RADIATION FROM DENSE MATTER

Jörg Knoll, GSI Darmstadt

ABSTRACT

The principle aspects and problems of Bremsstrahlung arising from collisions in dense matter are discussed. Once the mean collision rate in the matter becomes comparable or larger than the photon energy the incoherent quasi-free scattering approximation which is used in all kinetic simulation models is no longer valid since coherence effects are important. The lectures deal with this general problem, explain the principle physics at simple examples and gives a perspective on the theoretical tools to be used for a proper quantum description.

1. Introduction

With increasing beam energy the production of new particles becomes a dominant component of the reaction dynamics in nuclear collisions. Thus particles, like pions, other mesons or baryons as well as real and virtual photons are produced besides the original nucleons. For the theoretical description it is therefore important to have a reliable concept for the corresponding production and absorption rates in dense matter. However most of the models today use prescriptions that date back to the early days of the Intra Nuclear Cascade model. There one uses the incoherent quasi-free collision picture (IQF), where on the basis of the free production cross section and the quasi-particle approximation the contributions from different microscopic collisions are incoherently added. That is, one adds probabilities rather than amplitudes. In high energy physics one soon has realized that the IQF picture has its limitations once the collision rate among the constituents $\Gamma = 1/\tau_c$ ($h = 1$) becomes comparable or even larger than the energy $\omega$ of the produced particle. In physical terms: depending on its energy $\omega$ and momentum $\vec{q}$ the produced or absorbed particle looks at the interaction zone only with a limited space-time resolution due to the uncertainty principle $\Delta x \sim 1/q, \Delta t \sim 1/\omega$. Once two or more collisions fall into this limited space-time volume they are no longer resolved individually and the coherence between successive collisions becomes important. The resulting interference is generally destructive and yields a reduction of the production rates with decreasing $\omega$ relative to those rates estimated with the IQF prescription. For the bremsstrahlung from fast electrons traversing matter this effect has been studied by Landau, Pomeranchuk, Migdal and subsequently by many others.
2. Production Rates

While for scattering problems one normally first calculates the transition amplitude, which then squared gives the production cross section according to Fermi’s golden rule, for dense matter problems it is more appropriate to take a different route. Suppose the coupling between the produced particle and the source is given by a current density of the source. For instance in the electromagnetic case the interaction energy between the photon and a source of charged particles is given by \( \int d^3 x A_\mu(x) j^\mu(x) \).

In such cases the local production rate can be expressed through the current-current correlation function \( \langle j^\mu(x_1, t_1) j^\nu(x_2, t_2) \rangle \). Here the brackets \( \langle \ldots \rangle \) denote the ensemble average. The Wigner transformation of this correlation function

\[
C(\omega, q; \tilde{x}, \tilde{t}) = \int d^3 \xi d\tau e^{i\omega \tau - i\xi \cdot \tilde{x}} \langle \hat{j}(\tilde{x} - \tilde{\xi}/2, \tilde{t} - \tau/2) \hat{j}(\tilde{x} + \tilde{\xi}/2, \tilde{t} + \tau/2) \rangle
\]

defines the local gain and loss terms for a photon with energy momentum \( \omega, q \)

\[
\frac{d}{dt} n_\gamma(\tilde{x}, \tilde{q}, t) = \{\omega \partial_t + \tilde{q} \partial_x\} n_\gamma(\tilde{x}, \tilde{q}, t) - \frac{2\pi}{3} \{2C(\omega, q; \tilde{x}, \tilde{t})(1 - n_\gamma(\tilde{x}, \tilde{q}, t)) - C(-\omega, -q; \tilde{x}, \tilde{t})n_\gamma(\tilde{x}, \tilde{q}, t)\}.
\]

Here \( n_\gamma(\tilde{x}, \tilde{q}, t) \) denote the phasespace occupation of the photon at space-time \( \tilde{x}, \tilde{t} \) and momentum \( \tilde{q} \). The extra factor 2 in the gain term comes from the summation over the two transverse polarizations. For homogeneous systems in equilibrium \( C \) does no longer depend on \( \tilde{x}, \tilde{t} \). The advantage of this formulation is that for statistical systems such correlations die out for larger space-time distances such that the above Fourier transformation over \( \tilde{\xi}, \tau \) is limited to correspondingly small regions.

