Exotic Phenomena in Nuclei

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Shells, Clusters and Condensates

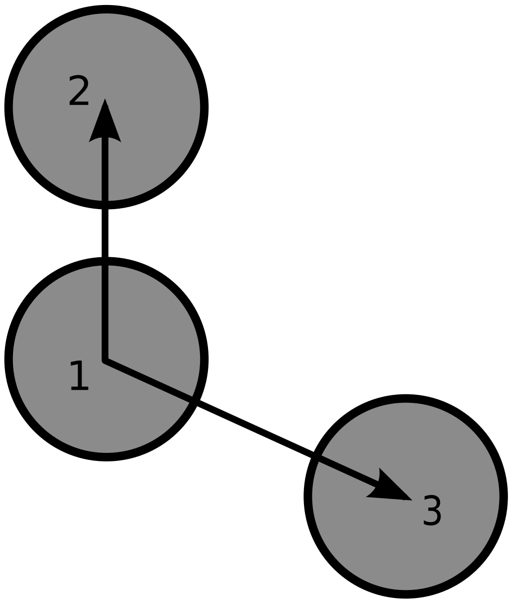
Nucleus as a Mesoscopic System

- **shell structure**
- **cluster structure**
- α**-cluster condensates close to the** α**-threshold**

Hoyle State in ^{12}C

- **microscopic** α**-cluster model**
- **microscopic** α**-cluster model with condensate wave functions**
- **Fermionic Molecular Dynamics model that can describe shell model like configurations as well as** α**-cluster configurations**

Microscopic α-Cluster Model



$$R_{12} = (2, 4, \dots, 10)\ \text{fm}$$
$$R_{13} = (2, 4, \dots, 10)\ \text{fm}$$
$$\cos(\vartheta) = (1.0, 0.8, \dots, -1.0)$$

alltogether 165 configurations

Kamimura, Nuc. Phys. **A351**, 456 (1981)

Basis States

- describe $^{12}\mathrm{C}$ as a system of 3 $^{4}\mathrm{He}$ nuclei
- Generator Coordinate Method: use Slater determinants to describe many-body state

$$\left|\Psi_{3\alpha}(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3)\right\rangle = \mathcal{A}\left\{\left|\psi_\alpha(\mathbf{R}_1)\right\rangle \otimes \left|\psi_\alpha(\mathbf{R}_2)\right\rangle \otimes \left|\psi_\alpha(\mathbf{R}_3)\right\rangle\right\}$$

Volkov Interaction

- simple central interacion
- parameters adjusted to reproduce α binding energy and radius, exchange parameter adjusted to reproduce $^{12}\mathrm{C}$ ground state energy and radius

✘ only reasonable for $^{4}\mathrm{He}$, $^{8}\mathrm{Be}$ and $^{12}\mathrm{C}$ nuclei

Condensate Wave Functions

Basis States

- Funaki *et al* proposed a description based on condensate wave functions
- all α-particles occupy the same spatially deformed center-of-mass orbit

$$\left|\Psi_{\mathrm{BEC}}(\beta_x,\beta_y,\beta_z)\right\rangle = \int d^3R_1\, d^3R_2\, d^3R_3\, \exp\left\{-\sum_{i=1}^{3}\left(\frac{R_{ix}^2}{\beta_x^2}+\frac{R_{iy}^2}{\beta_y^2}+\frac{R_{iz}^2}{\beta_z^2}\right)\right\}\left|\Psi_{3\alpha}(\mathbf{R}_1,\mathbf{R}_2,\mathbf{R}_3)\right\rangle$$

- Generator Coordinate Method:
  consider the deformation parameters $(\beta_x,\beta_y,\beta_z)$ as generator coordinates

➨ wave function has to be antisymmetrized, densities are not so low that antisymmetrization effects can be neglected

➨ detailed analysis by Matsumara *et al* finds 30% S-wave probability for the ground state and 70% S-wave probability for the Hoyle state

