

# Discerning the symmetry energy and neutron star properties from nuclear collective excitations

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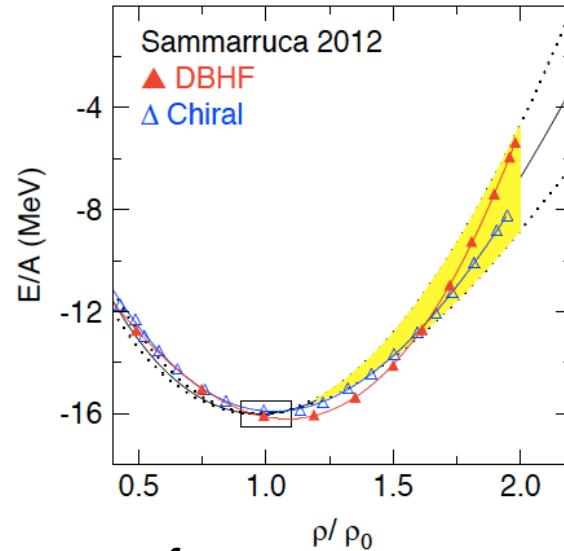
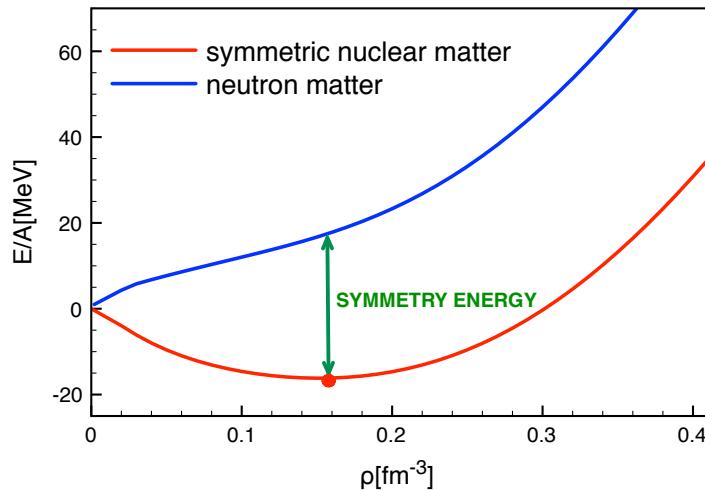
**QuantiXLie**

CENTER OF EXCELLENCE FOR THE THEORY OF QUANTUM AND COMPLEX SYSTEMS AND LIE ALGEBRA REPRESENTATION



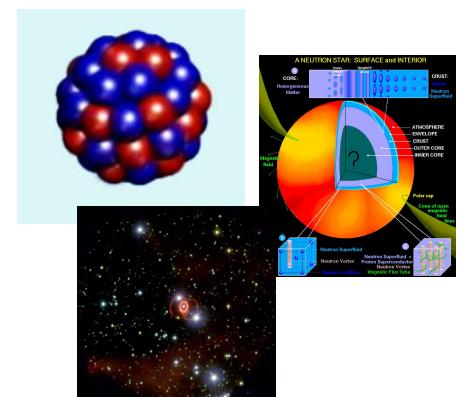
## INTRODUCTION

- Nuclear matter equation of state (EOS) plays important role in nuclear physics and astrophysics



- EOS beyond the saturation point?

- Symmetry energy  $S(p)$  describes the increase in the energy of the  $N \neq Z$  system as protons are turned into neutrons
  - Symmetry energy slope parameter determines the pressure of neutron matter
  - It is important for understanding the properties of neutron/rich nuclei, neutron-rich matter and neutron star radii, evolution of core collapse of massive star, explosive nucleosynthesis,...
  - Nuclear experiments provide important constraints on the symmetry energy at nuclear densities



## SYMMETRY ENERGY

- Nuclear matter equation of state:

