

Electrical conductivity of a warm neutron star crust in magnetic fields

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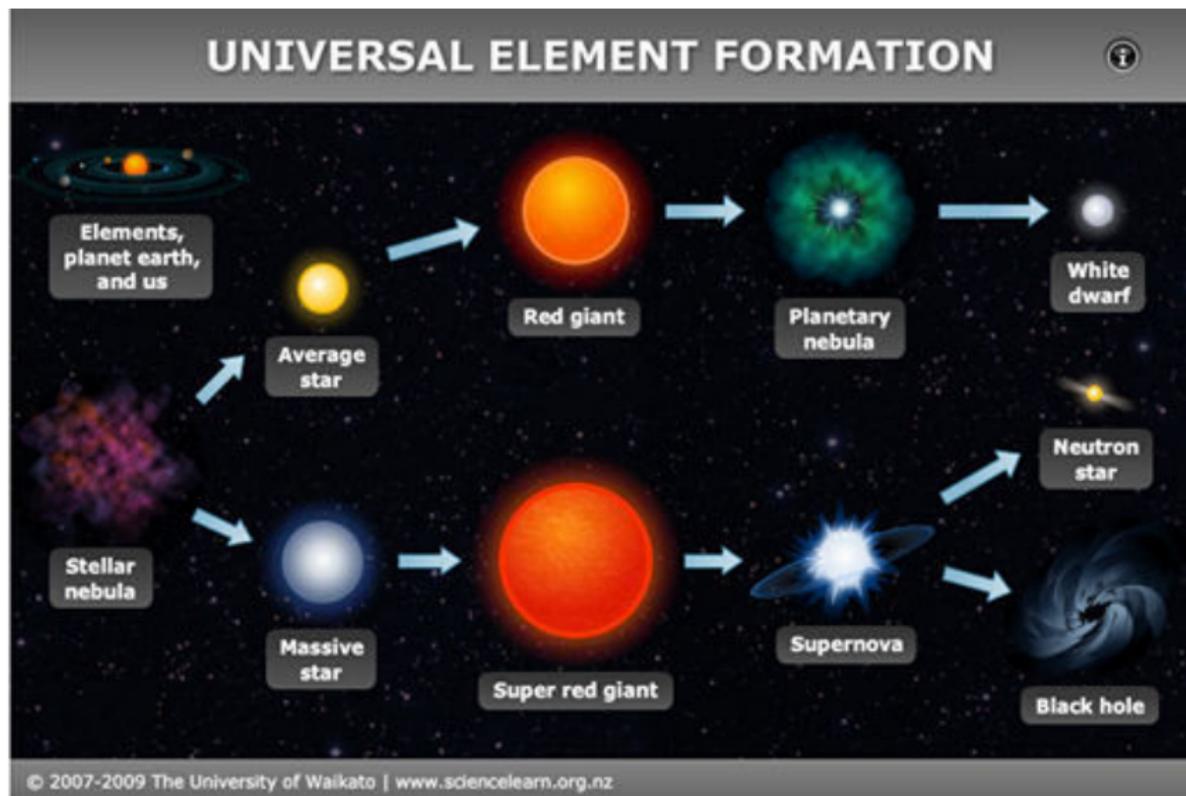


- Introduction & motivation
- Physical conditions
- Boltzmann kinetic theory
- Scattering amplitude & relaxation time
- Results
- Summary & outlook

Overview

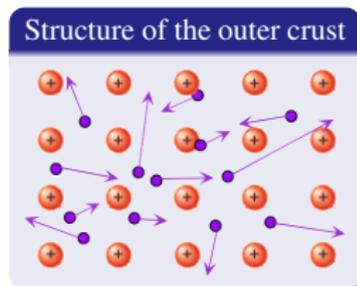
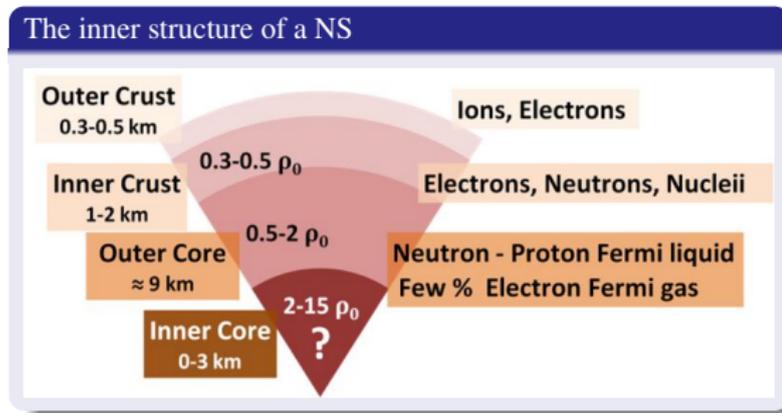
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Neutron stars



The inner structure of a neutron star

Neutron stars (NS) are compact stellar objects with masses $M \sim M_{\odot}$ and radii $R \sim 10$ km. They are embedded in strong magnetic fields $B \sim 10^{10} - 10^{15}$ G. Typical temperature after cooling is $T \sim 10^8$ K.



The outer crust of the star consists of nuclei and conducting Fermi gas of relativistic electrons.

Transport coefficients

Transport coefficients play important role in several astrophysical problems:

- Cooling or heating of a NS
- Magnetic field decay
- Oscillations & various instabilities
- Dissipation of magneto-hydrodynamic waves in NS crust

Transport properties of NS crust are well studied at low temperatures.

