\omega) = -\Gamma_{\rm b}(\omega), \quad n_{\rm b}(-\omega) = -\left[1+n_{\rm b}(\omega)\right], \tag{6.18}$$

and arrive at

$$(s^\nu_{\rm mem})_{\rm eq} = -\frac{1}{6}U^\nu\int \frac{\mathrm{d}^4p}{(2\pi)^4}\Gamma^3_{\rm b}\left[n_{\rm b}\left(1+n_{\rm b}\right)^2\frac{\omega}{T^2}\right]. \tag{6.19}$$

Taking the average of the r.h.s. in (6.19) and the last line of Eq. (6.17) and using the identity (B.7) we finally arrive at

$$(s^\nu_{\rm mem})_{\rm eq} = \frac{1}{12}U^\nu\int \frac{\mathrm{d}^4p}{(2\pi)^4}\Gamma^3_{\rm b}\frac{\partial n_{\rm b}}{\partial T}. \tag{6.20}$$

The 0-component of (6.20) precisely coincides with the thermodynamic quantity $S^{\rm qp}_{\rm mem}$ given by Eq. (6.12) in the quasiparticle approximation, provided we consider it in the rest frame of the matter, i.e. at $U^\nu = \{1, \mathbf{0}\}$. This fact again justifies the label "memory" for the thermodynamic quantity $S_{\rm mem}$.

Thus, we have demonstrated that our kinetic entropy, including memory contributions, coincides with the thermodynamic entropy in thermal equilibrium.

7 Conclusion and Prospects

We suggested to follow Baym's Φ-derivable principle to construct consistent transport theories which also apply to unstable particles. The Φ-derivable scheme has a couple of important conceptual advantages [33]. First, it leads to closed, i.e. self-consistent equations which can be closed at any order or loop level of the diagrams of Φ, this way defining an effective theory. We showed in ref. [1] that the original properties of Φ are also valid for genuine nonequilibrium systems described within the real-time formalism, namely that the so constructed approximation is conserving and at the same time thermodynamically consistent. The Φ-derivable energy momentum tensor has explicitly been constructed for the contour Dyson equation. As further shown in ref. [1] the scheme can be easily extended to include relativistic kinematics and dynamical classical boson fields, such as mean fields or condensates. The latter permit to include soft modes in terms of such classical fields, much in the spirit of the kinetic picture [69,70,13] of hard thermal loop approximations [71].

In this paper we showed that the conserving properties of a Φ-derivable scheme also hold for the quantum four-phase-space kinetic equations, if all phase-space distribution functions vary slowly across the space-time region. The usual restriction to small mass widths (mass-shell condition) for the particles involved was avoided. We demonstrated that the mass-shell equation is exactly equivalent to the quantum four-phase-space kinetic equation in the BM-choice while deviates from that of the KB-choice in second-order gradient terms. Besides the usual drift and collision terms, present in any transport equation, like in Landau's Fermi liquid theory [56,35,51], a genuine width and a fluctuation-dependent term appear in the quantum four-phase-space kinetic equation. The latter term, normally dropped within the quasiparticle approximation, gives rise to a back-flow contribution in the currents and produces width and fluctuation dependent contributions to the energy momentum. This term is indeed essential in order to preserve the conservation laws in the case of broad damping widths. Along with the kinetic equation a local retarded equation has to be solved which provides the dynamical information about the spectral functions of the particles.

The structure of the collision term was studied by means representing the Φ functional in terms of "$-+$" and "$+-$" Green functions which represent Wigner phase-space densities [13]. The advantage of the "$-+$" and "$+-$" representation is that it leads to a natural decomposition of the collision term into multi-particle processes with Feynman transition amplitudes which determine the partial rates. Furthermore, it has been discussed that sometimes it appears advantageous to account for such memory effects by including new "artificial" particles (e.g. bosonization of particle-hole excitations) which then lead to local collision terms.

We also addressed the question whether a closed nonequilibrium system approaches the thermodynamic equilibrium during its evolution. Investigating the structure of the collision term in the Φ-derivable scheme we obtained definite expressions for a local (Markovian) entropy flow and were able to explicitly demonstrate the H-theorem for some of the common choices of Φ approximations. The expression for the local entropy flow holds beyond the quasiparticle picture, and thus generalizes the well-known Boltzmann expression for the kinetic entropy. To demonstrate memory effects in the quantum four-phase-space kinetics, we considered a particular case of a system of fermions interacting via two-body zero-range potential. We calculated the memory (non-Markovian) contribution to the kinetic entropy, which merges the equilibrium limit with its famous correction to the specific heat of liquid ^{3}He [49–51]: $\sim T^3 \ln T$.

