Realistic interactions and short-range correlations in nuclei

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Overview

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Motivation

**Aim**

describe basic properties of nuclear system in terms of a realistic nucleon-nucleon interaction

**realistic interactions** $\tilde{H}$

- reproduce scattering data and deuteron properties
- meson-exchange (Bonn), phenomenological (Argonne), $\chi$-PT
- feature short-ranged repulsion and strong tensor force

**Many-Body State $|\Phi\rangle$**

- mean-field calculations (with effective interactions) $\langle \Phi|\tilde{H}_{\text{eff}}|\Phi\rangle$ (Slater determinant $|\Phi\rangle$)
  describe bulk properties (energies, radii) well
- realistic interactions induce short-ranged central and tensor correlations, Slater determinants cannot describe these

**Ansatz**

- use realistic interactions
- and include correlations with unitary correlation operator $\tilde{C}$
- in simple many-body states (shell-model, FMD) $\langle \Phi|\tilde{C}^\dagger \tilde{H}\tilde{C}|\Phi\rangle$
**Unitary Transformation**

Transform eigenvalue problem

\[ \hat{H} | \hat{\Psi}_n \rangle = E_n | \hat{\Psi}_n \rangle \]

with the unitary operator \( \hat{C} \)

\[ | \hat{\Psi}_n \rangle = \hat{C} | \Psi_n \rangle, \quad \hat{C}^{-1} = \hat{C}^\dagger \]

into the equivalent eigenvalue problem

\[ \hat{\tilde{H}} | \Psi_n \rangle = (\hat{C}^\dagger \hat{H} \hat{C}) | \Psi_n \rangle = E_n | \Psi_n \rangle \]

*“pre-diagonalization”*

include typical effects common to all states

**Nuclear System**

The nuclear system has different scales:

- **long range (low momenta)** – can be described by mean-field (Slater determinant)
- **short-range (high momenta)** – cannot be described by mean-field

Include short-range correlations by unitary transformation

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*Unitary correlator admixes components from outside the model space \( | \Psi_n \rangle \) does not project on model space*
The Unitary Correlation Operator

Two-Body Correlations

- two-body generator

\[ \mathcal{C} = e^{-i\mathcal{G}}, \quad \mathcal{G} = \sum_{i<j} g_{ij} \]

Cluster Expansion

correlated operators \( \hat{A} = \mathcal{C}^{\dagger} A \mathcal{C} \) are no longer operators with definite particle number

- decompose correlated operator into irreducible \( k \)-body operators

\[ \hat{A} = \hat{A}^{[1]} + \hat{A}^{[2]} + \ldots \]

Two-Body Approximation

\[ \hat{T}^{C2} = \hat{T}^{[1]} + \hat{T}^{[2]}, \quad \hat{V}^{C2} = \hat{V}^{[2]} \]

Correlator \( \mathcal{C} \)

should conserve translational, rotational and Galilei invariance

cluster decomposition principle should be fulfilled

Spin-Isospin Dependence

nuclear interaction strongly depends on spin and isospin

\[ \mathcal{v} = \sum_{S,T} v_{ST} \Pi_{ST} \]

- different correlations in the respective channels

\[ \mathcal{g} = \sum_{S,T} g_{ST} \Pi_{ST} \]

- correlated interaction in two-body space

\[ \hat{\mathcal{V}} = \sum_{S,T} (e^{ig_{ST}} \mathcal{v}_{ST} \mathcal{e}^{-ig_{ST}}) \Pi_{ST} \]
Central and Tensor Correlations

\[ C = C_\Omega C_r \]

\[ p = p_r + p_\Omega \]
\[ p_r = \frac{1}{2} \left\{ \frac{r}{r} (r \cdot p) + \left( p \frac{r}{r} \right) \frac{r}{r} \right\}, \quad p_\Omega = \frac{1}{2r} \left\{ l \times \frac{r}{r} - \frac{r}{r} \times l \right\} \]

