# Nuclear-matter–quark-matter phase diagram with strangeness

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A phenomenological equation of state of strongly interacting matter, including strange degrees of freedom, is presented. It is shown that the hyperon and kaon interactions must be included, in order to obtain a reasonable description of the deconfinement transition at high baryon densities. The consequences of kaon condensation on the nuclear-matter–quark-matter phase diagram are explored. The relative particle abundances obtained in an isentropic expansion of a blob of quark-gluon plasma are presented for different initial conditions. Implications for ultrarelativistic heavy-ion collisions are briefly discussed.

## I. INTRODUCTION

With increasing energy density, dense matter is expected to experience a phase transition from a system of strongly interacting hadrons into one of weakly interacting quarks and gluons. Although the behavior is predicted by finite-temperature lattice simulations of the underlying fundamental QCD (for recent reviews see Refs. 1 and 2), a tractable equation of state valid also at finite baryon densities is not yet available from first principles.

However, in many problems, e.g., in ultrarelativistic heavy-ion collisions or in the early Universe, an equation of state is very useful. One possibility is to construct a phenomenological one, which reproduces the essential features of the lattice simulations at zero density, and extrapolated to finite densities in a reasonable manner. A common approach, which we will also take, is to work with different models for the two phases, and join them at the phase transition using the Gibbs conditions for phase equilibrium. Such equations of states have been extensively studied in the literature and employed in many applications (see, e.g., Refs. 3–12). Usually the quark-gluon plasma is described within the bag model, while the hadronic equation of state has been parametrized by relatively simple forms.

For the hadronic equation of state reliable empirical information is available only at low temperatures near the saturation density, $\rho \simeq \rho_0$. The behavior of the equation of state at high energy densities depends strongly on which hadrons are actually included in the model and on how these particles are assumed to interact with each other. For example, pions, kaons, and hyperons are not relevant at the saturation point and yet they may generate qualitative changes in the behavior at high temperatures and densities. Very little empirical information is available in this regime.

Different aspects of strangeness production in scenarios involving a quark-gluon plasma in the early Universe and in ultrarelativistic heavy-ion collisions are currently of high interest.[13–22] A phenomenological model for strangeness production in heavy-ion collisions, where all particle abundances are allowed to deviate from their equilibrium ones, was recently investigated by us.[23] One of the basic ingredients of such studies is the corresponding equation of state, which, in addition to nucleons and pions, must also include strange particles.

Several such equations of state have been proposed. However, most of them suffer from more or less severe deficiencies; i.e., they are thermodynamically inconsistent or violate causality. In some early work, the nucleons were treated as noninteracting, pointlike, objects. This results in an equation of state, with the undesired feature that for very high baryon densities the hadronic phase is again the stable configuration.[10] By adding a short-range repulsion between the nucleons, e.g., due to the exchange of $\omega$ mesons, one can eliminate this problem. An alternative way is to take the finite size of the baryons into account, by means of an excluded volume.[24–26] We prefer the first alternative, which in contrast with the second one leads to a thermodynamically consistent equation of state. (In the excluded-volume approach, thermodynamic relations such as, e.g., $n = \partial p / \partial \mu$, where $p$ is the pressure and $\mu$ the chemical potential, are not satisfied in the hadronic phase.) As we will show, similar difficulties arise when the model is extended to include hyperons, if they are treated as noninteracting.

At high densities and low temperatures, kaon condensation is likely to occur. We show that, depending on the kaon interactions, the presence of a condensate may change the phase diagram considerably. In many models, this phenomenon has been excluded from the beginning, by treating the kaons as Boltzmann particles.

The use of equations of state with such deficiencies is unsatisfactory and may lead to unreliable results. In this paper we suggest a model for the equation of state of hadronic matter, which is practicable and void of the deficiencies mentioned above and at the same time incorporates the contribution of the strange particles in a semirealistic way. To this end, we shall extend Walecka's relativistic mean-field model,[27,28] to account for the interactions of hyperons in addition to those of nucleons (see also Refs. 29–31). We treat all mesons, except the kaons, as noninteracting. A repulsive kaon-kaon interaction is needed to obtain a reasonable phase diagram in the presence of a kaon condensate.

In Sec. II we present the equation of state, and discuss the kaon-condensation problem in detail. Applications of the equation of state to situations relevant for relativistic heavy-ion collisions are discussed in Sec. III. Concluding remarks and a summary are given in Sec. IV.

## II. EQUATION OF STATE FOR HADRONIC AND QUARK MATTER

In this section we first present the equation of state of hadronic matter, which is obtained by generalizing the Walecka model (see also Refs. 29–31) and then we describe how kaon-kaon interactions have been incorporated. Finally a brief presentation of the equation of state of the quark-gluon plasma is given.