Mass-width effects are important for description of various physical systems. As for immediate applications of the developed formalism, we see the description of wide resonances (such as ρ-meson, Δ-resonance, etc.) in nonequilibrium hadron matter produced in heavy-ion collisions. Since the widths of these resonances are of the order (or even larger) than the mean excitation energy per particle, a self-consistent treatment of such widths effects is required. Up to now, width effects were considered either within some simplified dynamics with phenomenological Landau–Migdal residual interaction [11,12] or within a simple Φ-derivable approximation at thermal equilibrium and in the dilute gas limit [66]. In particular, it was demonstrated in ref. [13], that the soft-photon production is sensitive to dynamical and width effects. The interplay between the width and the in-medium population of ρ-mesonic states may also simulate a shift of the ρ-meson mass in the nuclear medium, and thus may affect the production of di-leptons in relativistic heavy-ion collisions. It is of interest to study these effects within a dynamical approach, such as the scheme presented here.

Further applications concern relativistic plasmas, like QCD and QED plasmas. The plasma of deconfined quarks and gluons was present in the early Universe, it may exist in cores of massive neutron stars, and may also be produced in laboratory in ultra-relativistic nucleus–nucleus collisions. All these systems need a proper treatment of particle transport. Perturbative descriptions of soft-quanta propagation suffers from infrared divergences and one needs a systematic study of the mass-width effects in order to treat them, cf. ref. [13]. A thermodynamic Φ-derivable approximation for hot relativistic QED plasmas—a gas of electrons and positrons in a thermal bath of photons—was recently considered by Vanderheyden and Baym [6]. Their treatment may also be applied to the high-temperature super-conductors and the fractional quantum Hall effect [72,73]. Our approach allows for a natural generalization of such a Φ-derivable schemes to the dynamical case.

Another application, as we see it, concerns the description of the neutrino transport in supernovas and hot neutron stars during first few minutes of their evolution. At an initial stage, neutrinos typically of thermal energy, produced outside (in the mantel) and inside the neutron-star core, are trapped within the regions of production. However, coherent effects in neutrino production and their rescattering on nucleons [13] reduce the opacity of the nuclear-medium and may allow for soft neutrinos to escape the core and contribute

to the heating of the mantle. The extra energy transport may be sufficient to blow off the supernova's mantle in the framework of the shock-reheating mechanism [74]. The description of the neutrinos transport in the semi-transparent region should therefore be treated with the due account of mass-widths effects.

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APPENDICES

A Matrix Notation

In calculations that apply the Wigner transformations, it is necessary to decompose the full contour into its two branches—the *time-ordered* and *anti-time-ordered* branches. One then has to distinguish between the physical space-time coordinates $x, \ldots$ and the corresponding contour coordinates $x^{\mathcal{C}}$ which for a given x take two values $x^- = (x^-_\mu)$ and $x^+ = (x^+_\mu)$ $(\mu \in \{0,1,2,3\})$ on the two branches of the contour (see figure 1). Closed real-time contour integrations can then be decomposed as

$$\int\limits_{\mathcal{C}} \mathrm{d}x^{\mathcal{C}} \ldots = \int\limits_{t_0}^{\infty} \mathrm{d}x^- \ldots + \int\limits_{\infty}^{t_0} \mathrm{d}x^+ \ldots = \int\limits_{t_0}^{\infty} \mathrm{d}x^- \ldots - \int\limits_{t_0}^{\infty} \mathrm{d}x^+ \ldots , \tag{A.1}$$

where only the time limits are explicitly given. The extra minus sign of the anti-time-ordered branch can conveniently be formulated by a $\{-+\}$ "metric" with the metric tensor in $\{-+\}$ indices

$$\left(\sigma^{ij}\right) = \left(\sigma_{ij}\right) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.2}$$

which provides a proper matrix algebra for multi-point functions on the contour with "co"- and "contra"-contour values. Thus, for any two-point function F, the contour values are defined as

$$\begin{aligned} F^{ij}(x,y) &:= F(x^i, y^j), \quad i,j \in \{-,+\}, \quad \text{with} \\ F_i{}^j(x,y) &:= \sigma_{ik} F^{kj}(x,y), \quad F^i{}_j(x,y) := F^{ik}(x,y)\sigma_{ki} \\ F_{ij}(x,y) &:= \sigma_{ik}\sigma_{jl} F^{kl}(x,y), \quad \sigma_i^k = \delta_{ik} \end{aligned} \tag{A.3}$$

on the different branches of the contour. Here summation over repeated indices is implied. Then contour folding of contour two-point functions, e.g. in Dyson equations, simply becomes

$$H(x^i, y^k) = H^{ik}(x,y) = \int_{\mathcal{C}} \mathrm{d}z^{\mathcal{C}} F(x^i, z^{\mathcal{C}}) G(z^{\mathcal{C}}, y^k) = \int \mathrm{d}z F^i{}_j(x,z) G^{jk}(z,y) \tag{A.4}$$

in the matrix notation.

