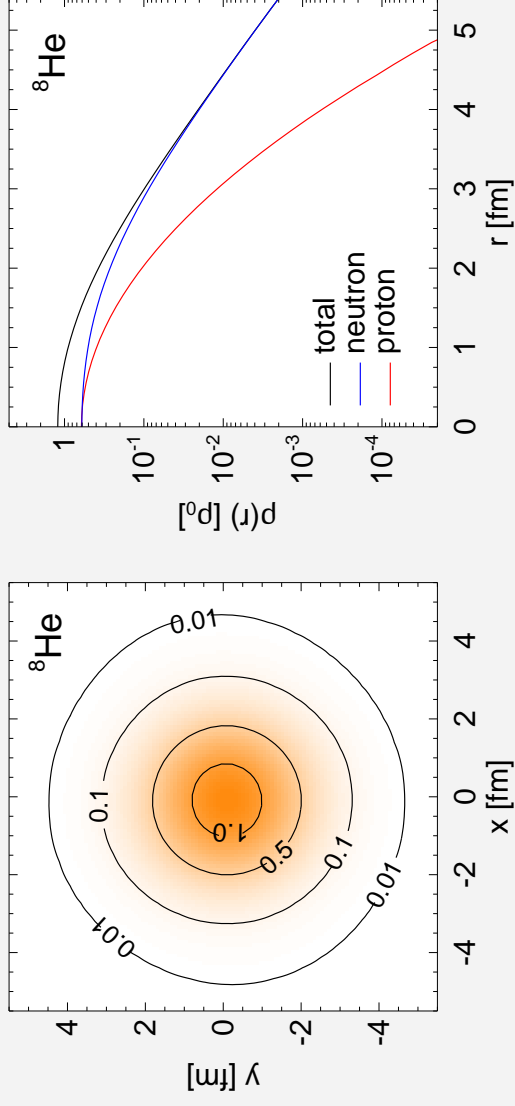


# Multiconfiguration Calculations

in

# Fermionic Molecular Dynamics



**Rauischholzhausen XII @ Burg Rieneck, June 19-21, 2000**

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# FMD attributes

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## Fermionic

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$$|\hat{Q}\rangle = \tilde{C} \mathcal{A} (|q_1\rangle \otimes \dots \otimes |q_A\rangle)$$

- Unitary Correlator  $\tilde{C}$
- antisymmetrized  $A$ -particle state

## Molecular

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$$\langle \vec{x} | q \rangle = \sum_i c_i \exp\left\{-\frac{(\vec{x} - \vec{b}_i)^2}{2a_i}\right\} |x_i\rangle \otimes |\xi_i\rangle$$

## Dynamics

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### Variational Principle

$$\delta \int dt \frac{\langle \hat{Q} | i \frac{d}{dt} - \tilde{H} | \hat{Q} \rangle}{\langle \hat{Q} | \hat{Q} \rangle} = 0$$

### Equations of motion

$$i \sum_{\nu} \mathcal{E}_{\mu\nu} \dot{q}_{\nu} = \frac{\partial \mathcal{H}}{\partial q_{\mu}^*}$$

$$\mathcal{E}_{\mu\nu} = \frac{\partial}{\partial q_{\mu}^*} \frac{\partial}{\partial q_{\nu}} \ln \langle \hat{Q} | \hat{Q} \rangle$$

$$\mathcal{H} = \frac{\langle \hat{Q} | \tilde{H} | \hat{Q} \rangle}{\langle \hat{Q} | \hat{Q} \rangle}$$

# Evaluation of Matrix-Elements

## One-Body Matrix-Elements

$$\mathcal{O} = (\langle \mathbf{q}_i | \mathbf{q}_j \rangle)^{-1}$$

$$\frac{\langle \hat{\mathcal{Q}} | \mathcal{O}^{[1]} | \hat{\mathcal{Q}} \rangle}{\langle \hat{\mathcal{Q}} | \hat{\mathcal{Q}} \rangle} = \sum_{k,l} \langle \mathbf{q}_k | \mathcal{O}^{[1]} | \mathbf{q}_l \rangle \mathcal{O}_{lk}$$