The correlation function obeys certain integral constraints in form of sum rules, e.g.

\[
\int_{-\infty}^{\infty} d\omega C(\omega, q = 0; t, \tilde{x}) d^3 x = 2\pi \langle \hat{J}(t)^2 \rangle, \quad \text{where} \quad \hat{J}(t) = \int d^3 x j(t, \tilde{x})
\]

which directly follows from definition (1), c.f. also ref.\(^2\) for further sum rules. They impose constraints on the analytical form of the production rate in dense matter which for a source system in equilibrium at temperature \( T \) can be formulated as

\[
\int_0^\infty d\omega \frac{d^2}{d\omega dt} N(\omega, t) \left(1 + e^{\omega/T}\right) = \frac{4}{3} \langle \hat{J}(t)^2 \rangle,
\]

where \( \frac{d^2}{d\omega dt} N \) is the space and angle integrated yield per energy \( \omega \) and time \( t \).

Therefore the question arises: what are the relevant scales which determine the form of the correlation function in dense matter and therefore the form of the photon spectrum, and how does this compare to the spectrum in the naive IQF approximation? Since the nature of the problem is entirely classical, that is, it is already present in the
coupling of a classical system of charged particles to the Maxwell field, it is instructive to first study such cases. For simplicity we consider now a single point charge which makes stochastic collisions with a neutral random medium, just like Brownian motion. Due to these random collisions it radiates photons. We describe the motion of this charge in two ways: i) macroscopically by means of a diffusion equation; and ii) microscopically by a stochastic Langevin process. These examples and the following quantum many body considerations are discussed in detail in a recent publication.

3. Incoherent Quasi-free Scattering Approximation (IQF)

The IQF approximation relies on the elementary $\gamma$ production cross section for the in-matter collisions. For dense nuclear matter and $\gamma$ energies below 200 MeV the most essential processes are proton-neutron (pn) collisions. As precise pn $\gamma$ cross sections are not available one relies on theoretical investigations. In ref. such cross sections have been calculated with realistic nuclear forces, where in particular the low energy properties of the pn system have been incorporated very carefully. The cross section shows two distinct features:

(a) a $1/\omega$ singularity in the limit of vanishing $\gamma$-energy $\omega \to 0$ (soft photon limit)

(b) an enhancement of the cross section towards the end of phase space $\omega \to \omega_{\text{max}}$ beyond the classical expected behavior.

The infra-red divergence (a) is generic to all photon production cross sections and relates to the infinite time scale available in free scattering (Low-theorem, c.f. lectures by O. Scholten, these proceedings). For the in-matter production rates in the IQF approximation the infra-red divergence remains, which is obviously in contradiction to the sum-rule requirement (3). The reason is that in dense matter such long time scales are not available and the corresponding in-matter rates will be quenched as we shall see. This infra-red problem is the main subject of this lecture. Feature (b), which is particular to the pn-system, is caused by a resonance close to zero kinetic energy in the pn channel (the anti-bound state), which becomes relevant at the kinematical limit, i.e. at the maximum photon energy in each collision. In IQF approximation for the dense matter rates also this structure causes problems with the sum-rule, since it augments the yield at large photon energies such that the resulting slope is more flat than that of $e^{-\omega/\gamma}$, c.f. ref. However such a resonance structure close to zero kinetic energies requires a very long length scale which again is not available in dense matter. Therefore we see that in both cases the sum rules point towards a defect of the underlying approximation used for the dense matter rates, namely the IQF approximation.

4. Bremsstrahlung from Classical Sources

For a clarification of the infra-red problem we first discuss two simple examples of classical electrodynamics, which both can be solved analytically to a certain extent:
a diffusion process and a random walk problem.