Calculation of high-temperature transport coefficients is motivated by some problems such as:

- Merging in binary NS
- Supernova explosions
- Accretion on NS from a companion

We calculate the electrical conductivity tensor of warm outer crust of a NS for non-quantizing magnetic fields and for arbitrary electron degeneracy in the parameter range

$$10^6 < \rho < 10^{11} \text{ g cm}^{-3}, \quad 0.1 < T < 10 \text{ MeV}, \quad 10^{10} < B < 10^{14} \text{ G}.$$

We use the natural (Gaussian) units $\hbar = c = k_B = k_e = 1$, $e = \sqrt{\alpha}$, $\alpha = 1/137$.

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Phase diagram of one-component plasma and electron scattering mechanism

The electron scattering mechanism is controlled by the ionic plasma parameter $\Gamma = Z^2 e^2 / a_i T$:

- If $\Gamma < 1$ - Boltzmann gas, the scattering is on individual, uncorrelated nuclei
- If $1 < \Gamma < \Gamma_m \simeq 160$ - liquid state, the scattering is on correlated nuclei
- If $\Gamma > \Gamma_m$ - solid state, the scattering is on phonons and impurities

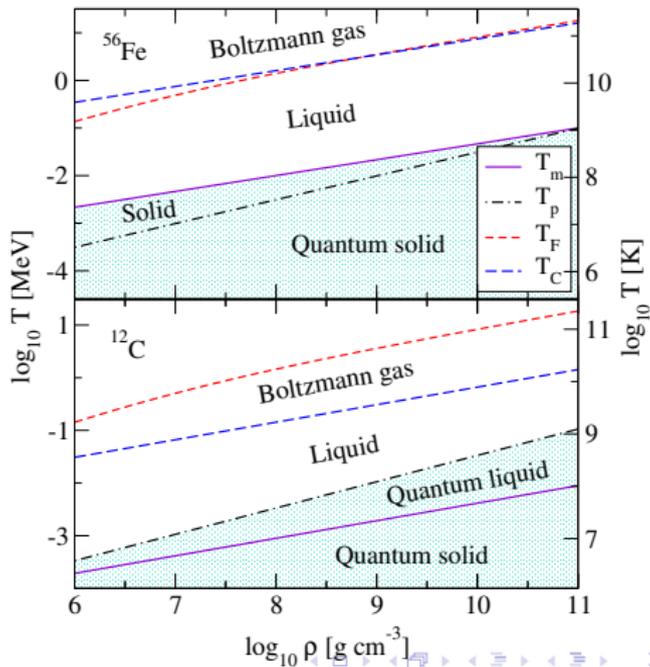
$$a_i = \left(\frac{3}{4\pi n_i} \right)^{1/3}$$

$$T_C = \frac{Z^2 e^2}{a_i}$$

$$T_m = \frac{Z^2 e^2}{a_i \Gamma_m}$$

$$T_p = \sqrt{\frac{4\pi n_i Z^2 e^2}{M}}$$

$$T_F = \sqrt{p_F^2 + m^2} - m$$

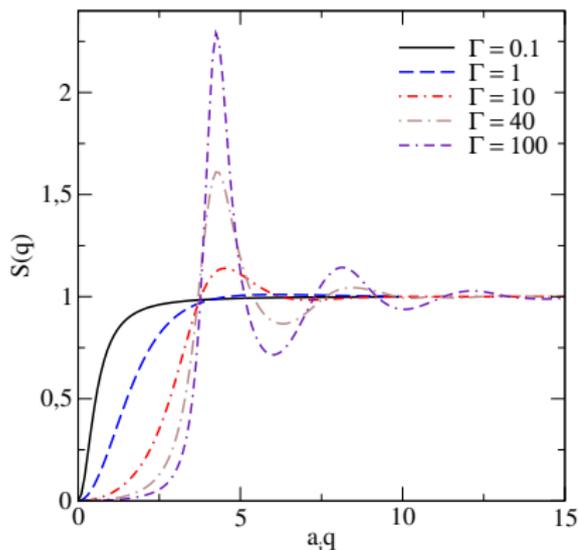


The ion structure factor and nuclear formfactor

The ion-ion correlations and finite size of nuclei are taken into account via 2-point structure function $S(q)$ and nuclear formfactor $F(q)$, respectively

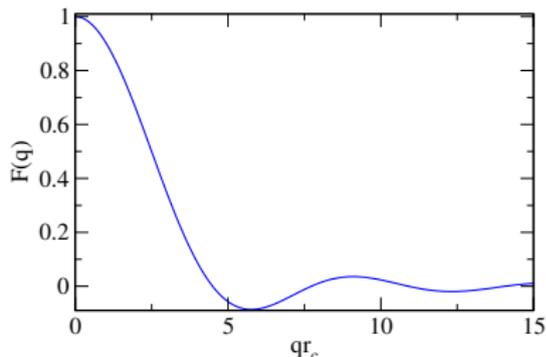
$$|\mathcal{M}_{12 \rightarrow 34}|^2 \rightarrow |\mathcal{M}_{12 \rightarrow 34}|^2 S(q) F^2(q).$$

We adopt the Monte-Carlo results from *S. Galam, J. P. Hansen, Phys.Rev.A.14, 816 (1976)* ($\Gamma \geq 2$) and the analytical expressions from *M. N. Tamashiro, Y. Levin, M. C. Barbosa, Physica A 268, 24 (1999)* ($\Gamma \leq 2$) for $S(q)$.



$$F(q) = -3 \frac{qr_c \cos(qr_c) - \sin(qr_c)}{(qr_c)^3}$$

$$r_c = 1.15 A^{1/3} \text{fm}$$



The ion structure factor suppresses the scattering with small q . In $a_i q \gg 1$ limit $S(q) \rightarrow 1$.