$$E(\rho, \delta) = E_{SNM}(\rho) + E_{sym}(\rho)\delta^2 + \dots$$

$$\rho = \rho_n + \rho_p \quad \delta = \frac{\rho_n - \rho_p}{\rho}$$

- Symmetry energy term

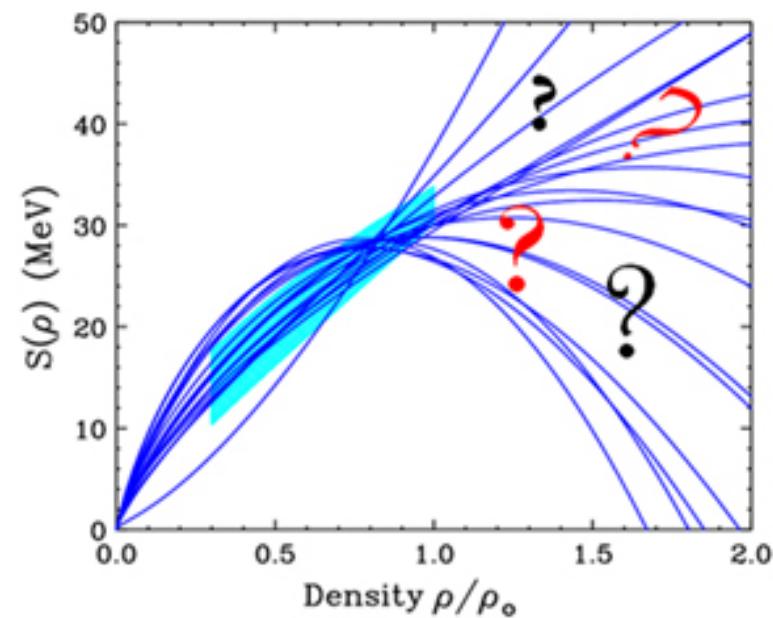
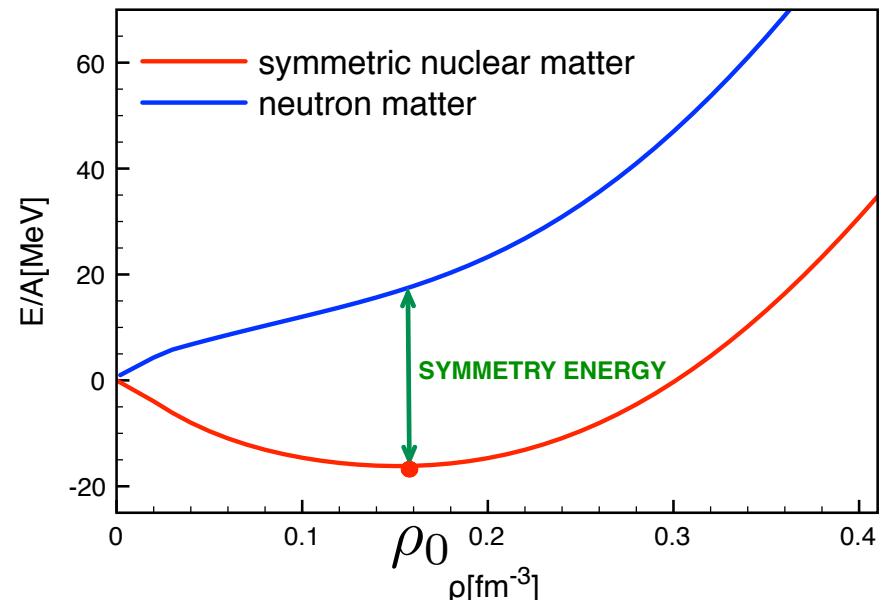
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \frac{dS_2(\rho)}{d\rho} \Big|_{\rho_0}$$