The \textit{diffusion process} is assumed to be described by a Fokker-Planck equation for the probability distribution $f$ of position and velocity $\vec{x}, \vec{v}$:

$$
\frac{\partial}{\partial t} f(\vec{x}, \vec{v}, t) = \left( D \frac{\partial^2}{\partial \vec{x}^2} + \Gamma_x \frac{\partial}{\partial \vec{v}} - \vec{v} \frac{\partial}{\partial \vec{x}} \right) f(\vec{x}, \vec{v}, t).
$$

(5)

Likewise fluctuations evolve in time by this equation and this way determine the correlations. The two macroscopic parameters are the spatial diffusion coefficient $D$ and a friction constant $\Gamma_x$ which determines the relaxation rates of velocities (friction due to collisions with the medium). In the equilibrium limit ($t \to \infty$) the distribution attains a Maxwell-Boltzmann velocity distribution where $T = \frac{m}{3} \frac{\langle \vec{v}^2 \rangle}{\Gamma_x}$. The correlation function can be obtained in closed form and one can discuss the resulting time correlations of the current at different fixed values of the photon momentum $q$, fig. 1 (details are given in ref.3). For the transverse part of the correlation tensor this correlation decays exponentially as $\sim e^{-\Gamma_x \tau}$ at $q = 0$, and its width further decreases with increasing momentum $q = |\vec{q}|$. Besides trivial kinematical factors, the in-medium production rate is given by the time Fourier transform $\tau \to \omega$. Fig. 2 displays the corresponding total production rates $d^2 N/(d\omega dt)$ of on-shell photons (number per time and energy; which is dimensionless) in units of $4\pi \epsilon^2 \langle \vec{v}^2 \rangle / 3$. One sees that the hard part of the spectrum behaves as expected, namely, like in the IQF approximation the rate grows proportional to $\Gamma_x$ and this way proportional to the microscopic collision rate $\Gamma$ (c.f. below). However independent of $\Gamma_x$ the rate saturates at a value of $\sim 1/2$ in these units around $\omega \sim \Gamma_x$, and the soft part shows the inverse behaviour. That is,

$$
\text{current correlation function in units of } e^2 \langle \vec{v}^2 \rangle \text{ as a function of time (in units of } 1/\Gamma_x \text{) for different values of the photon momentum } q^2 = 3k^2 \Gamma_x / \langle \vec{v}^2 \rangle \text{ with } k = 0, 1, 2, 3.
$$

$$
\text{photon rate (diffusion model) in units of } 4\pi \epsilon^2 \langle \vec{v}^2 \rangle / 3 \text{ for a non-relativistic source for } \Gamma_x = 50, 100, 150 \text{ MeV; for comparison the IQF results (dashed lines) are also shown.}
$$

with increasing collision rate the production rate is more and more suppressed! This is in line with the picture that such photons cannot resolve the individual collisions any more. Since the soft part of the spectrum behaves like $\omega / \Gamma_x$, it shows a genuine
non-perturbative feature which cannot be obtained by any power series in $\Gamma_x$. For comparison: the dashed lines show the corresponding IQF yields, which agree with the correct rate for the hard part while they completely fail and diverge towards the soft end of the spectrum. For non-relativistic sources $\langle \vec{v}^2 \rangle \ll 1$ one can ignore the additional $q$-dependence (dipole approximation; c.f. fig. 1) and the entire spectrum is determined by one macroscopic scale, the relaxation rate $\Gamma_x$. This scale provides a quenching factor

$$C_0(\omega) = \frac{\omega^2}{\omega^2 + \Gamma_x^2}.$$  

by which the IQF results have to be corrected in order to account for the finite collision time effects in dense matter.

In the microscopic Langevin picture one considers a classical process, where hard scatterings occur at random with a constant mean collision rate $\Gamma$. These scatterings consecutively change the velocity of a point charge from $\vec{v}_m$ to $\vec{v}_{m+1}$ to $\vec{v}_{m+2}$, ... (in the following subscripts $m$, and $n$ refer to the collision sequence). In between scatterings the charge moves freely. For such a multiple collision process some explicit results can be given, since the correlated probability to find the charge at time $t_1$ and $t_2$ at two different segments with $n$ scatterings in between follows from the iterative folding of the exponential decay law with decay time $1/\Gamma$. Therefore the space integrated current-current correlation function takes a simple Poissonian form (fig. 3)

$$\int d^3x_1 d^3x_2 \langle j^i(x_1, t - \frac{1}{\tau}) j^k(x_2, t + \frac{1}{\tau}) \rangle = e^2 \langle v_i(0) v_i(\tau) \rangle$$

$$= e^2 e^{-\Gamma \tau} \sum_{n=0}^{\infty} \frac{\Gamma \tau^n}{n!} \langle v_i^n v_i^k \rangle_m,$$

which represents a genuine multiple collision description of the correlation function. Here $\langle \ldots \rangle_m$ denotes the average over the discrete collision sequence $\{m\}$. This form, which one writes down intuitively, directly includes what one calls damping in the corresponding quantum case. Fourier transformed it determines the spectrum in completely regular terms (void of any infra-red singularities) where each term describes the interference of the photon being emitted at a certain time or $n$ collisions later.