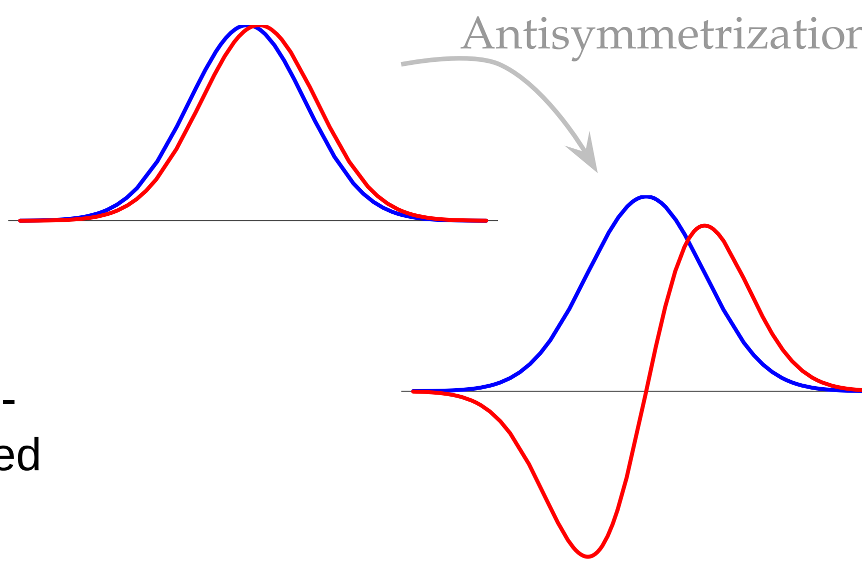
Volkov Interaction

- use same interaction as for the microscopic α-cluster model

Y. Funaki *et al*, Phys. Rev. C**67**, 051306(R) (2003)
H. Matsumara and Y. Suzuki, Nuc. Phys. **A739**, 238 (2004)

Fermionic Molecular Dynamics

Wave Functions



- describe many-body system by Slater determinants

$$\left| Q \right\rangle = \mathcal{A}\Big(\left| q_1 \right\rangle \otimes \cdots \otimes \left| q_A \right\rangle\Big)$$

- single-particle states are Gaussian wave-packets in phase-space (complex parameter $\mathbf{b}_i$ encodes mean position and mean momentum), spin is free, isospin is fixed

$$\left\langle \mathbf{x} \middle| q \right\rangle = \exp\left\{-\frac{(\mathbf{x}-\mathbf{b})^2}{2a}\right\} \otimes \left| \chi^{\uparrow}, \chi^{\downarrow} \right\rangle \otimes \left| \xi \right\rangle$$

Interaction

- derive effective interaction from Argonne V18 with the **Unitary Correlation Operator Method** which introduces short-range central and tensor correlations
- augment with phenomenological two-body correction that contains momentum-dependent central and spin-orbit terms fitted to doubly-magic nuclei to mimic effects of three-body correlations and genuine three-body forces

Rev. Mod. Phys. **72** (2000) 655

Nucl. Phys. **A745** (2004) 3

Fermionic Molecular Dynamics

Basis States

- 20 FMD states obtained in **Variation after Angular Momentum and Parity Projection** on 0^+ and 2^+ with constraints on the radius
- 42 FMD states obtained in **Variation after Parity Projection** with constraints on radius and quadrupole deformation
- 165 explicit α-cluster configurations

$$P^{\mathbf{P}} = \frac{1}{(2\pi)^3} \int d^3X \, \exp\{-i(\mathbf{P}-\mathbf{P})\cdot\mathbf{X}\}$$

$$P^J_{MK} = \frac{2J+1}{8\pi^2} \int d^3\Omega \, D^{J\,\star}_{MK}(\Omega)\, R(\Omega)$$