**Central Correlations**

\[ g_r = \frac{1}{2} \{ p_r s(r) + s(r) p_r \} \]

- probability density shifted out of the repulsive core

**Tensor Correlations**

\[ g_\Omega = \partial(r) \left\{ \frac{3}{2} (\sigma_1 \cdot p_\Omega) (\sigma_2 \cdot r) + \frac{3}{2} (\sigma_1 \cdot r) (\sigma_2 \cdot p_\Omega) \right\} \]

- tensor force admixes other angular momenta

**S = 0, T = 1**

**S = 1, T = 0**
determination of $s(r)$ und $\vartheta(r)$ by variational principle

$$\min_{s(r), \vartheta(r)} \langle \phi^{ST}_{\text{trial}} | C^\dagger_C^\dagger H C C^\dagger C r | \phi^{ST}_{\text{trial}} \rangle$$

Central Correlations

$$s(r) \text{ [fm]}$$

$4\text{He}$

$const$

$s(r)$

$r \text{ [fm]}$

$\Rightarrow$ correlator depends only weakly on the trial state

Tensor Correlations

$$\vartheta^d(r)$$

$$\vartheta(r)$$

$\alpha \beta \gamma$

$r \text{ [fm]}$

$\Rightarrow$ correlation range has to be restricted
Many-Body Calculations

- Central correlator shifts density out of the repulsive core
- Tensor correlator aligns density with spin orientation

\[ \rho_{S,T}^{(2)}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0 \]

Both central and tensor correlations are essential for binding

\[ \langle T \rangle \quad \langle V \rangle \quad \langle H \rangle \]
Many-Body Calculations

Benchmark AV8’

**Binding Energy**

![Graph showing binding energy vs. rms radius for 4He]

- Extremely simple trial state, only one Slater determinant
- Big cancelations between kinetic and potential energy, therefore binding energy very sensitive

**Contributions to the Binding Energy**

![Graph showing contributions to the binding energy vs. rms radius for 4He]

ref: PRC64 (2001) 044001
The figure shows the momentum distributions for 16O in the context of nucleon and other observables. The distributions are given in units of $A^{-1} [fm^3]$. The distributions are plotted against momentum $k$ in $[fm^{-1}]$.

- **Bonn-A**
  - The plot shows the momentum distribution with contributions from central and tensor correlations.
  - Correlations induce high-momentum components.
  - Contributions of tensor correlations are very big.
  - Different correlator ranges are relevant, especially at the Fermi surface.

- **Argonne V18**
  - The plot includes VMC and LDA contributions.
  - The central correlation range is highlighted.

The plots illustrate the importance of tensor correlations and the relevance of different correlator ranges in understanding nucleon momentum distributions.
Interaction in Momentum Space

\[ \langle klm \mid \hat{H}^{[2]} \mid k' l' m' \rangle = i^{l'-l} M \int d^3 x \ Y_{lm}^*(\hat{x}) j_l(kx) \langle x \mid \hat{H}^{[2]} \mid x \rangle j_{l'}(k'x) Y_{l'm'}(\hat{x}) \]

1\(^S\)\(_0\) channel

![Graph showing 1\(^S\)\(_0\) channel](image1)

3\(^S\)\(_1\) channel

![Graph showing 3\(^S\)\(_1\) channel](image2)

**unique effective potential** – identical to \(V_{\text{lowk}}\)
Kuo, Schwenk, nucl-th/0108041

**\(V_{\text{lowk}}\) Cutoff** \(\Lambda = 1.0 - 2.0\ \text{fm}^{-1}\)
AV18 Interaction in Momentum Space
Off-diagonal Matrix Elements

\[ \langle ^1S_0;k| V | ^1S_0;k' \rangle \]

\[ \langle ^1S_0;k| \hat{H}^{(2)} | ^1S_0;k' \rangle \]

\[ \langle ^3S_1;k| V | ^3D_1;k' \rangle \]

\[ \langle ^3S_1;k| \hat{H}^{(2)} | ^3D_1;k' \rangle \]