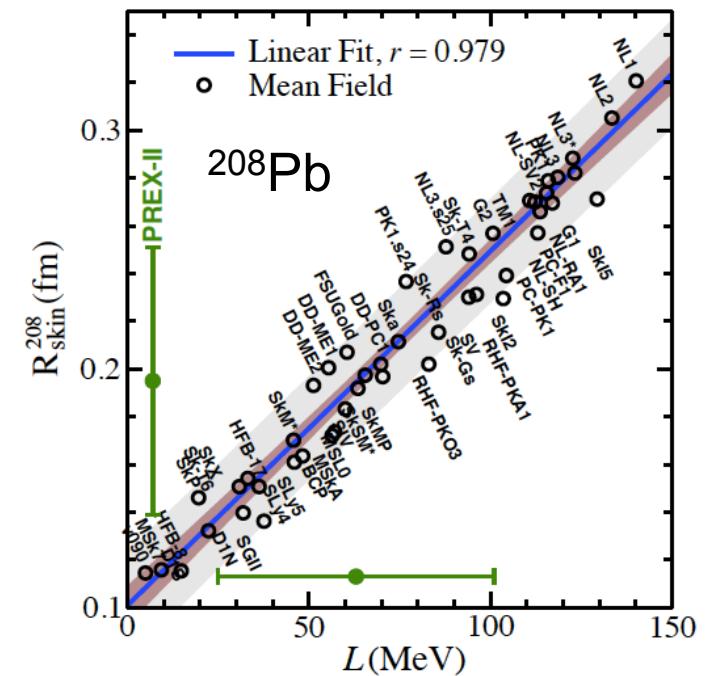
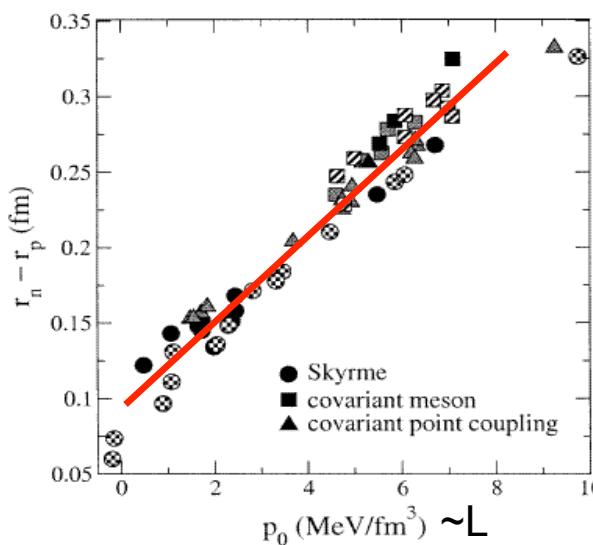
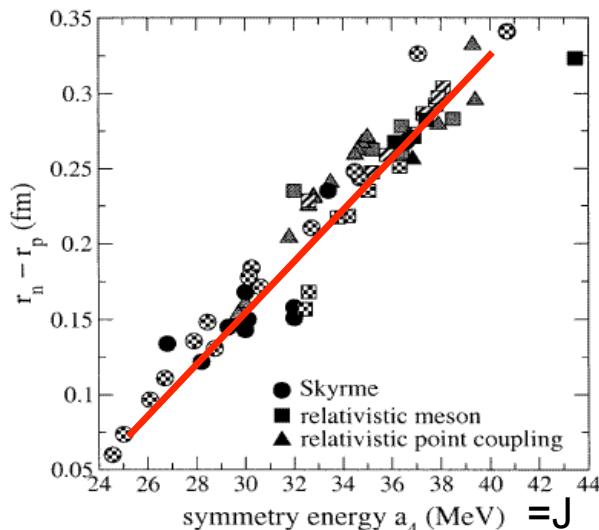
**J** – symmetry energy at saturation density

**L** – slope of the symmetry energy



B. Tsang, NSCL

# SYMMETRY ENERGY AND NEUTRON SKIN



- strong linear correlation between neutron skin thickness and symmetry energy parameters  $J(a_4)$ ,  $L(p_0)$

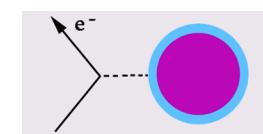
R.J.Furnstahl, Nucl. Phys. A 706, 85 (2002)

X. Roca-Maza et al.,  
Phys. Rev. Lett. 106, 252501 (2011)

- Parity violating electron scattering – Lead Radius Experiment (PREEx) @ JLab:

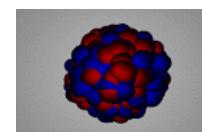
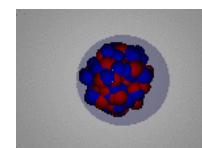
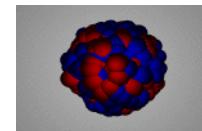
$$R_n - R_p = 0.33^{+0.16}_{-0.18}$$

Abrahamyan et al. PRL 108, 112502 (2012)



## COLLECTIVE EXCITATIONS AND THE SYMMETRY ENERGY

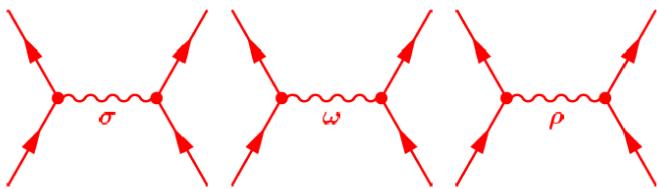
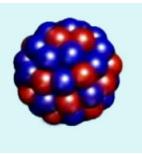
- There are various (isovector) modes of collective excitations that also provide constraints on the neutron skin thickness, recent experimental data available
  - Isovector giant dipole resonances
    - Dipole polarizability A. Tamii et al., PRL 107, 062502 (2011)  
 $\alpha_D \sim m_{-1}$  D.M. Rossi et al., PRL 111, 242503 (2013)  
T. Hashimoto et al., Phys. Rev. C 92, 031305(R) (2015)
    - Pygmy dipole resonances A. Carbone et al., PRC 81, 041301(R) (2010)  
A. Klimkiewitz et al., PRC 76, 051603(R) (2007)
    - Isovector giant quadrupole resonances S.S. Henshaw, M.W. Ahmed, et al, PRL 107, 222501 (2011)
    - Charge-exchange excitations - Anti-analog GDR, Gamow-Teller A. Krasznahorkay et al., PLB 720, 428 (2013)
    - ...
  - The goal: use the knowledge on collective excitations to constrain the symmetry energy