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The Boltzmann equation

- The kinetics of electrons is described by the Boltzmann equation for the distribution function f in the presence of electric and magnetic field

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} - e(\mathbf{E} + [\mathbf{v} \times \mathbf{B}]) \frac{\partial f}{\partial \mathbf{p}} = I[f]. \quad (1)$$

- The collision integral for electron-ion scattering has the form

$$I[f] = -(2\pi)^4 \sum_{234} |\mathcal{M}_{12 \rightarrow 34}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) [f_1(1 - f_3)g_2 - f_3(1 - f_1)g_4].$$

- For small perturbations the solution of Eq. (1) can be searched in the form

$$f = f_0 + \delta f, \quad f_0 = \frac{1}{e^{\beta(\varepsilon - \mu)} + 1}, \quad g = n_i(2\pi\beta/M)^{3/2} e^{-\beta\varepsilon}.$$

- In the relaxation time approximation $I[f] = -\delta f/\tau$ the solution has the form

$$\delta f = \frac{e\tau}{1 + (\omega_c\tau)^2} \frac{\partial f_0}{\partial \varepsilon} v_i [\delta_{ij} - \omega_c\tau \varepsilon_{ijk} b_k + (\omega_c\tau)^2 b_i b_j] E_j, \quad b_i = B_i/B, \quad \omega_c = eB/\varepsilon.$$

- The relaxation time is given by the formula

$$\tau^{-1}(\varepsilon_1) = (2\pi)^{-5} \int d\omega d\mathbf{q} d\mathbf{p}_2 \frac{\mathbf{q} \cdot \mathbf{p}}{p^2} |\mathcal{M}_{12 \rightarrow 34}|^2 \delta(\varepsilon_1 - \varepsilon_3 - \omega) \delta(\varepsilon_2 - \varepsilon_4 + \omega) g_2 \frac{1 - f_3^0}{1 - f_1^0}.$$

The electrical conductivity tensor

- The electrical conductivity tensor is defined as the coefficient of proportionality between the electrical current and electrical field

$$j_i = - \int \frac{2d\mathbf{p}}{(2\pi)^3} e v_i \delta f = \sigma_{ij} E_j.$$

- If magnetic field is directed along z axis, then σ_{ij} tensor has the form

$$\hat{\sigma} = \begin{pmatrix} \sigma_0 & -\sigma_1 & 0 \\ \sigma_1 & \sigma_0 & 0 \\ 0 & 0 & \sigma \end{pmatrix}.$$

- Longitudinal conductivity does not depend on magnetic field (isotropic conduction)

$$\sigma = - \frac{e^2}{3\pi^2} \int_0^\infty dp p^2 v^2 \tau \frac{\partial f_0}{\partial \varepsilon}.$$

- Transverse (σ_0) and Hall (σ_1) conductivities depend on magnetic field via the Hall parameter $\omega_c \tau$, where ω_c is the cyclotron frequency of electrons.

$$\sigma_0 = - \frac{e^2}{3\pi^2} \int_0^\infty dp p^2 v^2 \frac{\tau}{1 + (\omega_c \tau)^2} \frac{\partial f_0}{\partial \varepsilon}, \quad \sigma_1 = - \frac{e^2}{3\pi^2} \int_0^\infty dp p^2 v^2 \frac{\omega_c \tau^2}{1 + (\omega_c \tau)^2} \frac{\partial f_0}{\partial \varepsilon}.$$

Recovering limiting cases

Limiting cases of strongly degenerate and non-degenerate electrons

- At low temperature limit $T \ll T_F$ one comes to the famous Drude formulae ($\epsilon \rightarrow \epsilon_F$):

$$\sigma = \frac{ne^2\tau_F}{\epsilon_F}, \quad \sigma_0 = \frac{\sigma}{1 + (\omega_{cF}\tau_F)^2}, \quad \sigma_1 = \frac{\omega_{cF}\tau_F}{1 + (\omega_{cF}\tau_F)^2}\sigma.$$

- At high temperature limit $T \gg T_F$ Drude-type formulae are obtained with $\bar{\epsilon} \simeq 3T$:

$$\sigma \simeq \frac{ne^2\bar{\tau}}{\bar{\epsilon}}, \quad \sigma_0 \simeq \frac{\sigma}{1 + (\bar{\omega}_c\bar{\tau})^2}, \quad \sigma_1 \simeq \frac{\bar{\omega}_c\bar{\tau}}{1 + (\bar{\omega}_c\bar{\tau})^2}\sigma.$$

Limiting cases of isotropic and strongly anisotropic conduction

- If $\omega_c\tau \ll 1$ (weak magnetic fields), the conduction is isotropic:

$$\sigma_0 \simeq \sigma, \quad \sigma_1 \simeq \omega_c\tau\sigma \ll \sigma \quad \Rightarrow \quad \sigma_{kj} \rightarrow \delta_{kj}\sigma.$$

- If $\omega_c\tau \gg 1$ (strong magnetic fields), the transverse conduction is strongly suppressed:

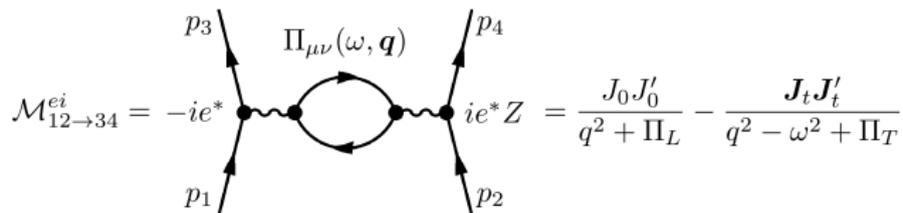
$$\sigma_0 \simeq \frac{\sigma}{(\omega_c\tau)^2} \ll \sigma, \quad \sigma_1 \simeq \frac{\sigma}{\omega_c\tau} \ll \sigma.$$