Solve the Many-Body Schrödinger Equation

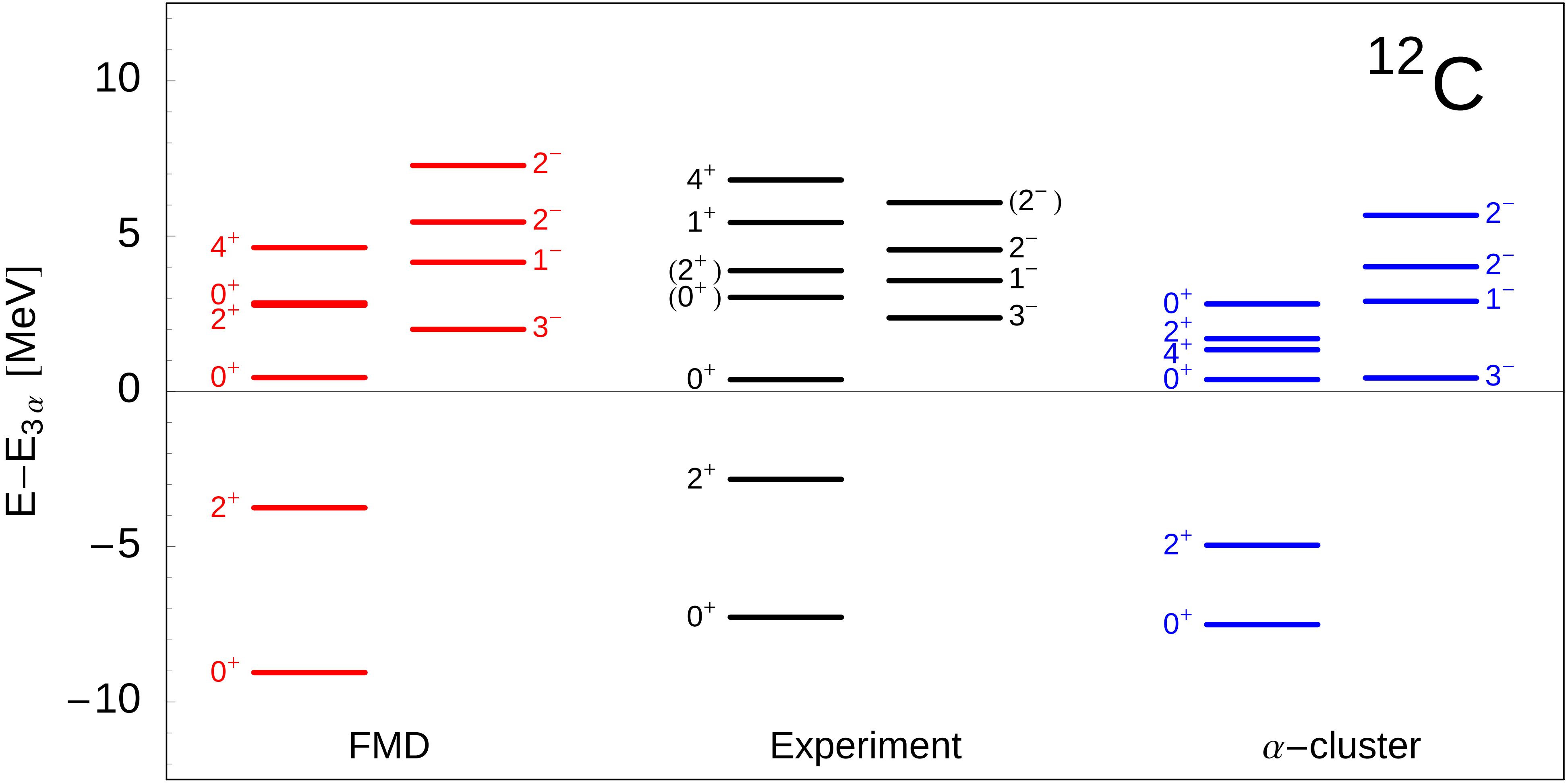
- Generalized Eigenvalue problem
- matrix elements are projected on parity, angular momentum and linear momentum

$$\sum_{K'b} \left\langle Q^{(a)} \middle| H P^{J\pi}_{KK'} P^{\mathbf{P}=0} \middle| Q^{(b)} \right\rangle \cdot c^{(i)}_{K'b} = E^{J^\pi(i)} \sum_{K'b} \left\langle Q^{(a)} \middle| P^{J\pi}_{KK'} P^{\mathbf{P}=0} \middle| Q^{(b)} \right\rangle \cdot c^{(i)}_{K'b}$$