## THEORY FRAMEWORK

### RELATIVISTIC NUCLEAR ENERGY DENSITY FUNCTIONAL

i) Nucleons are Dirac particles coupled by the exchange mesons and the photon field



ii) Four-fermion contact interaction (Point-coupling model)



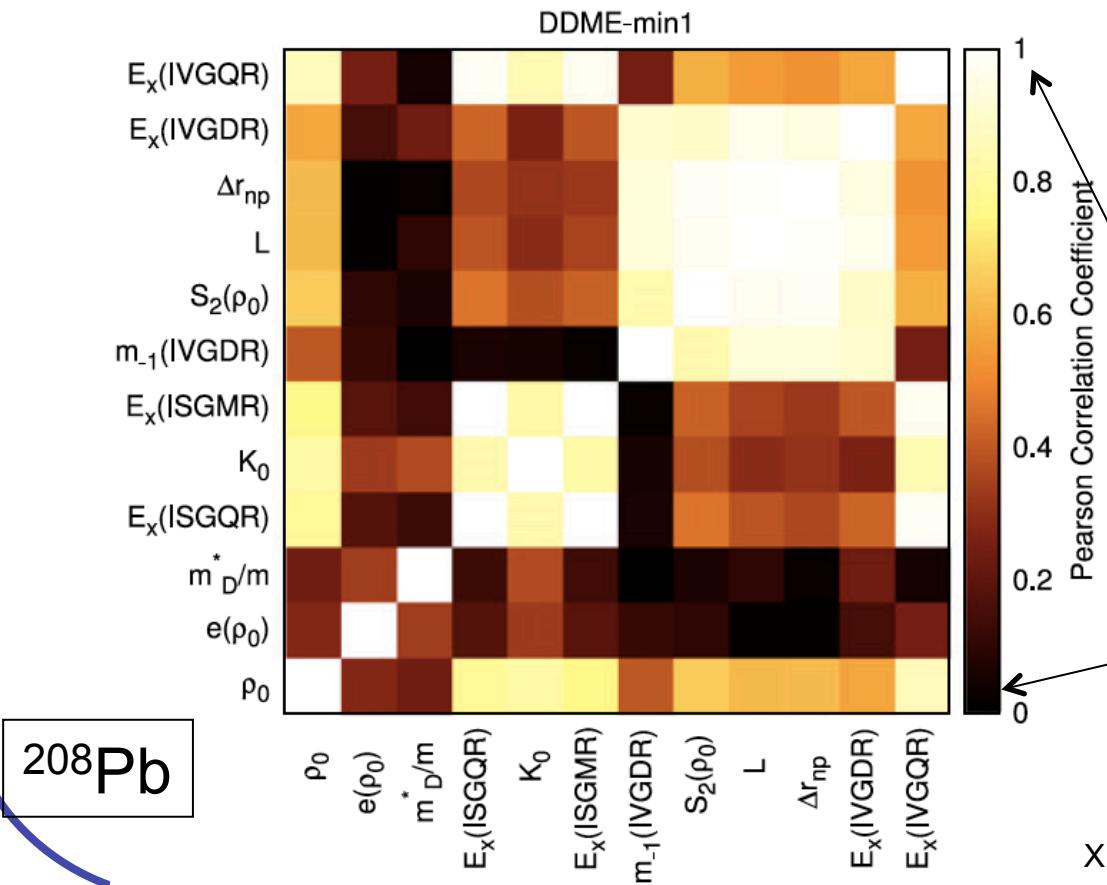
- Density dependent couplings
- The model parameters are constrained directly by many-body observables (masses, charge radii, pseudo-data, ...)
- Complicated many body dynamics encoded in the functional and its empirical constants
- DIRHB -- a relativistic self-consistent mean-field framework for atomic nuclei  
Relativistic Hartree Bogoliubov model [T. Niksic et al., Comp. Phys. Comm. 185, 1808 \(2014\).](#)
- In the small amplitude limit, self-consistent quasiparticle random phase approximation (QRPA) is used to describe nuclear excitations, etc.

## CORRELATIONS: NUCLEAR MATTER vs. PROPERTIES OF NUCLEI

- Covariance analysis in the EDF framework - information on relevant correlations and statistical uncertainties
- **Pearson product-moment correlation coefficient**  
provides a measure of the correlation (linear dependence) between two variables A and B.