## Two-Body Matrix-Elements

$$\frac{\langle \hat{\mathcal{Q}} | \mathcal{O}^{[2]} | \hat{\mathcal{Q}} \rangle}{\langle \hat{\mathcal{Q}} | \hat{\mathcal{Q}} \rangle} = \frac{1}{2} \sum_{k,l,m,n} a \langle \mathbf{q}_k, \mathbf{q}_l | \mathcal{O}^{[2]} | \mathbf{q}_m, \mathbf{q}_n \rangle a \mathcal{O}_{mk} \mathcal{O}_{nl}$$

## Gaussian Integrals

$$G(|\vec{x}_1 - \vec{x}_2|) = \exp \left\{ -\frac{(\vec{x}_1 - \vec{x}_2)^2}{2\kappa} \right\}$$

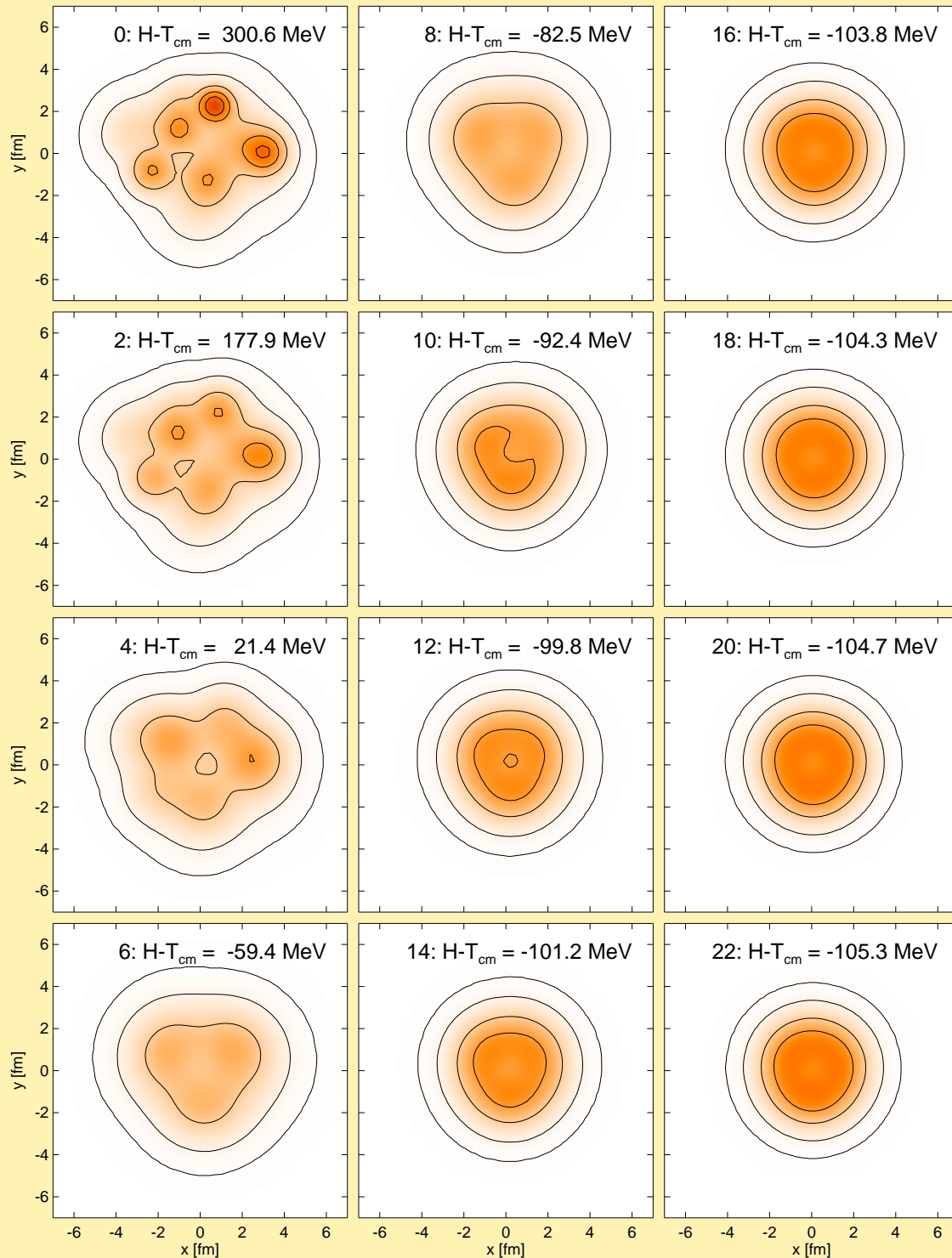
$$\langle a_k \vec{b}_k, a_l \vec{b}_l | \mathcal{G} | a_m \vec{b}_m, a_n \vec{b}_n \rangle = R_{km} R_{ln} \left( \frac{\kappa}{\alpha_{klmn} + \kappa} \right)^{3/2} \exp \left\{ -\frac{\vec{p}_{klmn}^2}{2(\alpha_{klmn} + \kappa)} \right\}$$