Spectra

FMD and α-cluster reproduce Hoyle state close to threshold

differences for 0_3^+ and 2_2^+ states



Comparison

	Exp[1]	Exp[2]	Exp[3]	FMD	α-cluster	'BEC'
$E(0_1^+)$	-92.16			-92.64	-89.56	-89.52
$E^*(2_1^+)$	4.44			5.31	2.56	2.81
$E(3\alpha)$	-84.89			-83.59	-82.05	-82.05
$\mathbf{E(0_2^+) - E(3\alpha)}$	**0.38**			**0.43**	**0.38**	**0.26**
$E(0_3^+) - E(3\alpha)$	(3.0)	2.7(3)	3.96(5)	2.84	2.81	
$E(2_2^+) - E(3\alpha)$	(3.89)	2.6(3)	6.63(3)	2.77	1.70	
$r_{\text{charge}}(0_1^+)$	2.47(2)			2.53	2.54	
$r(0_1^+)$				2.39	2.40	2.40
$\mathbf{r(0_2^+)}$				**3.38**	**3.71**	**3.83**
$r(0_3^+)$				4.62	4.75	
$r(2_1^+)$				2.50	2.37	2.38
$r(2_2^+)$				4.43	4.02	
$M(E0, 0_1^+ \to 0_2^+)$	5.4(2)			6.53	6.52	6.45
$B(E2, 2_1^+ \to 0_1^+)$	7.6(4)			8.69	9.16	
$B(E2, 2_1^+ \to 0_2^+)$	2.6(4)			3.83	0.84	

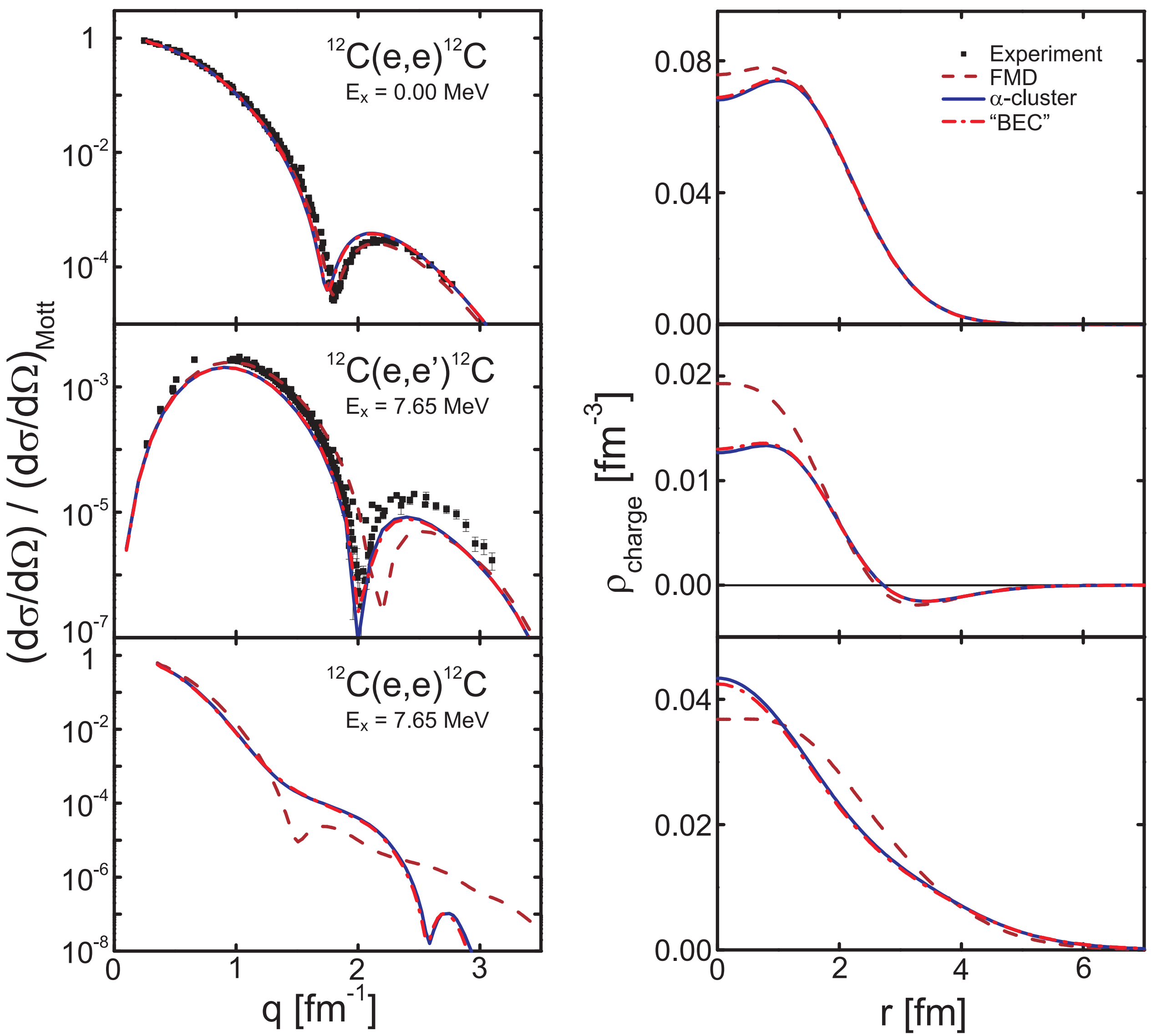
very large radii for 0_2^+, 0_3^+ and 2_2^+ states