Curvature matrix:

$$\mathcal{M}_{ij} = \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2|_{\mathbf{p}_0}$$



Correlation matrix between nuclear matter properties and several properties for  $^{208}\text{Pb}$  (DDME-min1)

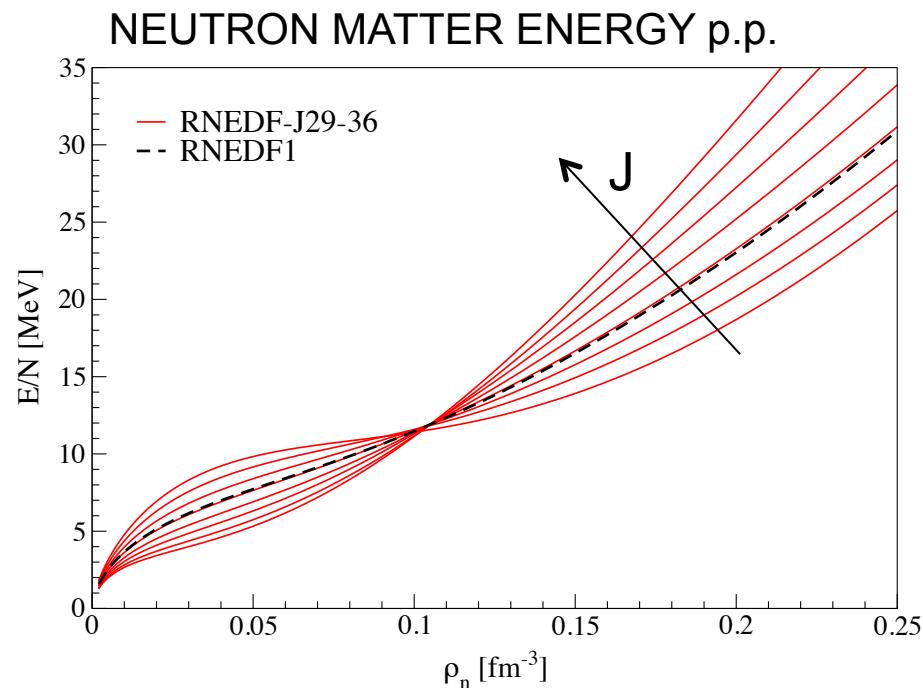
- neutron skin thickness,  
properties of giant  
resonances,...

strongly correlated

uncorrelated