For any multi-point function the external point x_{max}, which has the largest physical time, can be placed on either branch of the contour without changing the value, since the contour-time evolution from x^-_{max} to x^+_{max} provides unity. Therefore, one-point functions have the same value on both sides on the contour.

Due to the change of operator ordering, genuine multi-point functions are, in general, discontinuous, when two contour coordinates become identical. In particular, two-point functions like $\mathrm{i}F(x,y) = \left\langle \mathcal{T}_{\mathcal{C}} \hat{A}(x) \hat{B}(y) \right\rangle$ become

$$\mathrm{i}F(x,y) = \begin{pmatrix} \mathrm{i}F^{--}(x,y) & \mathrm{i}F^{-+}(x,y) \\ \mathrm{i}F^{+-}(x,y) & \mathrm{i}F^{++}(x,y) \end{pmatrix} = \begin{pmatrix} \left\langle \mathcal{T} \hat{A}(x) \hat{B}(y) \right\rangle & \mp \left\langle \hat{B}(y) \hat{A}(x) \right\rangle \\ \left\langle \hat{A}(x) \hat{B}(y) \right\rangle & \left\langle \mathcal{T}^{-1} \hat{A}(x) \hat{B}(y) \right\rangle \end{pmatrix}, \tag{A.5}$$

where $\mathcal{T}$ and $\mathcal{T}^{-1}$ are the usual time and anti-time ordering operators. Since there are altogether only two possible orderings of the two operators, in fact given by the Wightman functions F^{-+} and F^{+-}, which are both continuous, not all four components of F are independent. Eq. (A.5) implies the following relations between nonequilibrium and usual retarded and advanced functions

$$\begin{aligned} F^R(x,y) &= F^{--}(x,y) - F^{-+}(x,y) = F^{+-}(x,y) - F^{++}(x,y) \\ &:= \Theta(x_0 - y_0) \left(F^{+-}(x,y) - F^{-+}(x,y) \right), \\ F^A(x,y) &= F^{--}(x,y) - F^{+-}(x,y) = F^{-+}(x,y) - F^{++}(x,y) \\ &:= -\Theta(y_0 - x_0) \left(F^{+-}(x,y) - F^{-+}(x,y) \right), \end{aligned} \tag{A.6}$$

where $\Theta(x_0 - y_0)$ is the step function of the time difference. The rules for the co-contour functions F_{--} etc. follow from Eq. (A.3).

For such two point functions complex conjugation implies

$$\begin{aligned}
\Big(\mathrm{i}F^{-+}(x,y)\Big)^* &= \mathrm{i}F^{-+}(y,x) \quad &\Rightarrow \quad & \mathrm{i}F^{-+}(X,p) = \text{real}, \\
\Big(\mathrm{i}F^{+-}(x,y)\Big)^* &= \mathrm{i}F^{+-}(y,x) \quad &\Rightarrow \quad & \mathrm{i}F^{+-}(X,p) = \text{real}, \\
\Big(\mathrm{i}F^{--}(x,y)\Big)^* &= \mathrm{i}F^{++}(y,x) \quad &\Rightarrow \quad & \Big(\mathrm{i}F^{--}(X,p)\Big)^* = \mathrm{i}F^{++}(X,p), \\
\Big(F^{R}(x,y)\Big)^* &= F^{A}(y,x) \quad &\Rightarrow \quad & \Big(F^{R}(X,p)\Big)^* = F^{A}(X,p),
\end{aligned} \tag{A.7}$$

where the right parts specify the corresponding properties in the Wigner representation. Diagrammatically these rules imply the simultaneous swapping of all + vertices into − vertices and vice versa together with reversing the line arrow-sense of all propagator lines in the diagram.