As we mentioned in the Introduction, nucleon-nucleon interactions must be taken into account, in order to obtain a reasonable phase diagram. When the nucleons are treated as free, pointlike, particles then the free energy of the hadronic phase is lower than that of the quark-gluon plasma at high densities. Similar difficulties occur, if one simply adds a gas of noninteracting hyperons to a system of interacting nucleons. This is illustrated in Fig. 1, where we show the pressure of the two phases as a function of the baryon chemical potential at constant temperature ($T=10$ MeV). Note that, because of the principle of maximum pressure, the hadronic phase is stable at low

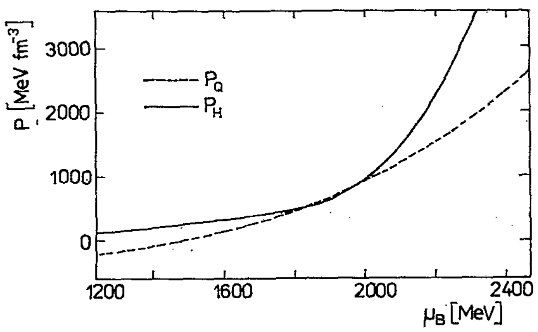


FIG. 1. The pressure $p_H$ of the hadronic gas, with only nucleon-nucleon interactions taken into account, and the pressure of the quark-gluon plasma $p_Q$ as functions of the baryon chemical potential $\mu_B$ at given temperature $T$.

and at high densities, while the plasma phase is stable only at intermediate densities. This unphysical feature emerges because at large densities the nucleons feel the short-range repulsion, while the noninteracting hyperons do not. Therefore, almost all nucleons are converted into hyperons, e.g., through the reaction $\bar{K}+N\to\pi+Y$, thus lowering the energy below that of a quark-gluon plasma. Consequently, with such an approximate equation of state, one still faces the same difficulties as with noninteracting nucleons.

Furthermore, in order to obtain a consistent treatment of kaon condensation, kaon interactions must be taken into account. Therefore, we have constructed a new equation of state, where nucleon and hyperon, as well as kaon, interactions are taken into account.

### A. Hadronic matter

A straightforward extension of the Walecka model is made by incorporating hyperon interactions mediated by the exchange of vector and scalar mesons, such as the nucleon interactions in the original model. For simplicity, the hyperons ($\Lambda$ and $\Sigma$) are represented by an effective $Y$ particle of mass $m_Y=1170$ MeV and degeneracy factor $g=8$. Neglecting the contribution of the negative-energy states we find for the pressure of the nucleons and hyperons in the mean-field approximation ($\hbar=c=1$)

$$p_{NY}(\mu_N,\mu_Y,T)=\frac{1}{2m_N^2}(C_V^2 n_B^2-C_S^2 n_s^2)+\sum_{j=N,\bar{N},Y,\bar{Y}} p_j^{\mathrm{kin}}(\nu_j,T)\ . \tag{1}$$

[We note that in this approximation, nuclear matter at high density exhibits unphysical instabilities with respect to fluctuations of the meson fields at short wavelengths.[32,33] However, since we consider only long-wavelength (bulk) properties, these instabilities play no role in the present calculation.] The coupling constants $C_V^2=195.7$ and $C_S^2=266.9$ are the original ones of the Walecka model,[27,28] which have been adjusted to fit binding energy and density of nuclear matter in its ground state. For simplicity we assume that the coupling constants are universal, i.e., the nucleon and hyperon couping constants are equal. Since the binding energies of $\Lambda$ hypernuclei are not very different from the average binding energy of a nucleon, this assumption seems reasonable as a first approximation.

In Eq. (1) the kinetic pressure of particle $j$ is

$$p_j^{\mathrm{kin}}(\nu_j,T)=\frac{g_j}{6\pi^2}\int_0^\infty dk\frac{k^4}{(k^2+m_j^{*2})^{1/2}}f(k,m_j^*,\nu_j) \tag{2}$$

and the vector and scalar densities of the nucleons and hyperons are

$$\begin{aligned} n_B&=(n_N-n_{\bar{N}})+(n_Y-n_{\bar{Y}})\ ,\\ n_s&=n_{s,N}+s_{s,\bar{N}}+n_{s,Y}+n_{s,\bar{Y}}\ . \end{aligned} \tag{3}$$