## VARIATION OF THE SYMMETRY ENERGY IN CONSTRAINING THE EDF

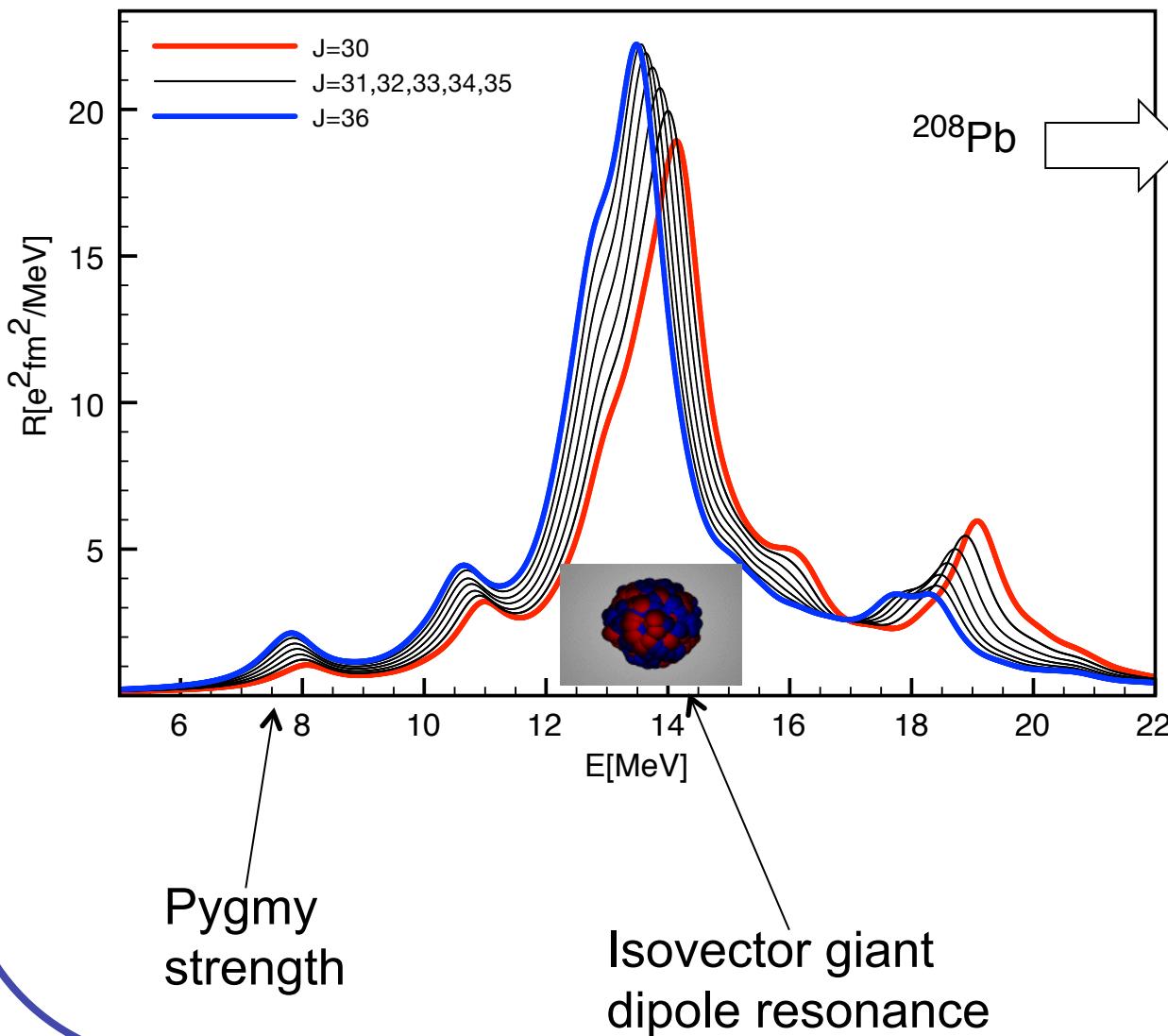
- Establish a set of 8 relativistic point coupling interactions that span the range of values of the **symmetry energy at saturation density**:  $J=29,30,\dots,36$  MeV
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- Each interaction is determined independently using the same dataset supplemented with an additional constraint on  $J$