$$\alpha_{klmn} = \frac{a_k^* a_m}{a_k^* + a_m} + \frac{a_l^* a_n}{a_l^* + a_n},$$

$$\vec{p}_{klmn} = \frac{a_m \vec{b}_k^* + a_k^* \vec{b}_m}{a_k^* + a_m} - \frac{a_n \vec{b}_l^* + a_l^* \vec{b}_n}{a_l^* + a_n},$$

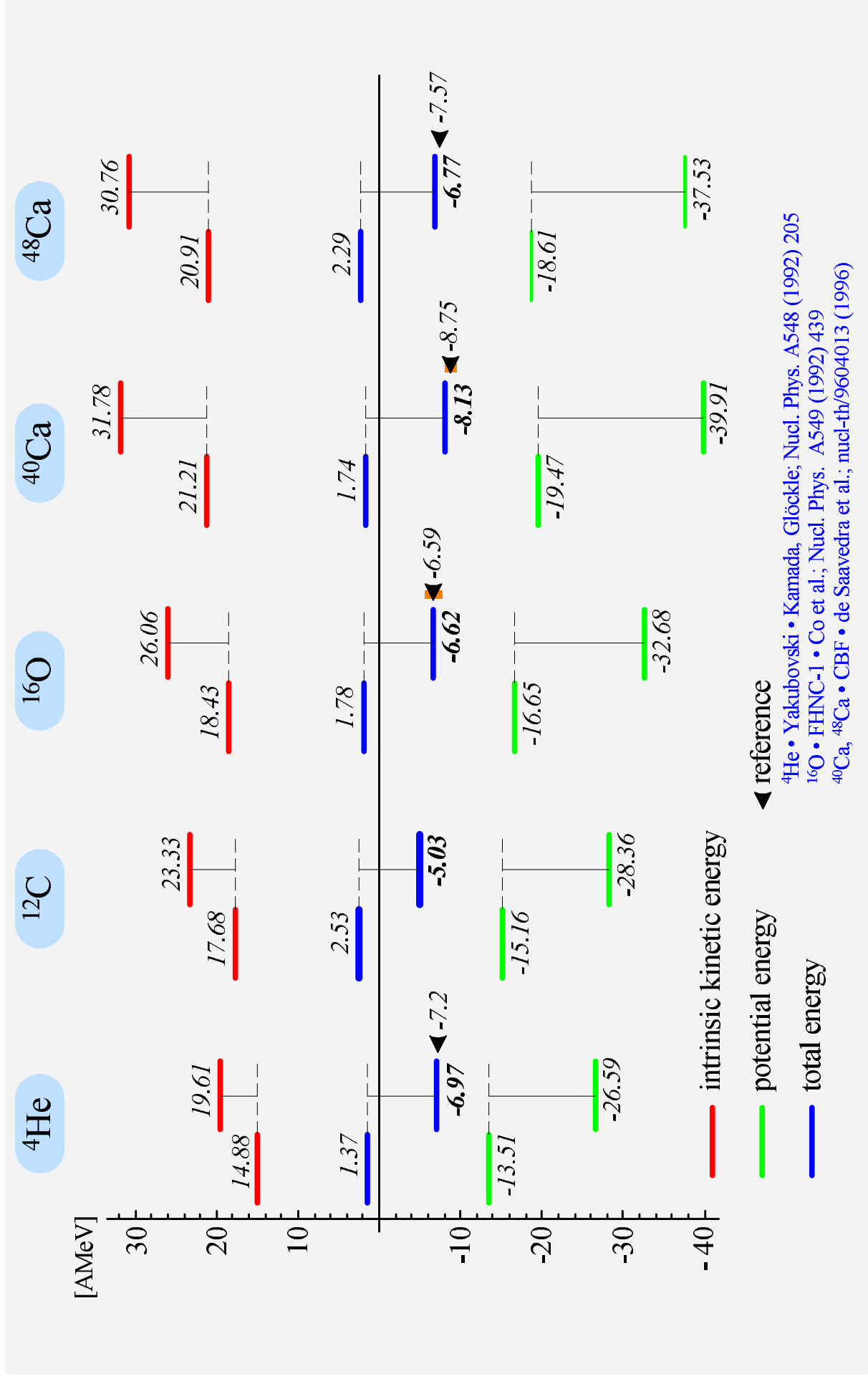
$$R_{km} = \langle a_k \vec{b}_k | a_m \vec{b}_m \rangle$$