The partial density of particle $j$, with effective mass $m_j^*$ and degeneracy $g_j$, is

$$n_j(\nu_j,T)=\frac{g_j}{2\pi^2}\int_0^\infty dk\, k^2 f(k,m_j^*,\nu_j)\ ,$$
$$n_{s,j}(\nu_j,T)=\frac{g_j m_j^*}{2\pi^2}\int_0^\infty dk \frac{k^2}{(k^2+m_j^{*2})^{1/2}} f(k,m_j^*,\nu_j)\ , \tag{4}$$

where

$$f(k,m,\nu)=\{\exp[(\sqrt{k^2+m^2}-\nu)/T]+1\}^{-1} \tag{5}$$

is the momentum distribution function. The auxiliary potentials $\nu_N=-\nu_{\bar{N}}$ and $\nu_Y=-\nu_{\bar{Y}}$, which are introduced for convenience, are shifted with respect to the nucleon and hyperon chemical potentials by the self-energy due to the vector field:

$$\mu_N=-\mu_{\bar{N}}=\nu_N+\frac{C_V^2}{m_N^2}n_B,\quad \mu_Y=-\mu_{\bar{Y}}=\nu_Y+\frac{C_V^2}{m_N^2}n_B\ . \tag{6}$$

The effective masses $m_j^*$ satisfy the implicit equation

$$m_j^*=m_j-\frac{C_S^2}{m_N^2}n_s\ . \tag{7}$$

In the case that the coupling constants are not universal, i.e., when the hyperon coupling constants differ from the nucleon ones, the equation for the effective mass is somewhat more complicated, involving the ratio of the two coupling constants. (This would be the case if we were to adjust the coupling constants, e.g., according to Refs. 34 and 35, where properties of hypernuclei have been fitted.)

Besides nucleons and hyperons we consider the following hadrons: pions ($g_\pi=3$, $m_\pi=139$ MeV), kaons ($g_K=2$, $m_K=490$ MeV), cascade particles ($g_\Xi=4$, $m_\Xi=1320$ MeV), $\Omega$'s ($g_\Omega=4$, $m_\Omega=1670$ MeV), and $\eta$'s ($g_\eta=1$, $m_\eta=550$ MeV, $g_{\eta'}=1$, $m_{\eta'}=950$ MeV). Except for the kaon, these hadrons are treated as free particles. Their contribution to the pressure is simply a sum of terms of the form (2).

The cascade resonance ($\Xi$) and the $\Omega$'s are, because of their larger masses, much less abundant than the nucleons or the hyperons. Consequently, one does not encounter difficulties with the phase diagram, even for noninteracting $\Xi$'s and $\Omega$'s. However, one should keep in mind that for nonvanishing net strangeness, the strangeness chemical potential may become so large that it becomes energetically favorable to convert nucleons and hyperons into noninteracting $\Xi$'s and $\Omega$'s, thus again making the hadron gas the energetically favored state at high densities. Consequently, when studying the possible formation of strangelets (see Refs. 12, 21, 22, 36, and 37), the interactions of the multistrange baryons must also be taken into account. In this paper we restrict ourselves to systems of vanishing net strangeness, so for simplicity we neglect the $\Xi$ and $\Omega$ interactions.

### B. Kaon condensation

For noninteracting kaons, condensation sets in, when $\mu_K=m_K$, which in our model happens in the coexistence region. Beyond the condensation threshold, the kaon chemical potential remains fixed: $\mu_K=m_K$. Since $\mu_K=\mu_q-\mu_s$ in equilibrium, the hadronic phase cannot exist for $\mu_q>m_K+\mu_s$. In particular, for densities corresponding to a quark chemical potential $\mu_q>m_K$, the strange-quark chemical potential $\mu_s$ cannot be equal to zero. Consequently, in this case, phase equilibrium between a quark-gluon plasma and an infinitesimal admixture of hadron matter is not possible for vanishing net strangeness, since this would require $\mu_s=0$. Thus the system remains in the coexistence region and the transition to quark matter cannot be completed.

In order to take care of this problem, we add a repulsive kaon-kaon interaction term to the Lagrangian density

$$\mathcal{L}_{\text{int}}=-\lambda(\Psi_K\Psi_K^\dagger)^2\ , \tag{8}$$

where $\Psi_K$ is the kaon field and $\lambda$ the kaon-kaon coupling constant. We treat the kaon interactions in a mean-field approximation; i.e., we take into account only the interactions of the kaons in the condensate with themselves and with the thermal kaons, while the interactions between the thermal kaons are neglected. We also neglect the kaon-baryon interaction, which, as shown by Kaplan and Nelson,[38] lowers the critical density for kaon condensation. We note that kaon condensation is strongly favored in the coexistence region. There the strange-quark chemical potential decreases towards zero, while the light-quark chemical potential remains roughly constant. Thus the kaon chemical potential grows, approaching $\mu_K=\mu_q$ on the quark-matter side of the coexistence region.