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The scattering probability

The electron-ion scattering matrix element can be calculated according to the Feynman rules



$$\mathcal{M}_{12 \rightarrow 34}^{ei} = -ie^* \cdot \text{Diagram} \cdot ie^* Z = \frac{J_0 J'_0}{q^2 + \Pi_L} - \frac{\mathbf{J}_t \mathbf{J}'_t}{q^2 - \omega^2 + \Pi_T}$$

J^μ, J'^μ are electronic and ionic 4-currents respectively

$$J^\mu = -e^* \bar{u}^{s_3}(p_3) \gamma^\mu u^{s_1}(p_1), \quad J'^\mu = Ze^* v_2^\mu = Ze^* p_2^\mu / M.$$

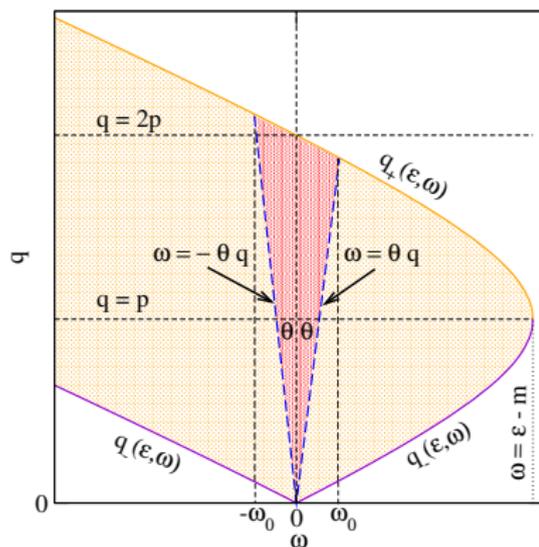
The relaxation time in the case of inelastic electron-ion scattering is given by the formula

$$\tau^{-1}(\varepsilon) = \frac{\pi Z^2 e^4 n_i}{\varepsilon p^3} \int_{-\infty}^{\varepsilon - m} d\omega e^{-\omega/2T} \frac{f^0(\varepsilon - \omega)}{f^0(\varepsilon)} \int_{q_-}^{q_+} dq (q^2 - \omega^2 + 2\varepsilon\omega) S(q) F^2(q) \times$$

$$\times \frac{1}{\sqrt{2\pi}\theta} e^{-\omega^2/2q^2\theta^2} e^{-q^2/8MT} \left\{ \frac{(2\varepsilon - \omega)^2 - q^2}{|q^2 + \Pi_L|^2} + \theta^2 \frac{(q^2 - \omega^2)[(2\varepsilon - \omega)^2 + q^2] - 4m^2 q^2}{q^2 |q^2 - \omega^2 + \Pi_T|^2} \right\}$$

$$e^* = \sqrt{4\pi e}, \quad \theta = \sqrt{T/M}, \quad q_{\pm} = \left| \pm \sqrt{\varepsilon^2 - m^2} + \sqrt{(\varepsilon - \omega)^2 - m^2} \right|$$

Limit of elastic scattering



Due to the suppression factor θ the scattering is more effective via exchange of virtual photons with energies and momenta lying inside the triangle $\omega/q < \theta \ll 1$ on the plane (ω, q) . In the limit of infinite massive ions ($\theta \rightarrow 0$) the electron-ion interaction is pure electrostatic, and we come to the limit of elastic scattering using the formula:

$$\frac{1}{\theta\sqrt{2\pi}} e^{-x^2/2\theta^2} \rightarrow \delta(x), \quad \text{as } \theta \rightarrow 0$$

In the limit of elastic scattering one obtains the well-known formula for the relaxation time

$$\tau^{-1}(\varepsilon) = \frac{4\pi Z^2 e^4 n_i \varepsilon}{p^3} \int_0^{2p} dq q^3 \frac{S(q)F^2(q)}{|q^2 + \Pi_L|^2} \left(1 - \frac{q^2}{4\varepsilon^2}\right)$$

The photon polarization tensor

- The polarization tensor $\Pi_{\mu\nu}(\omega, \mathbf{q})$ is decomposed into longitudinal and transverse modes

$$\Pi_{\mu\nu}(\omega, \mathbf{q}) = \Pi_L(\omega, q) P_{\mu\nu}^L(\omega, \mathbf{q}) + \Pi_T(\omega, q) P_{\mu\nu}^T(\omega, \mathbf{q})$$

- In the plasma rest frame projectors $P_{\mu\nu}^T$ and $P_{\mu\nu}^L$ have the components ($Q^2 = \omega^2 - q^2$)

$$P_{00}^T = P_{0i}^T = 0, \quad P_{ij}^T = -\delta_{ij} + \frac{q_i q_j}{q^2}, \quad P_{00}^L = -\frac{q^2}{Q^2}, \quad P_{0i}^L = -\omega q_i / Q^2, \quad P_{ij}^L = -\frac{\omega^2}{Q^2} \frac{q_i q_j}{q^2}$$

- Within the **HTL (hard-thermal-loop)** effective field theory ($q \ll p$) Π_L, Π_T are given by

$$\Pi_L = q_D^2 \left[1 - \frac{x}{2\bar{v}} \log \frac{x + \bar{v}}{x - \bar{v}} \right], \quad \Pi_T = \frac{1}{2} q_D^2 \left[x^2 + (\bar{v}^2 - x^2) \frac{x}{2\bar{v}} \log \frac{x + \bar{v}}{x - \bar{v}} \right]$$

$x = \omega/q, \bar{v} = v_F$ in degenerate and $\bar{v} = 1$ in ultra-relativistic limits, respectively.