# searching the $^{16}\text{O}$ ground state

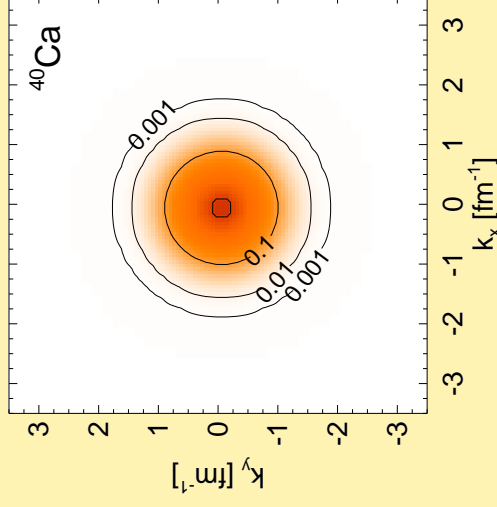
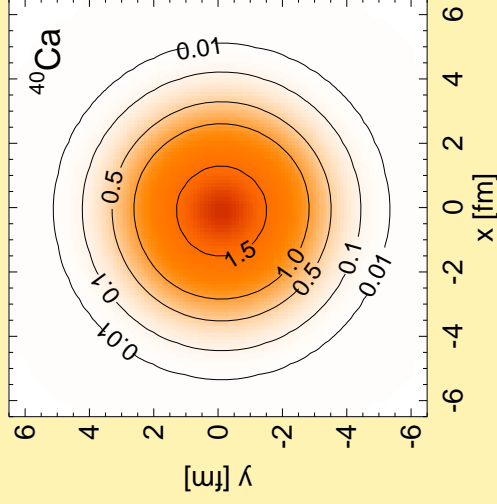
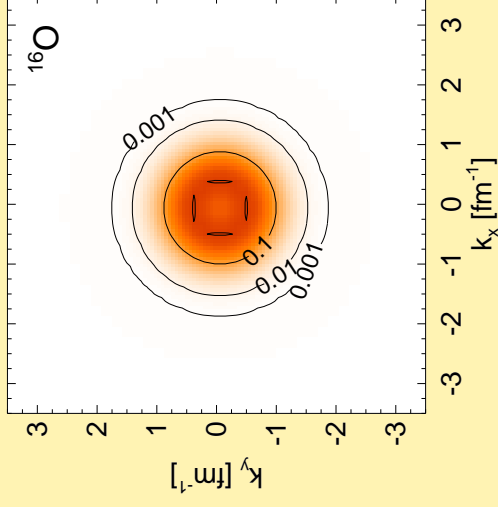
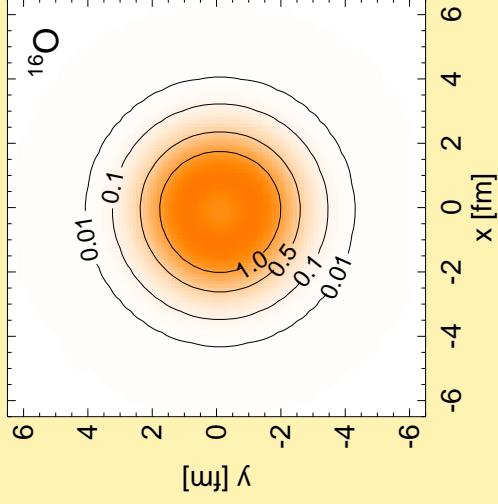