In special cases where velocity fluctuations are degraded by a constant fraction $\alpha$ in each collision, such that $\langle \vec{v}_m \cdot \vec{v}_{m+n} \rangle_m = \alpha^n \langle \vec{v}_m \cdot \vec{v}_m \rangle_m$, one can re-sum the whole
series in (7) and thus recover the relaxation result with \(2\Gamma_x \langle \bar{v}^2 \rangle = \Gamma (\bar{v}_{m} - \bar{v}_{m+1})^2\) at least for \(\vec{q} = 0\) and the corresponding quenching factor (6).

This clarifies that the diffusion result represents a resummation of the Langevin multiple collision picture and altogether only macroscopic scales are relevant for the form of the spectrum and not the details of the microscopic collisions.

5. Radiation on the Quantum level

We have seen that on the classical level the problem of radiation from dense matter can be solved quite naturally and completely at least for simple examples, and figs. 1 and 2 display the main physics. On the quantum level this problem is a challenge, since it requires techniques, that go beyond the standard repertoire of perturbation theory or the quasi-particle approximation. The classical examples above show, that the damping of the particles due to scattering is an important feature, which in particular has to be included right from the beginning of the description. This does not only assure results which no longer diverge, but also provides a systematic and convergent scheme.

As a consequence the mass spectrum of the particles in the dense matter is no longer a sharp delta function but rather acquires a width due to collisions. The corresponding quantum propagators (Green's functions) are no longer the ones as in the standard text books for fixed mass, but rather have to be folded over a so called spectral function \(A(\epsilon, \vec{p})\) which takes a Lorentz shape \(A(\epsilon, \vec{p}) \sim \Gamma/((\epsilon - \epsilon(\vec{P}))^2 + (\Gamma/2)^2)\) of width \(\Gamma/2\) in simple approximations. One thus comes to a picture which unifies resonances which have already a width in vacuum due to decay modes with the "states" of particles in dense matter, which obtain a width due to collisions (collisional broadening).

The theoretical concepts for a proper many body description in terms of a real time non equilibrium field theory have already been devised by Schwinger, Kadanoff, Baym and Keldysh in the early 60's. First investigations of the quantum effects on the Boltzmann collision term were given Danielewicz, the principle conceptual problems on the level of quantum field theory were investigated by Landsmann, while applications which seriously include the finite width of the particles in transport descriptions were carried out only in recent times, e.g. the delta resonance, it was natural to consider broad mass distributions and ad hoc recipes have been invented to include this in transport simulation models. However, many of these recipes are not correct as they violate some basic principle like detailed balance, and the description of resonances in dense matter has to be improved.

6. Metamorphosis of diagrams

In the following I like to illustrate the steps that lead to the proper diagrammatic formulation of non-equilibrium processes. We all are used to think in terms of Feynman diagrams which describe amplitudes in terms of certain "in" and "out" states. This
formulation, however, is linked to the concept of asymptotic states, hence states with infinite life time. Such concepts are no longer appropriate in dense matter.

Nevertheless I like to introduce the non-equilibrium diagrams at an example with asymptotic in and out states. Consider a single particle system such a H-atom (the generalization to many-body systems is straightforward) which undergoes transitions from some initial state $|i\rangle$ with occupation $n_i$ to final states $|f\rangle$ due to the coupling to the photon field. The transition rate is given by the absolute square of the following diagram

$$W = \sum_{i,f} n_i(1-n_f) \left| \begin{array}{c} f \\ i \end{array} \right| \frac{2}{(1+n_\omega)\delta(E_i - E_f - \omega_F)}$$