- If the contribution of positrons is neglected, for the Debye momentum we have

$$q_D^2 \simeq 4\alpha \begin{cases} p_F \varepsilon_F / \pi, & T \ll T_F, \\ \pi n_e / T, & T \gg T_F. \end{cases}$$

- We use low-frequency $x = \omega/q \ll 1$ expansions for Π_L, Π_T to order $O(x^2)$

$$\text{Re}\Pi_L(q, \omega) = \left(1 - \frac{x^2}{\bar{v}^2} \right) q_D^2, \quad \text{Im}\Pi_L(q, \omega) = -\frac{\pi x}{2\bar{v}} q_D^2,$$

$$\text{Re}\Pi_T(q, \omega) = x^2 q_D^2, \quad \text{Im}\Pi_T(q, \omega) = \frac{\pi}{4} x \bar{v} q_D^2.$$

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Relaxation time and the Hall parameter $\omega_c\tau$ as functions of density

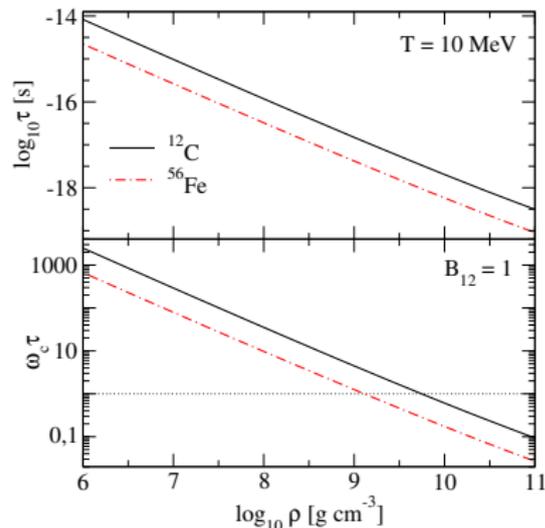
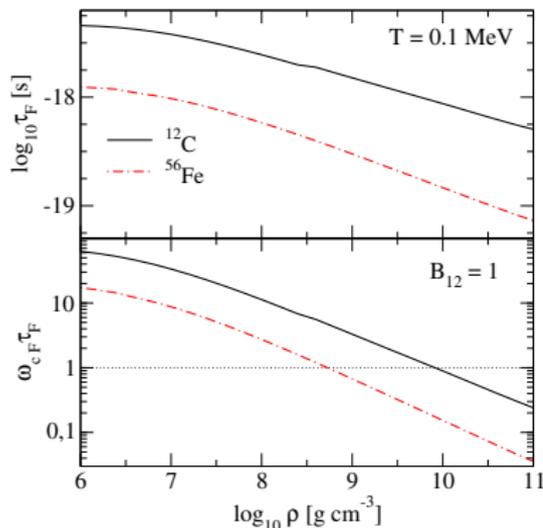
Relaxation time scales as $\tau \propto \varepsilon^2 Z^{-1} \rho^{-1} T^{-\delta}$ with $0.1 \leq \delta \leq 0.3$.

Degenerate regime

$$\varepsilon_F \propto \rho^{1/3}, \quad \tau \propto \rho^{-1/3}, \quad \omega_c\tau \propto \rho^{-2/3}$$

Non-degenerate regime

$$\bar{\varepsilon} \simeq 3T, \quad \tau \propto \rho^{-1}, \quad \omega_c\tau \propto \rho^{-1}$$



Relaxation time and $\omega_c\tau$ decrease faster in the non-degenerate regime.

Relaxation time as a function of temperature

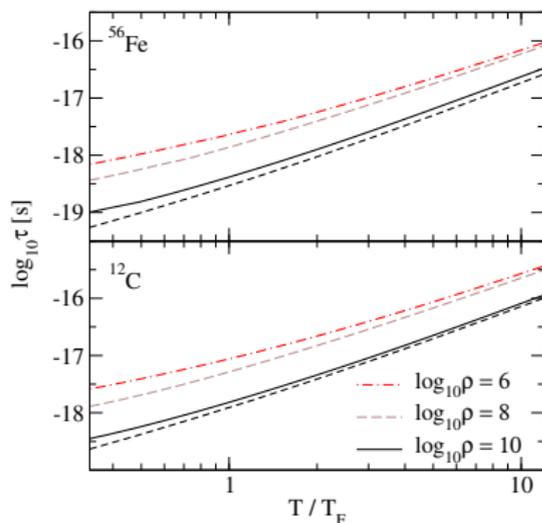
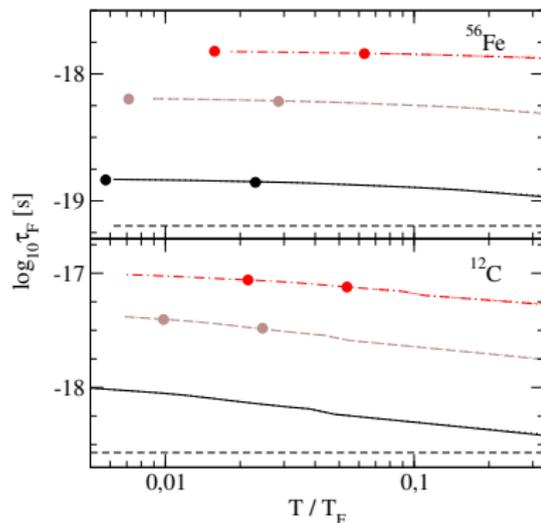
Relaxation time decreases with the temperature in the degenerate and increases in the non-degenerate regimes. The temperature dependence in the first case arises solely from the ion-ion correlation function.