- Minimization of ground state energy with quasi-Newton-method

# Afnan-Tang S3M results



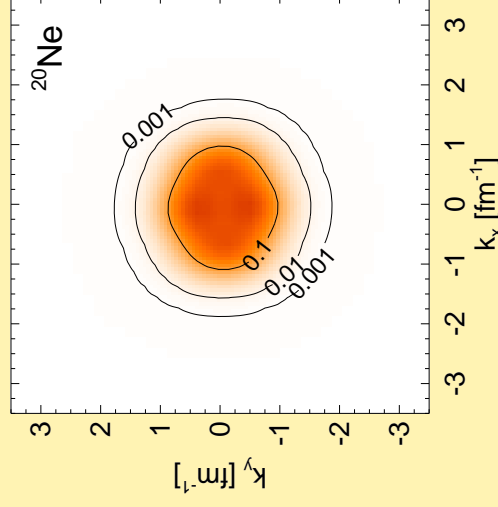
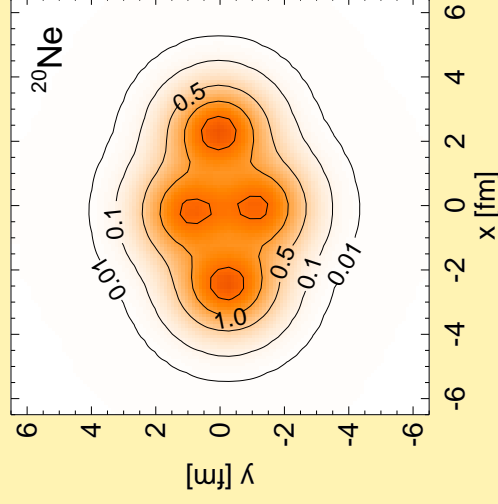
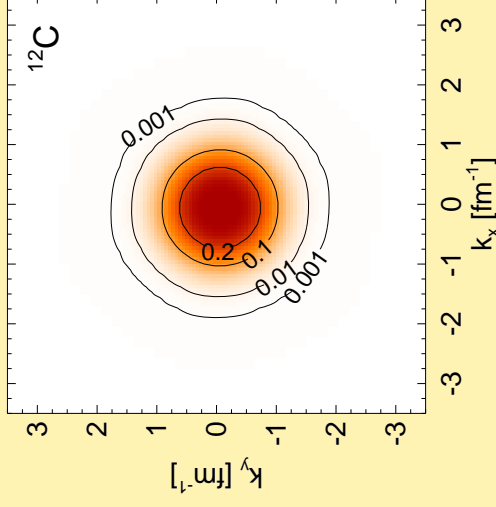
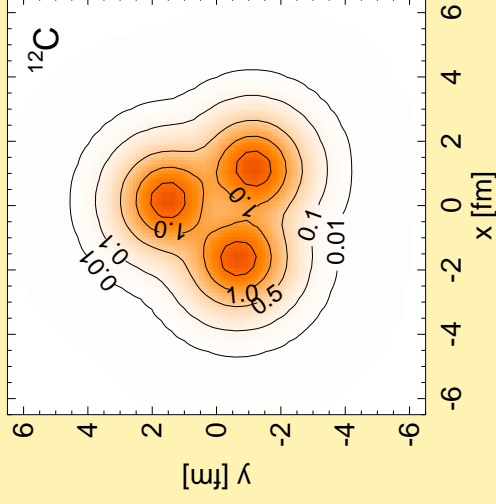
# shell-model-states