experimental situation for 0_3^+ and 2_2^+ states still unsettled

2_2^+ resonance at 1.8 MeV above treshold included in NACRE compilation

[1] Ajzenberg-Selove, Nuc. Phys. **A506**, 1 (1990)
[2] Itoh et al., Nuc. Phys. **A738**, 268 (2004)
[3] Fynbo et al, Nature **433**, 137 (2005). Diget et al., Nuc. Phys. **A738**, 760 (2005)

Form factors and Densities



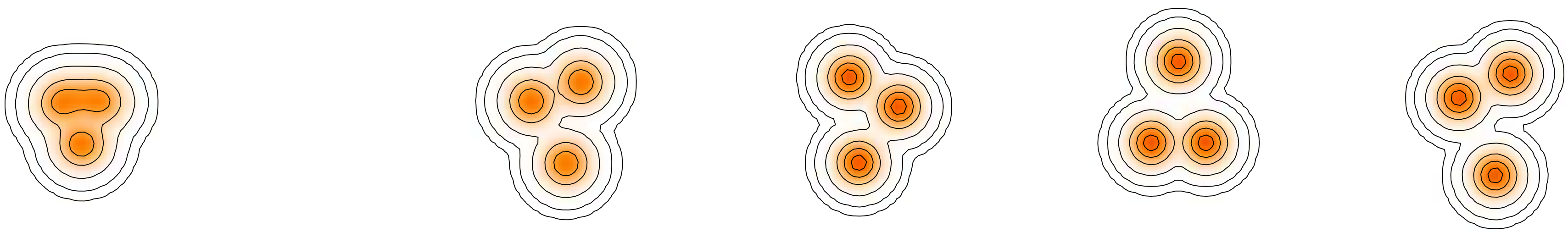
- compare with electron scattering data in Distorted Wave Born Approximation
- elastic form factor described very well by FMD
- transition form factor in first maximum better described by FMD, position of minimum and second maximum better described by cluster model

use intrinsic density

$$\rho(\mathbf{x}) = \sum_{k=1}^{A} \left\langle \Psi \middle| \delta(\mathbf{x}_k - \mathbf{X} - \mathbf{x}) \middle| \Psi \right\rangle$$

Important Configurations

- Calculate the overlap with FMD basis states to find the most important contributions to the Hoyle state



$\left\langle \cdot \middle| 0_1^+ \right\rangle = 0.94$, $\left\langle \cdot \middle| 0_2^+ \right\rangle = 0.04$

$\left\langle \cdot \middle| 0_1^+ \right\rangle = 0.30$, $\left\langle \cdot \middle| 0_2^+ \right\rangle = 0.72$

$\left\langle \cdot \middle| 0_1^+ \right\rangle = 0.25$, $\left\langle \cdot \middle| 0_2^+ \right\rangle = 0.71$

$\left\langle \cdot \middle| 0_1^+ \right\rangle = 0.15$, $\left\langle \cdot \middle| 0_2^+ \right\rangle = 0.61$

$\left\langle \cdot \middle| 0_1^+ \right\rangle = 0.08$, $\left\langle \cdot \middle| 0_2^+ \right\rangle = 0.61$