For a uniform condensate of the form $\langle\Psi_K(t)\rangle=\exp(i\mu_K t)\langle\psi_K\rangle$, where $\mu_K$ is the kaon chemical potential, the expectation value of the kaon-field $\langle\Psi_K\rangle$ satisfies the equation of motion[39]

$$(\mu_K^2-m_K^2-2\lambda|\langle\Psi_K\rangle|^2)\langle\Psi_K\rangle=0\ . \tag{9}$$

The contribution of the condensate to the pressure is

$$\begin{aligned} p_{\text{cond}}&=(\mu_K^2-m_K^2)|\langle\psi_K\rangle|^2-\lambda(|\langle\psi_K\rangle|^2)^2\\ &=\frac{1}{4\lambda}(\mu_K^2-m_K^2)^2\ , \end{aligned} \tag{10}$$

where the last equation is valid only when $\langle\psi_K\rangle\neq0$, i.e., when $\mu_K\geq m_K$. The density of the kaons in the condensate is given by

$$\begin{aligned} n_{\text{cond}}&=\frac{1}{\lambda}(\mu_K^2-m_K^2)\mu_K \quad (\mu_K>m_K)\\ &=0 \qquad\qquad (\mu_K<m_K)\ . \end{aligned} \tag{11}$$

In the condensed state, the thermal kaons acquire an effective mass $\bar{m}_K^2=m_K^2+2\lambda|\langle\Psi_K\rangle|^2=\mu_K^2$ through the interaction with the condensate. In this way the chemical potential can assume values above the rest mass of a free kaon and no singularities arise in the pressure and density integrals, Eqs. (2) and (4).

The total pressure in the hadron phase is then the sum of the contributions from the interacting nucleons and hyperons, from the noninteracting hadrons $\pi$, $K$, $\bar{K}$, $\eta$, $\eta'$, $\Xi$, $\bar{\Xi}$, $\Omega$, $\bar{\Omega}$, and the pressure of the kaon condensate

$$p_H = p_{NY} + \sum_j p_j^{\text{kin}} + p_{\text{cond}} \; . \tag{12}$$

### C. Quark-gluon plasma

The quark-gluon plasma is assumed to consist of noninteracting quarks, antiquarks, and gluons. The nonperturbative effects associated with the confinement transition are as usual subsumed in a constant vacuum pressure $B$. We have used the value $B=(235\text{ MeV})^4$, which yields a transition temperature consistent with Monte Carlo simulations.[1] The total pressure of the plasma phase then takes the form

$$p_{\text{QG}} = p_g + \sum_j p_j^{\text{kin}} - B \; , \tag{13}$$

where the partial pressure $p_j^{\text{kin}}$ of the quarks of species $j$ is given by Eq. (2) and the index $j$ runs over all quarks and antiquarks. The quark degeneracy factors are $g_j=6$ and the quark masses $m_u=m_d=0$ and $m_s=150$ MeV, respectively. The gluon pressure is

$$p_g = \frac{g_g \pi^2}{90} T^4 \quad (g_g=16) \tag{14}$$

and the quark and gluon densities are

$$n_j(\mu_j,T) = \frac{g_j}{2\pi^2} \int_0^\infty dk\, k^2 (\exp\{[(k^2+m_j^2)^{1/2}-\mu_j]/T\}+1)^{-1} \tag{15}$$

and

$$n_g = \frac{g_g \zeta(3)}{\pi^2} T^3 \; . \tag{16}$$

## III. RESULTS AND DISCUSSION

Phase equilibrium between the plasma and the hadronic phase is determined by Gibbs conditions for thermal ($T_{\text{QG}}=T_H$), mechanical ($p_{\text{QG}}=p_H$), and chemical equilibrium:

$$\begin{aligned} &\mu_N = -\mu_{\bar{N}} = 3\mu_q, \quad \mu_\pi = \mu_\eta = \mu_{\eta'} = 0 \; , \\ &\mu_K = -\mu_{\bar{K}} = (\mu_q - \mu_s), \quad \mu_Y = -\mu_{\bar{Y}} = (2\mu_q + \mu_s) \; , \\ &\mu_\Xi = -\mu_{\bar{\Xi}} = (2\mu_s + \mu_q), \quad \mu_\Omega = -\mu_{\bar{\Omega}} = 3\mu_s \; . \end{aligned} \tag{17}$$

At given temperature $T$ and quark chemical potential $\mu_q$ the strange-quark chemical potential $\mu_s$ is determined by fixing the net strangeness of the total system $S_{\text{net}}$ (hadronic matter and quark-gluon plasma), i.e.,

$$\frac{V_H}{V} \frac{\partial p_H}{\partial \mu_s} + \frac{V_Q}{V} \frac{\partial p_Q}{\partial \mu_s} = S_{\text{net}} \; , \tag{18}$$

where $V$ is the total volume and $V_H$ and $V_Q$ are the partial volumes of the coexisting phases.