with occupation $n_\gamma$ for the photon. For the subsequent transformation of diagrams one applies the following rules: in any diagram each vertex has to be marked by a $-$ sign; for the conjugate complex of a diagram all the vertices are to be marked by a $+$ sign and all line senses are to be inverted. The absolute square in (8) then becomes

$$W = \sum_{i,f} n_i(1-n_f) \left\{ \begin{array}{c} f \\ i \end{array} \right\} \times \left\{ \begin{array}{c} f \\ i \end{array} \right\} \frac{2}{(1+n_\omega)\delta(E_i - E_f - \omega_F)},$$

where we have placed the two diagrams in between braces. Since one sums over initial and final states $i$ and $f$ one can equally well close the corresponding fermion lines in the two diagrams and comes to the following closed diagram

$$W = \left( 1+n_\omega \right) \delta(\omega - \omega_F).$$

We see that with the line sense and the $-$ and $+$ marks at the vertices a unique correspondence is provided between the oriented $\pm -$ and $\mp -$ propagator lines and the initial and final states. Therefore such propagator lines define the density of occupied states or that of available states, respectively. Extending therefore the diagrammatic rules to the two types of vertices with marks $-$ and $+$ and the corresponding 4 propagators, the usual Feynman propagator $\mp -$ between two $-$ vertices, its conjugate complex $\pm -$ between two $+$ vertices and the mixed $\pm -$ or $\mp -$ ones as Wigner densities of occupied and available states all standard diagrammatic rules can be used again. For details I refer to the textbook of Lifshitz and Pitaevskii.

Closed diagrams with one photon "in" and one "out" vertex directly determine the correlation function (1), and each such diagram describes the interference of two Feynman amplitudes. The latter can be recovered through the inverse procedure just cutting the diagram across any $-+$ or $+-$ line. The advantage of the formulation in terms of "correlation" diagrams, which no longer refer to amplitudes but directly relate to physical observables, is that now one is no longer restricted to the concept of asymptotic states. Rather all internal lines, also the ones which originally correspond
to the in or out states are now treated on equal footing. Therefore one now can deal
with "states" which have a broad mass spectrum and therefore appropriately account
for the damping of the particles. The corresponding Wigner densities \( \pm \rightarrow \) or \( \mp \rightarrow \) are
then no longer on-shell \( \delta \)-functions in energy (on-mass shell) but rather acquire a width
in terms of the spectral function, e.g. for fermions

\[
\pm \rightarrow = i n(\epsilon) A_F(\epsilon, \vec{p}) \\
\mp \rightarrow = i (1 - n(\epsilon)) A_F(\epsilon, \vec{p})
\]

\[ A_F(\epsilon, \vec{p}) = \frac{\Gamma(\epsilon, \vec{p})}{(\epsilon + \mu_F - c_\rho^2 - \mathfrak{R} \Sigma^R(\epsilon, \vec{p}))^2 + (\Gamma(\epsilon, \vec{p})/2)^2}. \]

Here \( n(\epsilon) \) is the phase-space occupation at energy \( \epsilon \), \( A_F \) is the fermion spectral function
with the damping width \( \Gamma \) and in-medium on-shell energy \( c_\rho^2 - \mathfrak{R} \Sigma^R(p) \) and \( \mu \) is the
chemical potential. In general all quantities are dependent on both, energy \( \epsilon \) and
momentum \( \vec{p} \). One further realizes that the production and absorption rate diagrams
have the same topology as photon self energy diagrams, except that now the new
rules apply and the two external photon vertices are marked with opposite signs. This
correspondence is equivalent to the standard relation between the imaginary part of
the retarded self energy and the damping rate of a particle. There are many other
advantages which open this technic to genuine non-equilibrium problems.