shell model like structure for the ground state

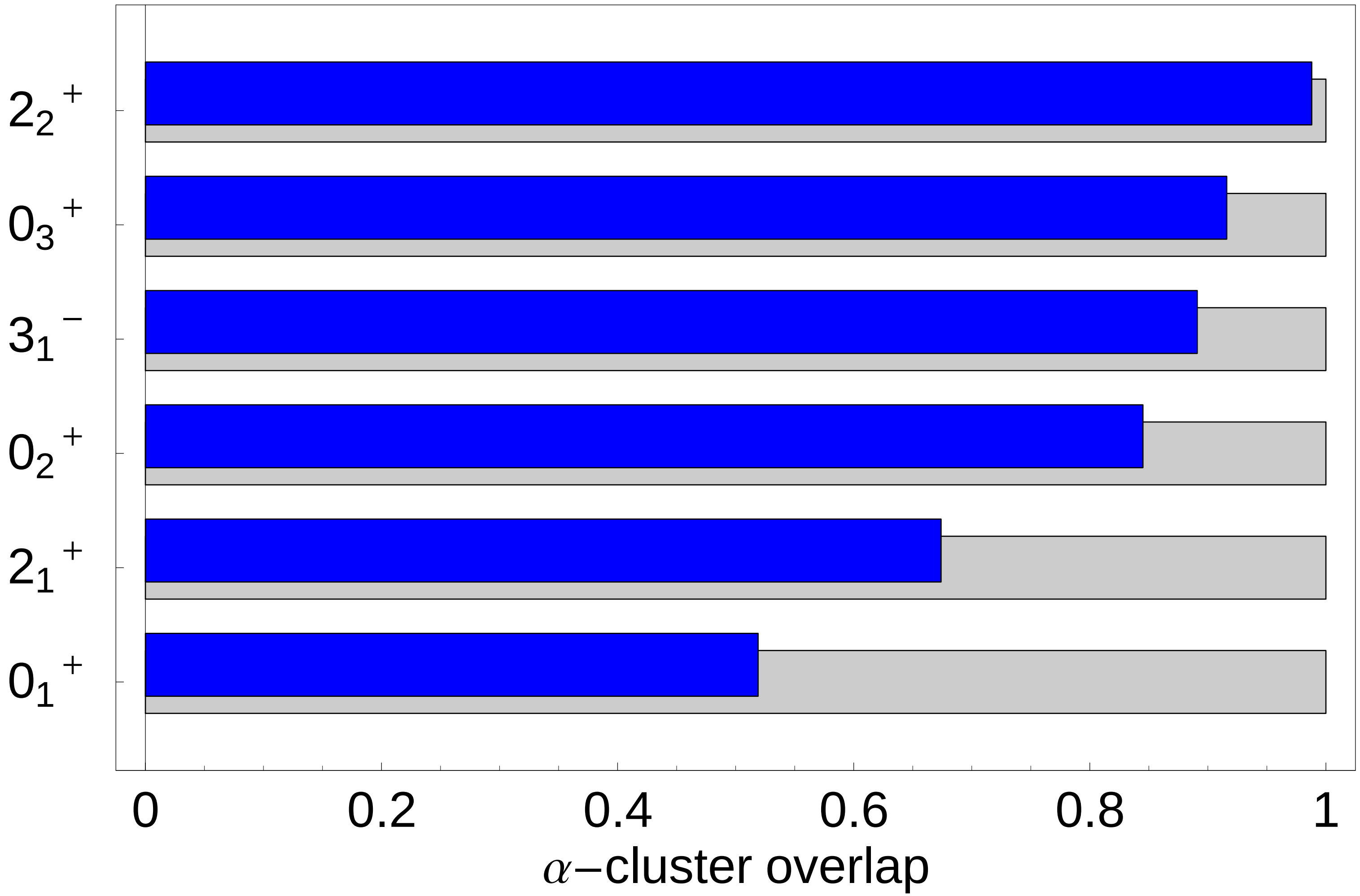
Hoyle state dominated by α-cluster configurations

FMD basis states are not orthogonal!

Overlap with Cluster Model Space

Calculate the overlap of FMD wave functions with pure α-cluster model space

$$N_\alpha = \left\langle \Psi \middle| P_{3\alpha} \middle| \Psi \right\rangle$$