The resulting phase diagram for vanishing total strangeness $S_{\text{net}}=0$ is shown in Fig. 2. For the parameters we use, the transition temperature at $\mu_q=0$ is $T_{\text{tr}}^0 \approx 161$ MeV. The phase boundary of the hadronic phase is labeled by $c_H$, while that of the quark phase is labeled by $c_Q$. We note that at low temperatures the phase boundary $c_Q$ depends on the strength of the kaon-kaon coupling. Details will be given below.

Let us begin by discussing the phase boundary of the hadronic gas $c_H$. In the hadronic phase at positive net baryon density, most of the antistrangeness is carried by the kaons, while the strangeness is predominantly carried by the hyperons. A nonzero chemical potential $\mu_s$ is then required to obtain vanishing net strangeness [Eq. (18)]. The relation between $\mu_s$ and $T$ along the hadronic phase boundary $c_H$ is shown in Fig. 3. We note that in hadron-

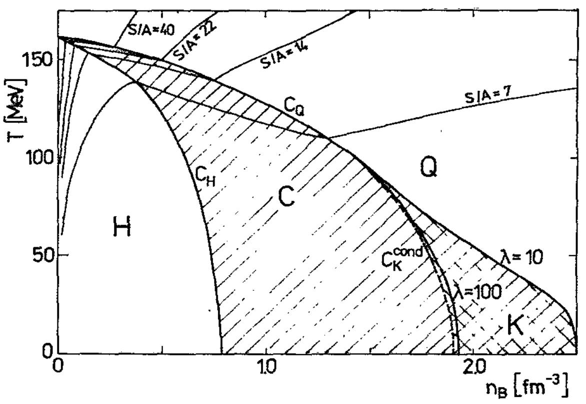


FIG. 2. The phase diagram of hadronic/quark matter for vanishing net strangeness. The phase boundary $c_H$ separates the pure hadronic phase from the coexistence region, while $c_Q$ separates the quark-gluon plasma from the coexistence region. We show $c_Q$ for two values of the kaon coupling constant $\lambda$. Finally, the phase boundary $c_K$ shows the threshold for kaon condensation.

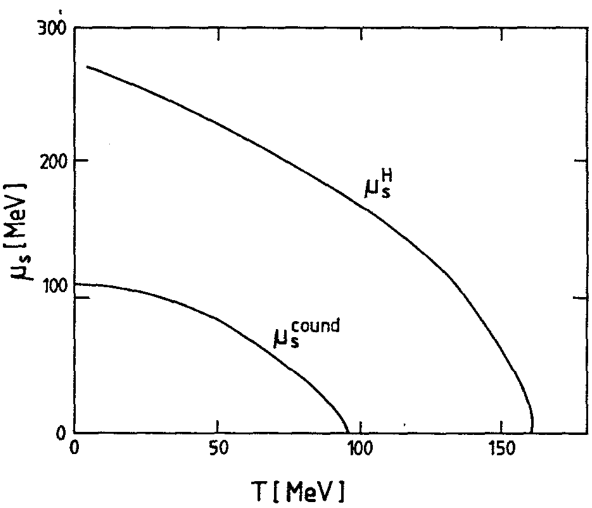


FIG. 3. Strange-particle chemical potentials $\mu_s^H$ along the hadronic phase boundary $c_H$ and the critical strange-quark chemical potential $\mu_s^{\text{cond}}$ below which the kaons start condensing. Both quantities are shown as a function of temperature.