In components the determination of the self-energy from the functional variation of Φ (cf. Eq. (2.7)) reads

$$-\mathrm{i}\Sigma_{ik}(x,y) = \mp\frac{\delta \mathrm{i}\Phi}{\delta \mathrm{i}G^{ki}(y,x)} \quad \Rightarrow \quad -\mathrm{i}\Sigma_{ik}(X,p) = \mp\frac{\delta \mathrm{i}\Phi}{\delta \mathrm{i}G^{ki}(X,p)}, \quad i,k \in \{-+\} \tag{A.8}$$

the right expression given in the Wigner representation. Note that the variation over a $\{+-\}$-"contra-variant" Green function G^{ki} produces a $\{+-\}$-"covariant" self-energy Σ_{ik}, cf. (A.3). This occurs due to the same reason discussed above, cf. Eq. (A.1), when dealing with $\{+-\}$ matrix notation with integrations over physical times, rather than contour times. The extra minus signs occurring for the anti-time ordered branches are precisely taken into account by the "covariant" notation (A.3). Fuctional variation, e.g. in Eqs. (A.8), differ for functions in coordinate and momentum space

$$\frac{\delta f(x')}{\delta f(x)} = \delta^4(x - x'), \qquad \frac{\delta f(p')}{\delta f(p)} = (2\pi)^4\delta^4(p - p'), \tag{A.9}$$

because of different integration measures d^4x and $\mathrm{d}^4p/(2\pi)^4$, respectively.

B Equilibrium Relations

For completeness of the thermodynamic consideration, we explicitly present here equilibrium relations between quantities on the real-time contour. Basically, they follow from the Kubo–Martin–Schwinger condition [75]

$$G^{-+}(p) = \mp G^{+-}(p) e^{-(\varepsilon-\mu)/T}, \quad \Sigma^{-+}(p) = \mp \Sigma^{+-}(p) e^{-(\varepsilon-\mu)/T}, \tag{B.1}$$

where $\varepsilon = p_\nu U^\nu$, while U^ν and μ are a global 4-velocity of the system and a chemical potential related to a conserved charge, respectively. All the Green functions can be expressed through the retarded and advanced Green functions

$$G^{ik}(p) = \begin{pmatrix} G^R(p) \pm \mathrm{i}nA(p) & \pm \mathrm{i}nA(p) \\ -\mathrm{i}\left[1 \mp n\right] A(p) & -G^A(p) \pm \mathrm{i}nA(p) \end{pmatrix}, \tag{B.2}$$

i, k mean $+$ or $-$, and the self-energies take a similar form

$$\Sigma_{ik}(p) = \begin{pmatrix} \Sigma^R(p) \pm \mathrm{i}n\Gamma(p) & \mp \mathrm{i}n\Gamma(p) \\ \mathrm{i}\left[1 \mp n\right] \Gamma(p) & -\Sigma^A(p) \pm \mathrm{i}n\Gamma(p) \end{pmatrix}. \tag{B.3}$$

Here

$$n = n(\varepsilon - \mu) = \left[\exp((\varepsilon - \mu)/T) \pm 1\right]^{-1} \tag{B.4}$$

are thermal Fermi/Bose–Einstein occupations. They obey some useful relations between fermion n_{f} and boson n_{b} occupation numbers, like

$$n_{\mathrm{f,b}}(\epsilon + \omega/2)\left[1 \mp n_{\mathrm{f,b}}(\epsilon - \omega/2)\right] = \left[n_{\mathrm{f,b}}(\epsilon - \omega/2) - n_{\mathrm{f,b}}(\epsilon + \omega/2)\right] n_{\mathrm{b}}(\omega), \tag{B.5}$$

or derivatives with respect to T

$$\frac{\partial n(\varepsilon-\mu)}{\partial T} = -\frac{\partial \sigma\left(n(\varepsilon-\mu)\right)}{\partial \varepsilon}, \quad \sigma(n) = \mp[1 \mp n]\ln[1 \mp n] - n \ln n, \tag{B.6}$$

$$\frac{\partial n(\varepsilon-\mu)}{\partial T} = \frac{\varepsilon - \mu}{T^2} n\left(1 \mp n\right). \tag{B.7}$$