Therefore the production or absorption rates are obviously related
to the photon self energy which can be given by the diagram to the right
with an in- and outgoing photon line (dashed), where the hatched loop
area denotes all strong interactions of the source. The latter give rise to a whole series
of diagrams. As mentioned, standard perturbation theory or the quasi-particle picture
do no longer apply to dense matter problems. Rather for the particles of the source,
e.g. the nucleons, one has to re-sum Dyson’s equation with the full complex self energy
of the nucleons in order to determine the full Green’s functions of the nucleons in dense
matter. Once one has these Green’s functions and the interaction vertices one can in
principle calculate the required diagrams. However both, the computational effort to
calculate a single diagram and the number of diagrams, are increasing dramatically
with the loop order of the diagrams, such that in practice only lowest order loop di-
agrams can be considered in the full quantum case. In certain limits some diagrams
drop out. We could show that in the classical limit of the quantum description only
the following set of diagrams survive

\[
\begin{array}{c}
+ \quad - + \\
+ \quad - + \\
\end{array} + \cdots
\]

Here the full lines denote the full nucleon Green’s functions which also include the
damping width, the black blocks denote the effective nucleon-nucleon interaction in
matter, and the full dots the coupling vertex to the photon. Each of these diagrams
with $n$ interaction loop insertions just corresponds to the $n^{th}$ term in the classical Langevin result (7). Thus the classical multiple collision example provides a quite intuitive picture about such diagrams. Thereby the diagram of order $n$ describes the interference of the amplitude where the photon is "emitted" at some time and that where it is "emitted" $n$ collisions later. Further details are given in

7. Qualitative changes going to a full quantum description

As the infra-red limit corresponds to the limit of classical electrodynamics one expects even quantitatively the same behavior for the here discussed quenching effect for the soft part of the spectrum. Beyond this one expects the following qualitatively changes going from the classical description to the full quantum field theory:

- far more diagrams than in the classical case contribute; still for specific couplings certain classes of diagrams drop or cancel out, like non-planar diagrams in SU(n) coupling; also if the relaxation rate $\Gamma_x$ is close to the collision rate $\Gamma$ (e.g. for isotropic scattering with heavy particles) higher order correlations drop and the one loop diagram gives the complete result;
- the radiated quantum has a finite energy $\hbar \omega$ and momentum $\hbar \vec{q}$ (in the classical case $\hbar \to 0$!); therefore one expects additional recoil corrections and phase space limitation factors, the latter leading to a suppression at large energies $\sim e^{-\omega/T}$ for the production rates from the available phase space in the final state, due to the energy $\omega$ taken by the quantum;
- occupations are no longer of Boltzmann type so that Pauli suppression and Bose-Einstein enhancement effects appear.

8. Conclusion

We have seen that the spectrum of photon resulting from collisions in dense matter cannot appropriately be described by the incoherent quasi-free scattering approximation, which leads to a divergent result in the soft limit. Rather the finite time between successive collisions and the ensuing relaxation rates $\Gamma_x$ in dense matter lead to a considerable quenching of the rate at small photon energies. This can be compiled in the simple quenching factor (6). Fig. 2 summarizes the main behavior, which also is relevant in a quantum treatment of the source.

Our considerations are of particular importance for the theoretical description of nucleus-nucleus collisions at intermediate to relativistic energies. Since the here discussed features are entirely of kinematical origin, they apply to the in-medium production and absorption rates of any kind of particle. With temperatures $T$ in the range of 30 to 100 MeV for dense nuclear matter, up to 200 MeV for hadronic matter and beyond 150 MeV for the quark gluon plasma or parton phase most of the kinetic models that are used infer collision rates $\Gamma$ for the constituents, which during the high density phase can reach the system's temperature, $\Gamma \lesssim T$. Such estimates make the use...
of on-shell concepts already rather questionable. The particles uncertainty in energy is comparable with their mean kinetic energy! In particular the bulk production and absorption rates of all particles with masses less than $T$, if calculated in standard IQF approximation, are seriously subjected to the here discussed effect. Therefore the corresponding quenching factors (6) should sensitively affect the production rates of quark pairs and gluons during the plasma phase, of low energy pions during hadronization and real and virtual photons with correspondingly low energies. Since our discussion was restricted to the production in dense matter with no incoming or outgoing asymptotic states, for the particular case of photon production in nuclear collisions one has to consider in addition the radiation caused by the incoming charged ions and outgoing charged fragments. Due to Low's theorem the latter give rise to an infra-red divergent $\sim 1/\omega$ component which interferes with the in-matter component discussed here.

In summary, the combined effort from many sides to include the finite width of the particles in dense matter, may give hope for a unified transport theory which appropriately describes both, the propagation of resonances and of off-shell particles in the dense matter environment.

References