Degenerate regime

$$\bar{\epsilon} \rightarrow \epsilon_F, \quad \tau \propto T^{-\delta}, \quad \omega_c \tau \propto T^{-\delta}$$

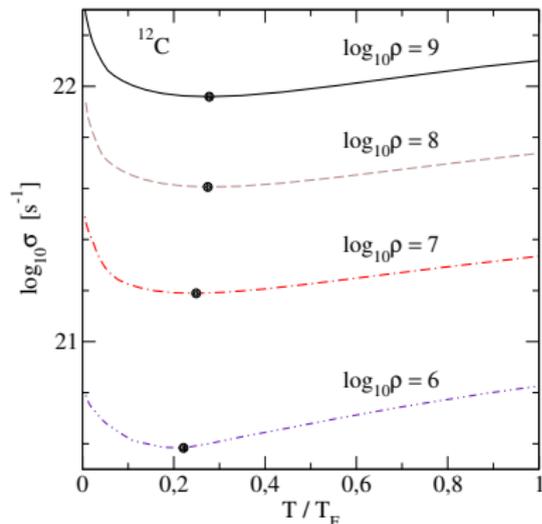
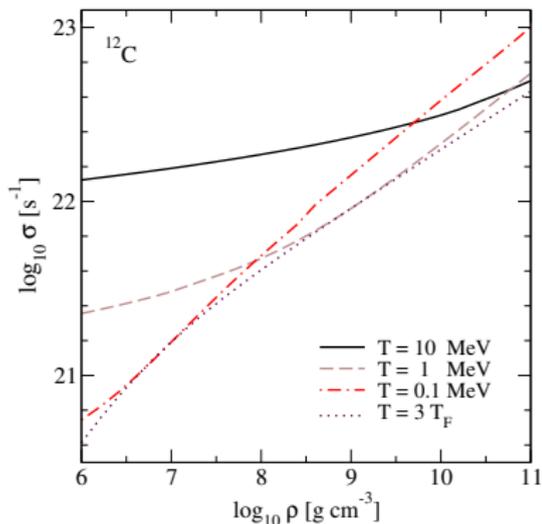
Non-degenerate regime

$$\bar{\epsilon} \simeq 3T, \quad \tau \propto T^{7/4}, \quad \omega_c \tau \propto T^{3/4}$$



The density and temperature dependence of the scalar conductivity

- The scalar conductivity σ shows a power-law increase with density $\sigma \propto \rho^\alpha$.
- In the deg. regime $\alpha \simeq 0.5$, in the non-deg. regime $\alpha \simeq 0.1$.
- σ decreases with temperature in the deg. regime and increases in the non-deg. regime.
- The minimum as a function of temperature arises around "transition" temperature $T^* \simeq T_F/3$ almost independent on density and type of nuclei.



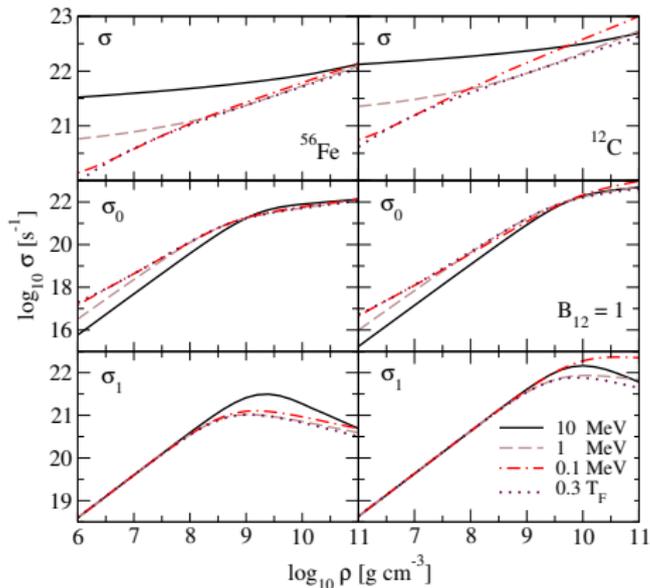
The density dependence of transverse and Hall conductivities

- At high densities or small magnetic fields $\omega_c \tau \ll 1$ (isotropic region)

$$\sigma_0 \simeq \sigma, \quad \sigma_1 \simeq \sigma \omega_c \tau \simeq \frac{B}{n_e e} \sigma^2 \ll \sigma.$$

- At low densities or strong magnetic fields $\omega_c \tau \gg 1$ (strongly anisotropic region)

$$\sigma_0 \simeq \frac{\sigma}{(\omega_c \tau)^2} \simeq \left(\frac{n_e e}{B} \right)^2 \sigma^{-1} \ll \sigma, \quad \sigma_1 \simeq \frac{\sigma}{\omega_c \tau} \simeq \frac{n_e e}{B} \ll \sigma.$$