ic matter of vanishing net strangeness, the strange-quark chemical potential is rather large, $\mu_s \gtrsim 150$ MeV, already at temperatures $T \lesssim 100$ MeV. On the other hand, along the quark-matter phase boundary $c_Q$, the strange-quark chemical potential vanishes, due to the strangeness symmetry in the quark phase. Thus, in the coexistence region $\mu_s$ varies between zero at $c_Q$ and some finite value at $c_H$. Consequently, the strangeness content of each of the two coexisting phases does not vanish separately. In fact, since $\mu_s \geq 0$ in the coexistence region, the $s$ quarks prefer the quark phase, while the $\bar{s}$ quarks accumulate in the hadronic phase.[21,22,36,37] Hence, there is a tendency to separate strangeness from antistrangeness during the hadronization of the plasma. The suppression of antistrangeness in the quark phase is more pronounced in a baryon-rich than in a baryon-poor plasma. This is illustrated in Fig. 3, where we show the strange-quark chemical potential $\mu_s$ as a function of $T$. We note that in the baryon-rich region ($T \rightarrow 0$) $\mu_s$ significantly exceeds the strange-quark mass. Thus, one expects a possible separation of strangeness to be strongest in cold matter (cf. Ref. 37).

Let us now discuss some details associated with the possible formation of a kaon condensate. For temperatures $T$ above 100 MeV the quark chemical potential $\mu_q$ does not exceed the kaon rest mass on $c_Q$. Since the kaon chemical potential $\mu_K = \mu_q$ for $\mu_s = 0$ this implies that $\mu_K < m_K$ and, consequently, that there are no complications due to kaon condensation in this regime with the present choice of parameters. However, for temperatures below $T_{\text{cond}} \approx 100$ MeV the kaon chemical potential exceeds the free kaon mass in the coexistence region. This signals the onset of kaon condensation; the threshold is determined by the condition $\mu_K = m_K$ [see Eq. (11)]. The corresponding phase boundary is labeled by $c_K$ in Fig. 2. We note that, in the mean-field approximation, the threshold density is independent of the coupling constant $\lambda$. In Fig. 3 we also show the strange-quark chemical potential $\mu_s^{\text{cond}}$ below which the kaons condense as a function of temperature.

However, the position of the phase boundary between the kaon-condensed coexistence region and the quark-gluon plasma does depend on the coupling strength $\lambda$. For large values of $\lambda$ the condensate exists only in a narrow band along $c_K$, while for $\lambda \rightarrow 0$ the coexistence region at low temperatures extend up to infinitely large baryon densities $n_B$ (cf. the two contours $c_Q$ for $\lambda = 10$ and 100 in Fig. 2). As we noted above, this is due to the fact that the equation $\mu_q = m_K + 2\lambda |\langle \Psi_K \rangle|^2 + \mu_s$, which holds in the condensate, cannot, for $\lambda = 0$ and $\mu_q > m_K$, be satisfied on the boundary $c_Q$, along which $\mu_s = 0$.

Thus, for small $\lambda$ ($\sim 1$), the position of the quark phase boundary $c_Q$ depends strongly on the coupling constant, while for $\lambda > 50$ the dependence is very weak. We also remark that the phase contour $c_Q$ obtained for large values of $\lambda$ is rather similar to that which emerges when the kaons are treated as Boltzmann particles. However, in doing so we would exclude the possibility of kaon condensation and at the same time unnecessarily distort the relation between $\mu_K$ and $\mu_s$ in the coexistence region. We stress that our investigation of kaon-condensation in connection with the hadronization transition is only preliminary. Further studies, employing more realistic models with kaon-baryon interactions are needed.

The equilibrium abundances of the different hadronic species along the phase border line $c_H$ are shown in Fig. 4 as a function of the entropy per baryon. These are of interest, since results of model calculations suggest that the relative hadron abundances, produced in the hadronization of the plasma, are close to the equilibrium ones of the hadronic phase.[18,23] In this context it is interesting to note that the strangeness-changing reactions in the hadronic phase do not significantly alter the abundances.[18,23]

We observe that the strange particle abundances reach maximum values at temperatures slightly below the maximum transition temperature $T_{\text{tr}}^0$, while the pion number increases exponentially with temperature. The maxima for the strange-particle densities are due to the fact that their chemical potentials vary from zero at $T = T_{\text{tr}}^0$ to some positive value at low temperature. For instance, the kaon density is approximately $n_K \propto T^{3/2} \exp[(\mu_K - m_K)/T]$. For temperatures below 90 MeV the kaon chemical potential is essentially constant, $\mu_K \approx 500$ MeV, along the $c_H$ border. Thus in this regime, the density grows with temperature, due to the $T^{3/2}$ factor. As the temperature is increased above 90 MeV, $\mu_K$ starts decreasing, but for temperatures up to $T \approx 140$ MeV, the decrease in the chemical potential is slow. In this regime, the density still grows with temperature, since the growth