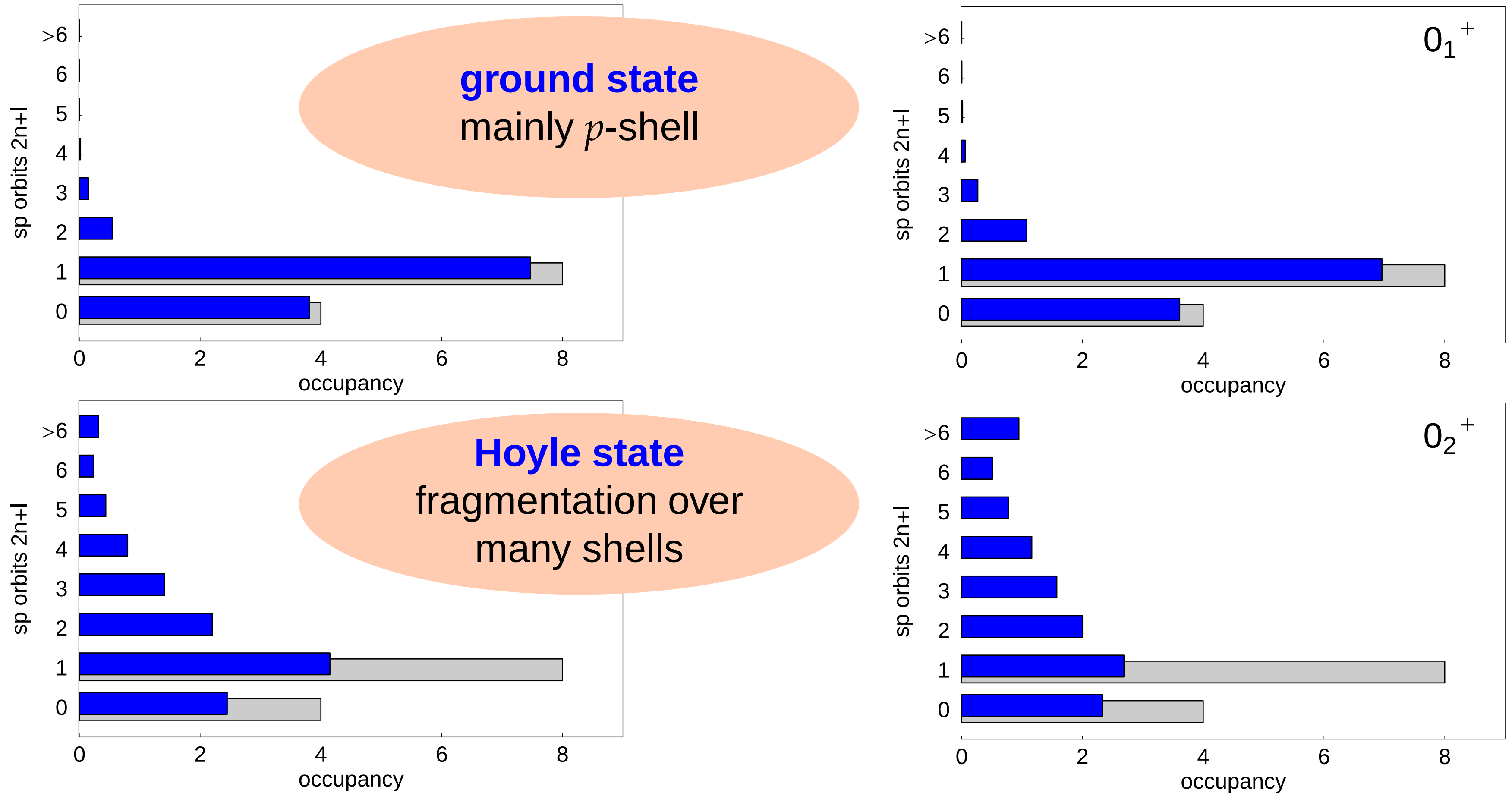


Harmonic Oscillator Occupation Numbers

- calculate one-body density in harmonic oscillator basis

$$n_{nlj} = \sum_m \left\langle \Psi \middle| a^{\dagger}_{nljm} a_{nljm} \middle| \Psi \right\rangle$$