- ✓ gaussian-basis generates perfect shell-model states
- ✓ linear combinations of slightly shifted gaussians result in harmonic oscillator  $p$ - and  $d$ -states – these linear combinations are a result of the antisymmetrization – the slater-determinant is invariant under unitary transformations of the single-particle states
- ✓ these states are the result of an unconstrained minimization of all ( $A$  · 8) Parameters

- Afnan-Tang S3M Interaction

# *intrinsically-deformed-states*



- ✓ FMD can also describe intrinsically deformed nuclei
- ✓ this is very difficult to do in other calculations
- ✓ in FMD this is just the result of the unconstrained minimization in the parameter space

- ✗ intrinsically deformed ground states have not the rotational symmetry of the Hamiltonian

# Multiconfiguration Calculations

## Improve FMD description

- long-range correlations beyond the meanfield
- address intrinsically deformed ground states
- discuss contributions of different configurations

## Multiconfiguration FMD

diagonalize **Hamiltonian** in a set

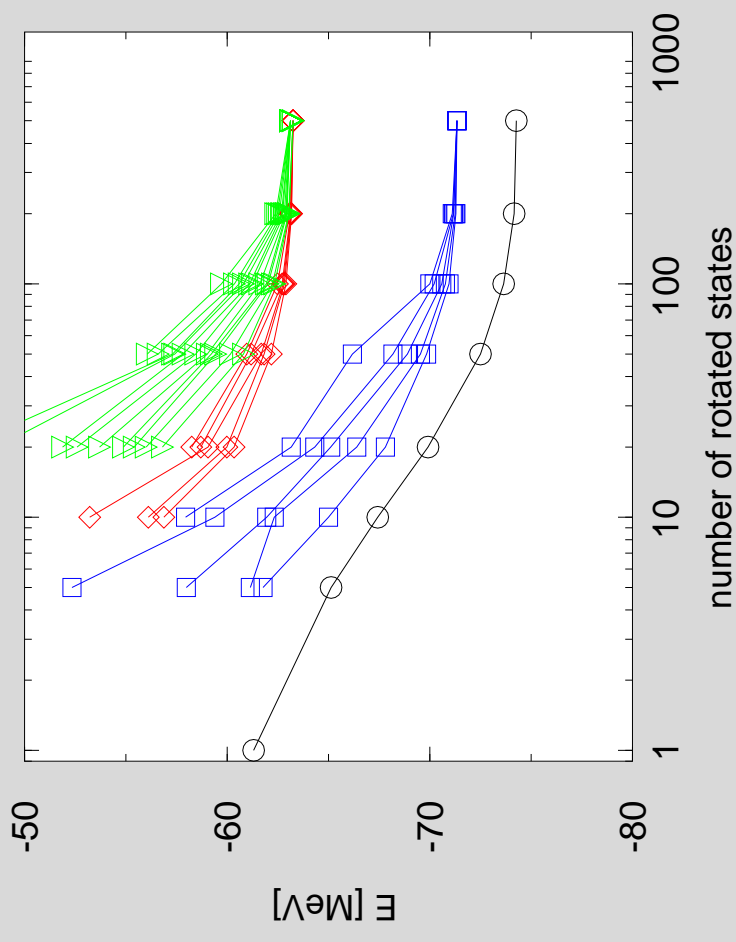
$$\{|\hat{Q}^i\rangle\}$$

of (nonorthogonal) FMD states, leads to generalized eigenvalue problem

$$\sum_j \langle \hat{Q}^i | \tilde{H} | \hat{Q}^j \rangle c_j^\alpha = E^\alpha \sum_j \langle \hat{Q}^i | \hat{Q}^j \rangle c_j^\alpha$$