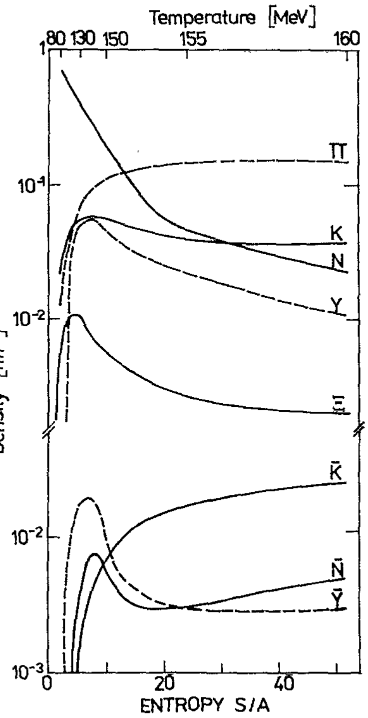


FIG. 4. Abundances of baryons and mesons along the hadronic phase contour $c_H$ as a function of the specific entropy. On top we also show the corresponding temperature scale.

of the $T^{3/2}$ factor compensates the decrease in the exponential factor. For $T > 140$ MeV the kaon chemical potential $\mu_K$ rapidly decreases and vanishes at $T = T_{tr}^0$. Thus, in this regime, the kaon density decreases, explaining the maximum at $T \approx 130$ MeV.

However, the maximum abundances are reached in a temperature-density regime, which is not likely to be reached in an isentropic expansion of a quark-gluon-plasma blob into the hadronic phase. This is illustrated in Fig. 2 where isentropes are displayed in the density-temperature plane. One sees that for rather different values of the initial entropy in the quark phase an isentropic expansion ends up in a relatively small density and temperature window in the hadronic phase. This behavior is a consequence of the requirement of entropy conservation during the phase transition. The high number of internal degrees of freedom in the plasma phase implies a high entropy per baryon. In the hadronic phase, the smaller number of degrees of freedom that can contribute to the entropy must be compensated by a decrease in the net baryon density, so as to yield the same entropy per baryon. Therefore, the baryon density is decreased, while the temperature increases during the hadronization. We note that if entropy is produced during the expansion, the system ends up on an isentrope of higher entropy per baryon and thus even further away from the region where the strange-particle abundances reach a maximum.

Thus it is evident that both a baryon-poor (high initial entropy values) and a baryon-rich plasma (low entropy values) after the hadronization transition end up in the hadronic phase, in a temperature range only slightly below the transition temperature ($T_{tr}^0 \approx 161$ MeV).

We also note that the yields for $\pi$, $K^+$, and $\bar{Y}$ are rather flat for specific entropy values $S/A > 10$ (see Fig. 4). Therefore, the $K^+/\pi^+$ ratio also remains fairly constant in this regime (see Fig. 5). In fact both the $K^+/\pi^+$ and $K^-/\pi^-$ ratios show a strong dependence on the specific entropy only for low values of $S/A$. For $S/A > 15$ the asymptotic ratios ($S/A \rightarrow \infty$) for $K^+/\pi^+$ and $K^-/\pi^-$ are quickly reached. The asymptotic ratios are relevant for a system of vanishing net baryon density. The results displayed in Fig. 5 indicate that the $K^+/K^-$ ratio is much more sensitive to the initial baryon content of the quark-matter blob than the other ratios. Furthermore, this ratio contains information on the net strangeness content of the quark blob in terms of the strange-quark chemical potential $\mu_s$, since $K^+/K^- \approx \exp(2\mu_s/T)$. Finally we note that the rather exotic ratio of antihyperons to antinucleons is on the order of unity and does not vary much for entropy values $S/A > 15$.

In summary, the results shown in Figs. 4 and 5 indicate that the ratios of particle yields should be rather independent of entropy for $S/A > 15$, provided the strangeness degrees of freedom are approximately equilibrated. From an experimental point of view, we expect that the above ratios should not change significantly as a function of the transverse energy deposition $E_\perp$ in ultrarelativistic nucleus-nucleus collisions.

As we already mentioned, there is a tendency to separate strangeness from antistrangeness in the coexistence region. Thus in small quark-matter blobs, which are in phase equilibrium with the hadronic phase, there is an excess of strange quarks. The relative strange-quark excess in an infinitesimal blob of quark-gluon plasma coexisting with hadronic matter on the phase boundary $c_H$, $\Delta_{\text{strange}} = (n_s - n_{\bar{s}})/(n_q - n_{\bar{q}} + n_s - n_{\bar{s}})$, is shown in Fig. 6 as a function of the temperature. The strange-quark excess $\Delta_{\text{strange}}(T)$ is a few percent at small temperatures and increases monotonously with $T$ up to values as high as 50%. Thus in our model there is an appreciable strangeness excess in the quark phase near the hadronic side of the coexistence region. This excess of strangeness has led to speculations about the formation of strangelets in relativistic heavy-ion collision at high temperatures[21,22] and even hadronic matter at zero temperature.[37] We note, however, that the present calculation and those in Refs. 21, 22, and 37, apply only to equilibrium situations. Whether strangeness separation can actually occur and generate observable effects in nonequilibrium situations, such as heavy-ion collisions, can only be ascertained in dynamic calculations, e.g., using the kinetic model of Ref. 23.