- $\sigma_0 \propto \rho^\alpha$, if $\omega_c \tau \ll 1$
degenerate: $\alpha \simeq 1/2$,
non-degenerate: $\alpha \simeq 1/10$
- $\sigma_0 \propto \rho^\beta$, if $\omega_c \tau \gg 1$
degenerate: $\beta \simeq 3/2$,
non-degenerate: $\beta \simeq 2$
- $\sigma_1 \propto \rho^{-\gamma}$, if $\omega_c \tau \ll 1$
degenerate: $\gamma \simeq 1/5$,
non-degenerate: $\gamma \simeq 3/4$
- $\sigma_1 \propto \rho$, if $\omega_c \tau \gg 1$
- $\sigma_{1\max} \simeq \sigma/2$ at $\omega_c \tau \simeq 1$

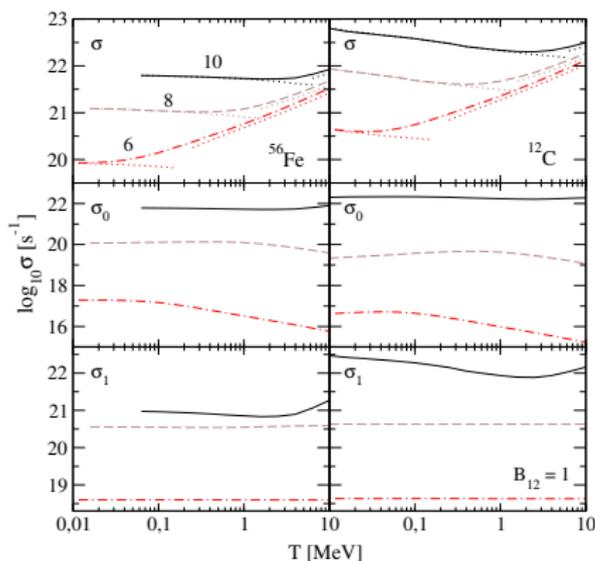
The temperature and Z-dependence of transverse and Hall conductivities

- At high densities or small magnetic fields $\omega_c\tau \ll 1$ (isotropic region)

$$\sigma_0 \simeq \sigma, \quad \sigma_1 \simeq \sigma\omega_c\tau \simeq \frac{B}{n_e e} \sigma^2 \ll \sigma.$$

- At low densities or strong magnetic fields $\omega_c\tau \gg 1$ (strongly anisotropic region)

$$\sigma_0 \simeq \frac{\sigma}{(\omega_c\tau)^2} \simeq \left(\frac{n_e e}{B}\right)^2 \sigma^{-1} \ll \sigma, \quad \sigma_1 \simeq \frac{\sigma}{\omega_c\tau} \simeq \frac{n_e e}{B} \ll \sigma.$$



- $\sigma_0 \sim \sigma, \sigma_1 \sim \sigma^2$ as functions of temperature and nucleus charge number in isotropic region.
- The temperature and Z dependence of σ_0 is reversed to that of σ in anisotropic region.
- σ_1 is independent on temperature and type of nuclei in anisotropic region.

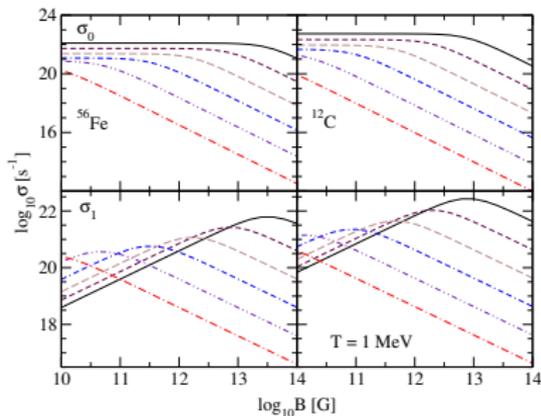
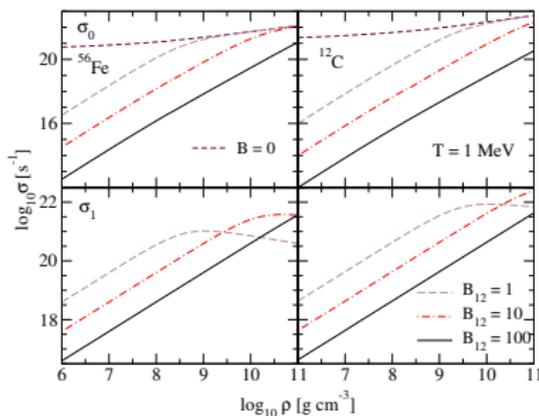
Dependence on magnetic field

- At high densities or small magnetic fields $\omega_c \tau \ll 1$ (isotropic region)

$$\sigma_0 \simeq \sigma, \quad \sigma_1 \simeq \sigma \omega_c \tau \simeq \frac{B}{n_e e} \sigma^2 \ll \sigma.$$

- At low densities or strong magnetic fields $\omega_c \tau \gg 1$ (strongly anisotropic region)

$$\sigma_0 \simeq \frac{\sigma}{(\omega_c \tau)^2} \simeq \left(\frac{n_e e}{B} \right)^2 \sigma^{-1} \ll \sigma, \quad \sigma_1 \simeq \frac{\sigma}{\omega_c \tau} \simeq \frac{n_e e}{B} \ll \sigma.$$