The link between the Matsubara technique and the real-time formulation used here can be provided by extending the real time contour by an imaginary tail going to $-\mathrm{i}\beta$, $\beta = 1/T$, this way defining the equilibrium density operator $\exp[-\beta(\widehat{H} - \mu\widehat{N})]$. Thus, the link is provided by considering analytic expressions like the "contour trace" of two-point functions $\mathrm{i}F_{eq}(x,y) = \left\langle \mathcal{T}_{\mathcal{C}}\, \widehat{A}(x)\widehat{B}(y) \right\rangle$

$$\int\limits_{\mathcal{C}\{\mathrm{eq}\}} F_{\mathrm{eq}}(t, t+0)\mathrm{d}t = \int\limits_{0}^{-\mathrm{i}\beta} F_{\mathrm{eq}}^{-+}(t,t)\mathrm{d}t = \mp \mathrm{i}\beta \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi}\, n(\varepsilon - \mu)\left(F_{\mathrm{eq}}^A(\varepsilon) - F_{eq}^R(\varepsilon)\right)$$

$$= \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \frac{F^S_{\mathrm{eq}}(\varepsilon)}{\mathrm{i}\varepsilon_m + \mu_{AB} - \varepsilon} = \sum_{m=-\infty}^{\infty} F^{Matsubara}(\mathrm{i}\varepsilon_m). \tag{B.8}$$

Here the contour time $t+0$ is placed infinitesimally behind t on the contour in order to specify a *fixed* operator ordering of the two external operators of F, μ_{AB} is the chemical potential associated to $\left\langle \widehat{A}(x)\widehat{B}(y)\right\rangle$, and $\mu_{AB} = -\mu_{BA}$. The step towards the discrete Matsubara sum is provided by standard residue technique, cf. ref. [76], fig. 25.4 ff. The sum runs over the Matsubara energies

$$\varepsilon_m = \begin{cases} (2m+1)\pi T & \text{for fermions} \\ 2m\pi T & \text{for bosons.} \end{cases} \tag{B.9}$$

Thereby, the Matsubara form of the two-point function F_{eq} has the spectral representation

$$F^{Matsubara}(z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\varepsilon}{2\pi} \frac{F^S_{\mathrm{eq}}(\varepsilon)}{z + \mu_{AB} - \varepsilon} = \begin{cases} F^R_{\mathrm{eq}}(z+\mu_{AB}) & \text{for Im } z > 0 \\ F^A_{\mathrm{eq}}(z+\mu_{AB}) & \text{for Im } z < 0 \end{cases} \tag{B.10}$$

in terms of the *real time contour* spectral function $F^S_{\mathrm{eq}} = -2\mathrm{Im}F^R_{\mathrm{eq}}$.

C Diagram rules

For relativistic theories with local vertices the diagrammatic rules on the contour are identical to the standard Feynman rules except that all time integrations are to be replaced by contour integrations. The diagrams contributing to Φ are calculated as diagrams with one external point, namely the interaction part of the Lagrangian $\left\langle \widehat{\mathcal{L}}^{\,\mathrm{int}}(x)\right\rangle$, then contour integrated over x and weighted with $1/n_\lambda$, where n_λ counts the number vertices in the diagram. These diagrams have to be two-particle irreducible with all lines representing full propagators. For details about the corresponding diagrammatic rules on the contour see ref. [1].

The rules for non-relativistic two-body interactions are also naturally extended to the contour $\mathcal{C}$ with

$$\widehat{H}^{\,\mathrm{int}}(t_1) = \frac{1}{2}\int \mathrm{d}^3x_1 \int_{\mathcal{C}} \mathrm{d}^4x_2\, \widehat{\Psi}^\dagger(x_1)\,\widehat{\Psi}^\dagger(x_2) V(x_1-x_2)\,\widehat{\Psi}(x_2)\,\widehat{\Psi}(x_1) \tag{C.1}$$

with $V(x_1-x_2) = U(\boldsymbol{x}_1-\boldsymbol{x}_2)\,\delta_{\mathcal{C}}(t_1-t_2)$ now defined for contour times t_1, t_2. One has however to observe that for the instantaneous two-body interactions the diagrammatic

elements are given by $= -\mathrm{i}V(x_1 - x_2)$, i.e. without the factor 1/2 contained in (C.1). With respect to the topological rules, indeed, the interaction lines are treated like bosons in the relativistic theory with local vertices. To this extend we recall that the expectation value for $\left\langle \widehat{H}^{\,\mathrm{int}}(t) \right\rangle = \frac{1}{2} \times \{diagrams\}$ has an explicit factor $\frac{1}{2}$ in front of the corresponding diagrams. Correspondingly, for the n_λ vertex counting for the diagrams of Φ each two-body interaction line counts as *two* vertices, cf. the rules given in ref. [35] for the thermodynamic potential in Matsubara formalism. However, the two-particle irreducibility of Φ-diagrams is defined with respect to cutting dynamical propagators only, i.e. leaving the V-interaction lines untouched. In $\{-+\}$-matrix notation the above rules imply, cf. [54] sect. X,

$$= -\mathrm{i}V(x_1 - x_2), \qquad = 0. \tag{C.2}$$

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