<b>J[MeV]</b>	<b>L[MeV]</b>
29	31.9
30	37.0
31	44.1
32	52.5
33	62.2
34	72.3
35	83.4
36	94.3

## CONSTRAINING THE SYMMETRY ENERGY

- Isovector dipole transition strength for  $^{208}\text{Pb}$  using a set of relativistic point coupling interactions which vary the symmetry energy properties ( $J=30,31,\dots,36$  MeV)



- Isovector giant dipole resonance
- Pygmy dipole strengths
- Dipole polarizability ( $\alpha_D \sim m_{-1}$ )

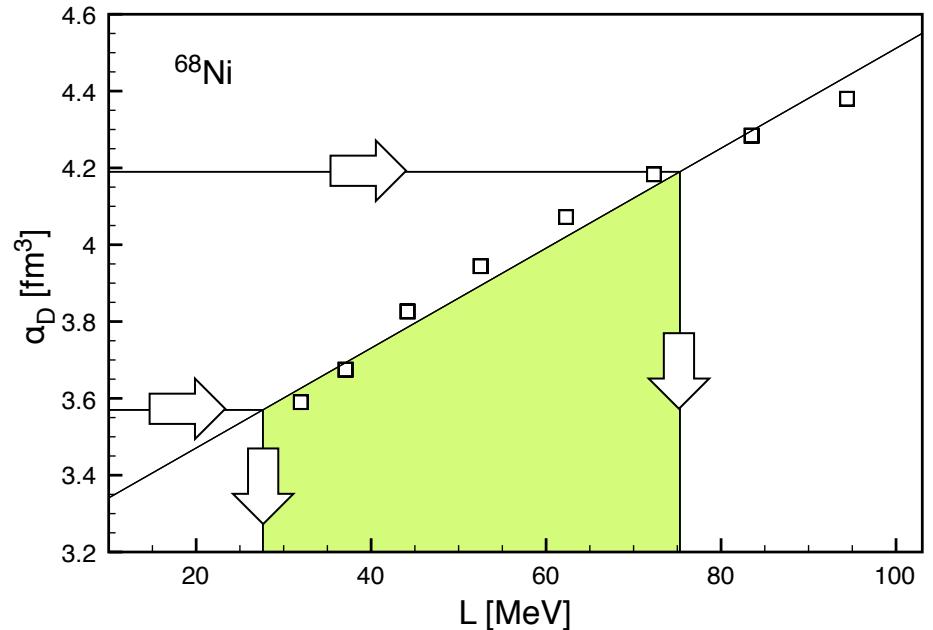
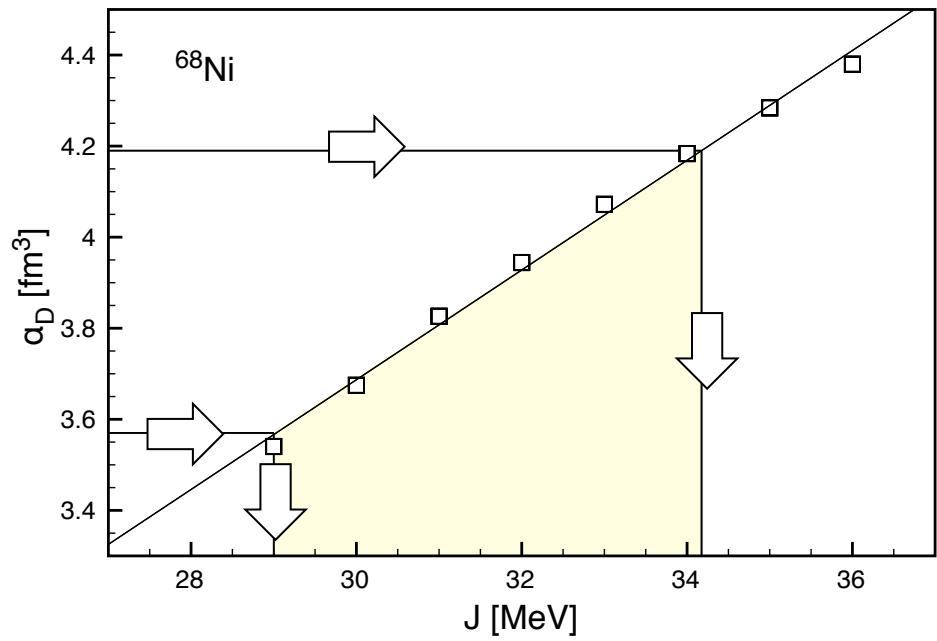
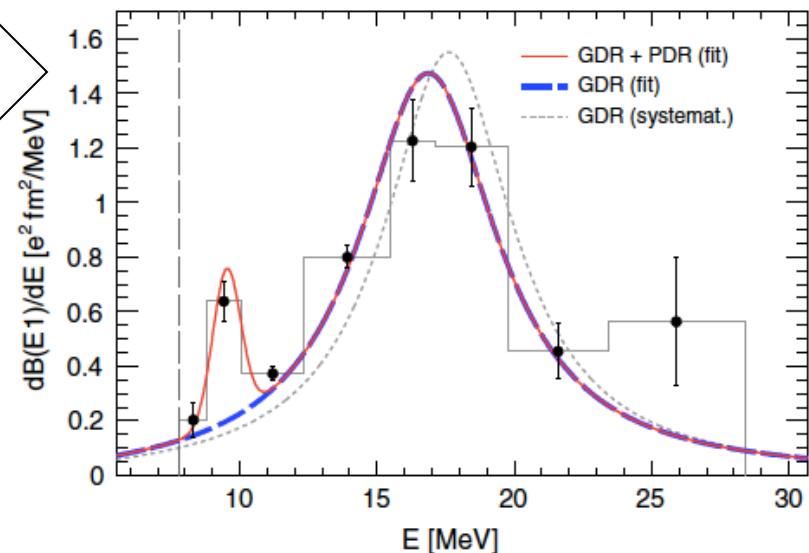
- The transition strength is sensitive on the properties of symmetry energy - ( $J,L$ )

→ Dipole response can be used to constrain effective nuclear interactions (isovector channel)

- There are exp. data available on the dipole response in nuclei ( $\alpha_D$ , IVGDR, pygmy strength)