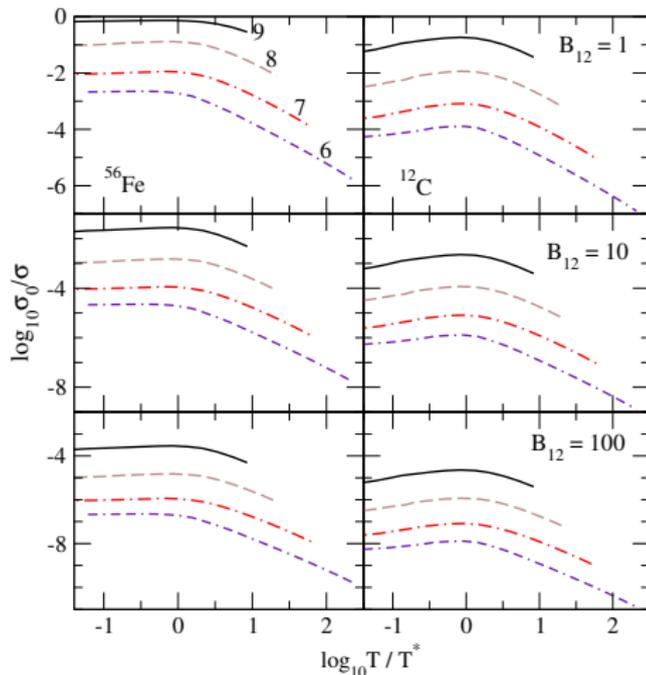


- σ_0 decreases with magnetic field: $\sigma_0 \sim B^{-2}$ in anisotropic region.
- $\sigma_1 \sim B$ in isotropic and $\sigma_1 \sim B^{-1}$ in anisotropic regions. Maximum occurs at $\omega_c \tau \simeq 1$.

The temperature dependence of the crust anisotropy

To characterize the anisotropy we consider the ratio σ_0/σ .

- All curves have a maximum at $T \simeq T^*$ independent of density, magnetic field and type of nuclei.
- At this maximum the anisotropy of the crust is the smallest.
- In the degenerate regime the anisotropy decreases with temperature.
- In the non-degenerate regime $\sigma_0/\sigma \propto T^{-3/2}$, and the crust is strongly anisotropic at very high temperatures.



Density-dependent crust composition

The composition of the crust depends on density and temperature. For not very high temperatures the β -equilibrium composition derived for $T = 0$ can be used.

For the conductivity we have the scaling

- In the degenerate regime

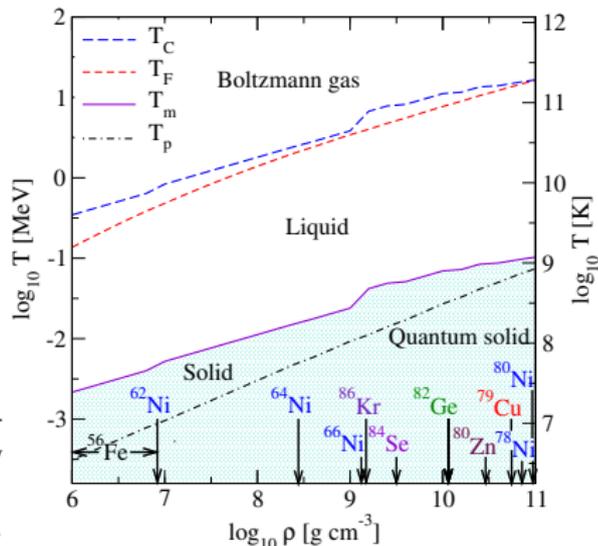
$$\sigma \propto \frac{n_e T_F}{\varepsilon_F} \propto \left(\frac{Z}{A}\right)^{1/3} Z^{-1}.$$

- In the non-degenerate regime

$$\sigma \propto \frac{n_e \bar{\varepsilon}}{\varepsilon} \propto Z^{-1}.$$

The results of density dependent composition differ from those of ^{56}Fe less than by a factor 1.4.

Composition from *J. M. Pearson, S. Goriely, N. Chamel, PhysRevC.83.065810 (2011)*.



It is interesting to study the conductivity of warm matter, which is composed of nuclei in statistical equilibrium, in which case the crust composition may become an important factor.

Fit formulae for three components of the conductivity tensor

We have performed fit to the longitudinal conductivity using the formula

$$\sigma^{fit} = 1.5Z^{-1}T_F^a \left(\frac{T}{T_F}\right)^{-b} \left(\frac{T}{T_F} + d\right)^{b+c} \times 10^{22} \text{ s}^{-1}$$

For the transverse and Hall components the following fit formulae can be used

$$\sigma_0^{fit} = \frac{\sigma'}{1 + \delta^2 \sigma'^2}, \quad \sigma_1^{fit} = \frac{\delta \sigma''^2}{1 + \delta^2 \sigma''^2}.$$

$$\delta = B(n_e e c)^{-1}, \quad \sigma' = \sigma^{fit} (T_F / \varepsilon_F)^g, \quad \sigma'' = \sigma^{fit} (1 + T / T_F)^h.$$

- The form of the fit formulae reproduces the correct temperature, density and magnetic field dependence of the conductivity tensor in all limiting cases discussed above.
- The fit parameters C, a, b, c, d, g, h depend on the ionic structure of the material.
- The relative error of the fit formulae vary from 7% to 15% depending on the composition.

Summary

- The behavior of electrical conductivity mostly depends on the electron degeneracy.
- The scalar conductivity of NS crust increases with density, but the slope of increase is much weaker in the non-degenerated regime.
- It has a minimum as a function of temperature around $T^* \simeq T_F/3$.
- The anisotropy in transport properties becomes sizable at low densities, high magnetic fields, high temperatures and small ion charge numbers.
- The crust is the most isotropic at the transition temperature T^* .

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Future outlook

- Consider temperature-dependent crust compositions of multi-component plasma (including light nuclear clusters).
- Take into account properly the contribution of positrons.
- Calculate other transport coefficients using the same formalism.
- Implementation of conductivities in MHD NS-NS merger codes.

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THANK YOU FOR YOUR ATTENTION!