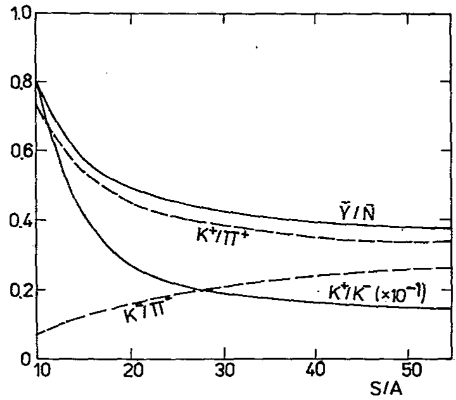


FIG. 5. Ratios of hadrons along the hadronic phase boundary $c_H$ as functions of the entropy per baryon.

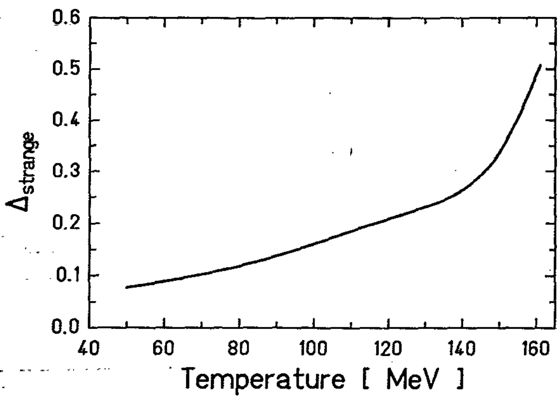


FIG. 6. Net strangeness content of an infinitely small quark-gluon blob in equilibrium with the hadronic phase, i.e., on the hadronic phase boundary, as a function of the temperature.

## IV. SUMMARY AND CONCLUSION

We have presented a model for the equation of state of strongly interacting matter, including strange degrees of freedom, which is practicable and at the same time includes the essential features of hadronic matter in a consistent way. The equation of state can be used both in static and dynamic calculations, which involve the phase transition between hadronic matter and quark-gluon matter. The main new features of this equation of state are a consistent treatment of the hyperon interactions and the possible formation of a kaon condensate. Although these effects have previously been considered by other authors, they are new in the context of the hadronization transition. The hyperon interaction is crucial in order to obtain a qualitatively correct behavior at baryon densities, where a transition to quark matter is expected.

For moderate temperatures ($T < 100$ MeV) kaon condensation occurs in the coexistence region. In order to obtain a reasonable phase diagram we have introduced a repulsive interaction among the kaons in the condensate. Depending on the details of the kaon interactions, the condensate may raise the deconfinement density at low temperatures considerably. This again may affect the structure of neutron stars and the possibility of strangeness separation in cold matter. Further studies of this effect, using more refined models of the kaon interactions in hadronic matter are needed before definitive conclusion can be drawn.

We have also found that, for a wide range of initial conditions, an isentropically expanding quark-matter blob ends up in a rather narrow temperature and density window in the hadronic phase. Consequently, particle ratios, such as $K^+/\pi^+$, $K^-/\pi^-$, and $\bar{Y}/\bar{N}$, are rather close to their asymptotic values already for specific entropy values $S/A > 15$. Since these ratios depend solely on the underlying equation of state, the measurement of these particle ratios could give us direct information on the hadronic equation of state at high temperatures.

Dynamic calculations indicate that, if a quark-gluon plasma is formed in an ultrarelativistic nucleus-nucleus collision, the final strangeness abundances are by and large determined in the hadronization transition. The strangeness production during the transition is probably very effective, so that the hadronic matter is produced near strangeness equilibrium. Furthermore, the strangeness changing processes in the hadronic phase are believed to be slow compared to the typical expansion time scale. Thus, a reliable calculation of the strangeness content of the hadronic phase near the phase transition region is useful for analyzing and interpreting experimental data.

The results represented in this work are the outcome of static equilibrium calculations. It is a major task for future studies to perform nonequilibrium phase transition calculations, e.g., along the lines of Ref. 23, where a less elaborate equation of state was used.

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