FMD | **α-cluster**

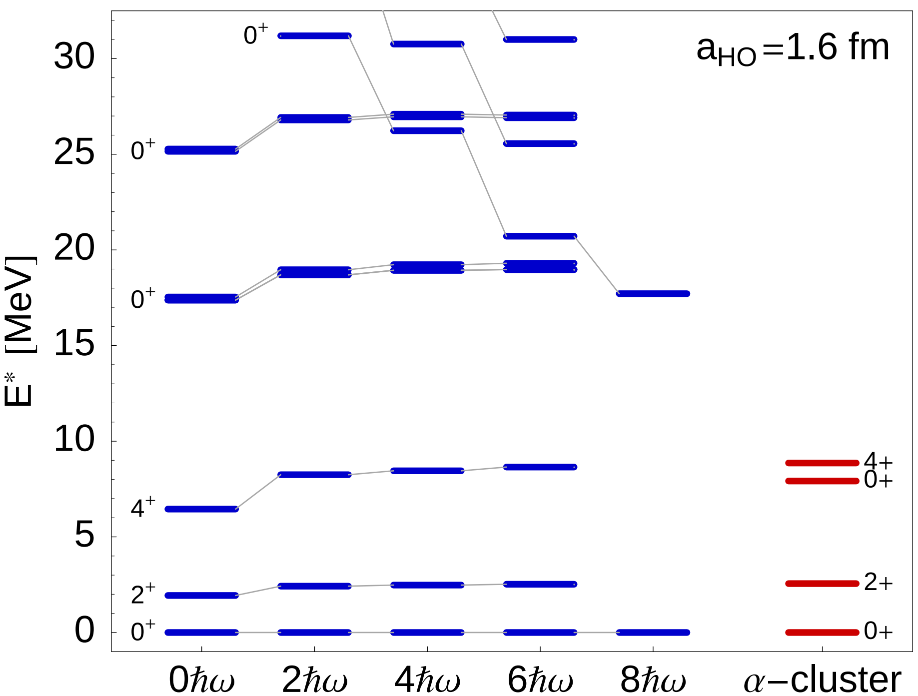


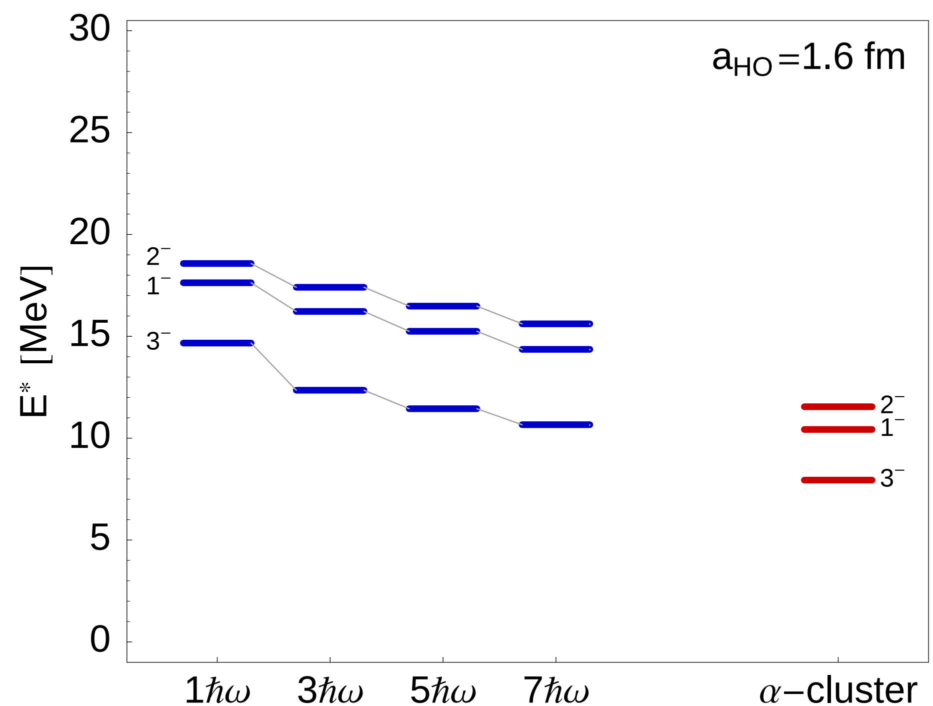
α-cluster states in the No-Core Shell Model ?

- compare spectra in NCSM and α-cluster model using the Volkov interaction
- bare interaction used in NCSM calculations

good agreement for ground state band $(0_1^+, 2_1^+, 4_1^+)$

very slow convergence for cluster states





Summary

- α-cluster model and "BEC" ansatz can describe $^{12}\mathrm{C}$ ground state and Hoyle state energies and form factors.
- Hoyle state is a dilute system of α-particles
- results confirmed by FMD calculations that make no explicit assumption about the α-cluster structure
- Harmonic Oscillator basis has a hard time describing the Hoyle state