$^{12}\text{C}$

diagonalization in a set of randomly rotated states

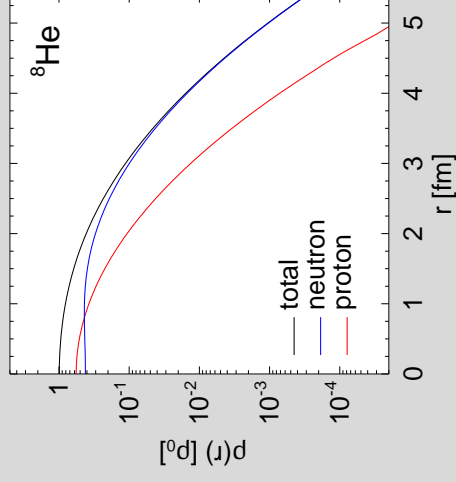
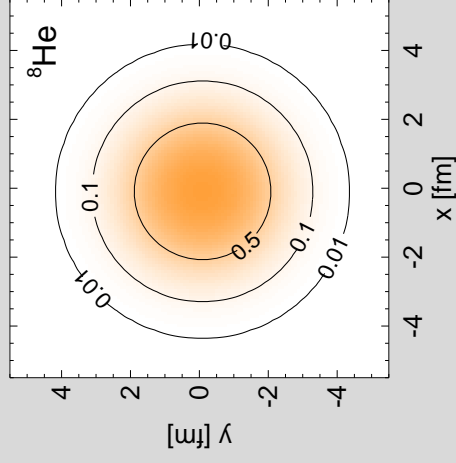


- ✓ big increase in binding energy
- ✓ eigenstates of angular momentum and parity
- ✓ spectrum of rotational excitations

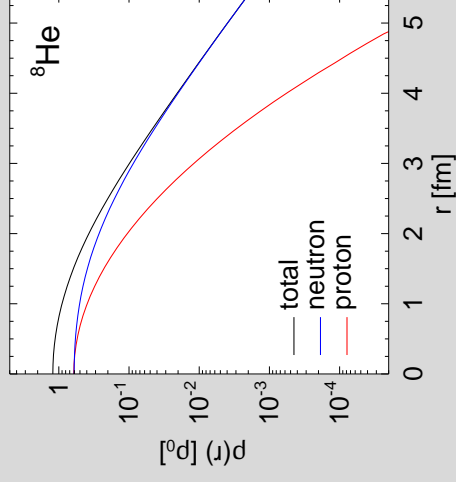
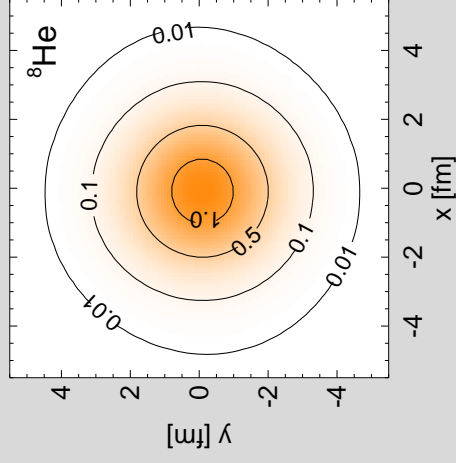
# Halo Nuclei

$^8\text{He}$

One gaussian per one-particle state



Two gaussians per one-particle state



$^8\text{He}$

	FMD		AVRGM
	1g	2g	
$E_B$	-21.67	-24.86	-24.22
$R_m$	2.27	2.38	2.73
$R_p$	1.79	1.70	2.08
$R_n$	2.41	2.55	2.91

diagonalization in rotated states

	2g	2g/2c	2g/25c
$E_B$	-24.86	-25.49	-26.70

AVRGM

three-cluster microscopic model

nuc1-th/0006001

# Summary & Outlook

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## Halo Nuclei

- study the importance of different configurations
- study spectrum and other properties of nuclei

## FMD

- ✓ FMD works well for nuclei in the valley of stability
- ✓ FMD seems to work also quite fine for halo-nuclei

## Choice of Interaction

- ✓ effective and (semi-) realistic interactions can be used
- ✓ Unitary Correlator works well for the hard-core of realistic interactions
- ✗ correlator for tensor- and spin-orbit interactions necessary ?