“pre-diagonalization”

bare potential

correlated interaction
No-Core Shell Model Calculations

- **3He**
  - correlated
  - bare
  - exact results from PRC52 (1995) 2885

- **4He**
  - correlated
  - bare

- use no-core shell model code from Pétr Navratil (LLNL)

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AV8' Interaction
No-Core Shell Model Calculations

\[ E \text{ [MeV]} \]
\[ r \text{ [fm]} \]
\[ N_{\text{max}} \]

\[ E \text{ [MeV]} \]
\[ r \text{ [fm]} \]
\[ N_{\text{max}} \]

\[ 4\text{He} \]

-30 -20 -10 0 10 20 30 40 50 60

\[ h\Omega \text{ [MeV]} \]

exact result from PRC64 (2001) 044001

\[ \alpha \]

\[ \beta \]

test of two-body approximation

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**Fermionic Molecular Dynamics**

**Fermionic**

single Slater determinant

\[ |Q\rangle = \mathcal{A}(|q_1\rangle \otimes \cdots \otimes |q_A\rangle) \]

- antisymmetrized \(A\)-body state

**Molecular**

Gaussian Wavepackets

\[ \langle x | q \rangle = \sum_i c_i \exp\left\{ -\frac{(x - b_i)^2}{2a_i} \right\} |\chi\rangle \otimes |\xi\rangle \]

**Dynamics**

Variational Principle

\[ \delta \int dt \frac{\langle \hat{Q} | i \frac{d}{dt} - \hat{H}_{\text{eff}} | \hat{Q} \rangle}{\langle \hat{Q} | \hat{Q} \rangle} = 0 \]

**Interaction**

\[ \hat{H}_{\text{eff}} = \hat{H}^{C2} + \hat{H}_{\text{correction}} \]

\[ \hat{H}^{C2} = \left[ \mathcal{C}_r^{\dagger} \mathcal{C}_\Omega \mathcal{C}_\Omega \mathcal{C}_r \right]^{C2} \]

- \(\hat{H}^{C2}\) – correlated interaction in two-body approximation

- \(\hat{H}_{\text{correction}}\) – adjusted on doubly magic nuclei (compensates for missing three-body contributions of the correlated interaction and genuine three-body forces)

**Variation**

minimize \( \langle Q | \hat{H}_{\text{eff}} | Q \rangle \) by

variation of the parameters of the single-particle states
Selected Nuclei

- $^4\text{He}$
- $^{12}\text{C}$
- $^{16}\text{O}$
- $^{20}\text{Ne}$
- $^{24}\text{Mg}$
- $^{40}\text{Ca}$

Spherical nuclei

Intrinsically deformed nuclei

$\rho^{(1)}(r)$
Multiconfiguration Calculations

- Include rotations
- Include vibrations

Diagonalize Hamiltonian in a set of FMD states

\[ \sum_j \langle Q^{(i)} | \hat{H}^{\text{eff}} | Q^{(j)} \rangle c_j^\alpha = E^\alpha \sum_j \langle Q^{(i)} | Q^{(j)} \rangle c_j^\alpha \]

\[ \begin{array}{c|c|c|c}
\text{Intrinsic} & +\text{rotations} & +\text{vibrations} & \text{Experiment} \\
1 & 128 & 128 \times 4 & \\
\end{array} \]
Summary and Outlook

- Unitary correlation operator can describe short-range central and tensor correlations
- Unitary correlator provides common low-momentum interaction
- Correlated interaction can be used in different many-body methods (HF, shell model, FMD)
- Other observables have to be correlated as well

Neff, Feldmeier, Nuc. Phys. A713 (2003), 311

- No-core shell model calculations for light nuclei
- FMD calculations with modified interaction for heavier nuclei

Outlook

- Study further observables (GT quenching, ...)
- Investigate three-body correlations and genuine three-body forces

Multiconfiguration calculations with FMD

- Cluster structure in nuclei and excited states
- Exotic nuclei, halos, ...