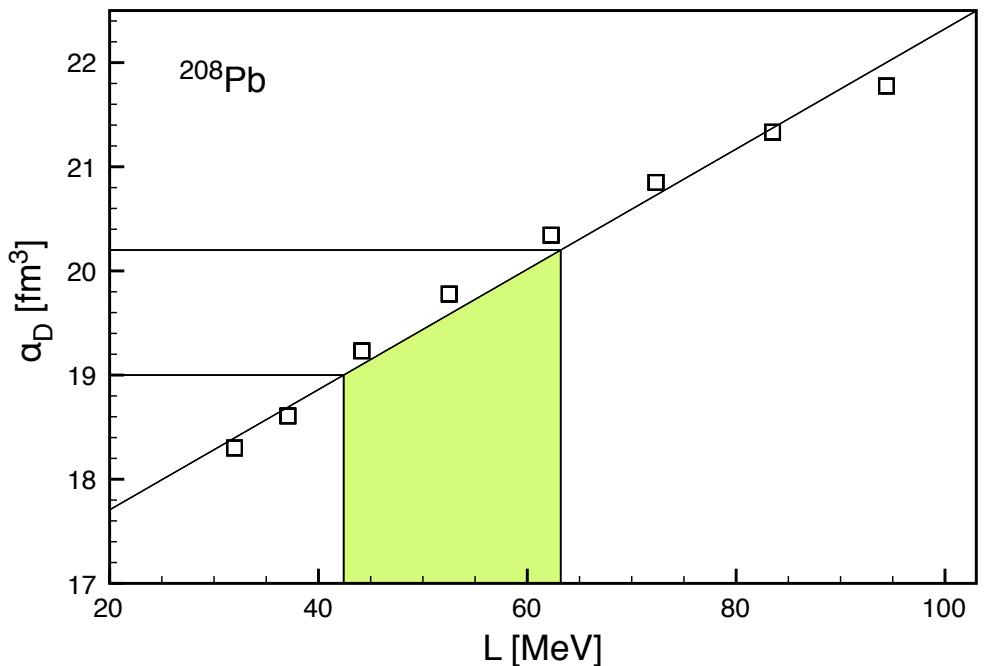
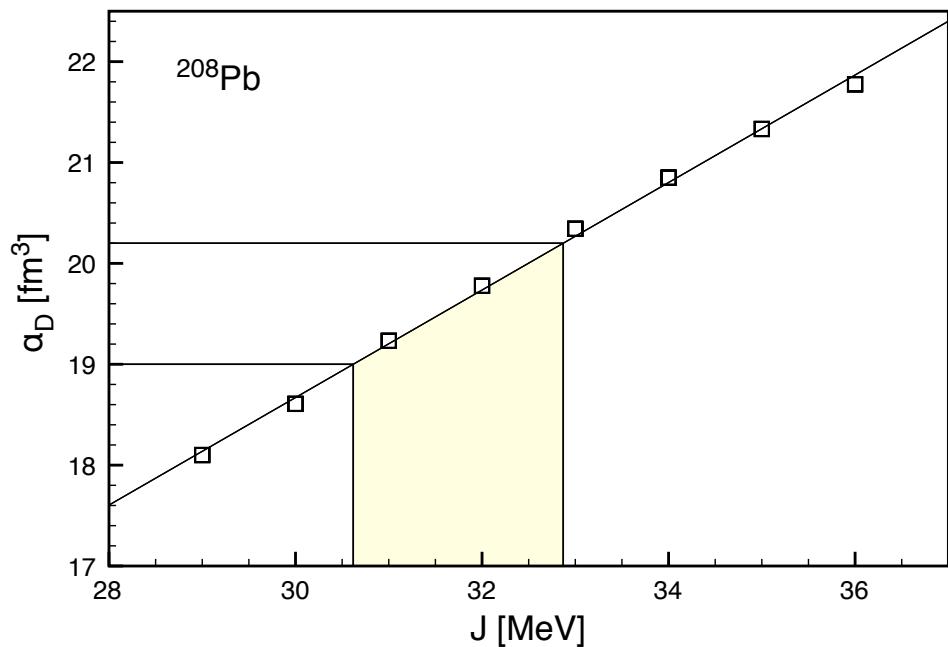
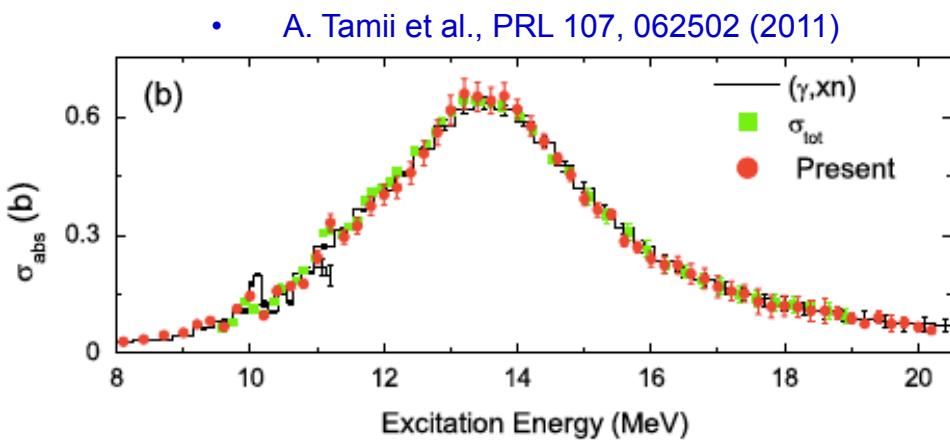
# DIPOLE POLARIZABILITY IN $^{68}\text{Ni}$ AND SYMMETRY ENERGY

- Measurement of dipole polarizability of unstable neutron rich  $^{68}\text{Ni}$   
D. Rossi et al, PRL 111, 242503 (2013)
- Implementation to constrain the symmetry energy using relativistic point coupling interactions



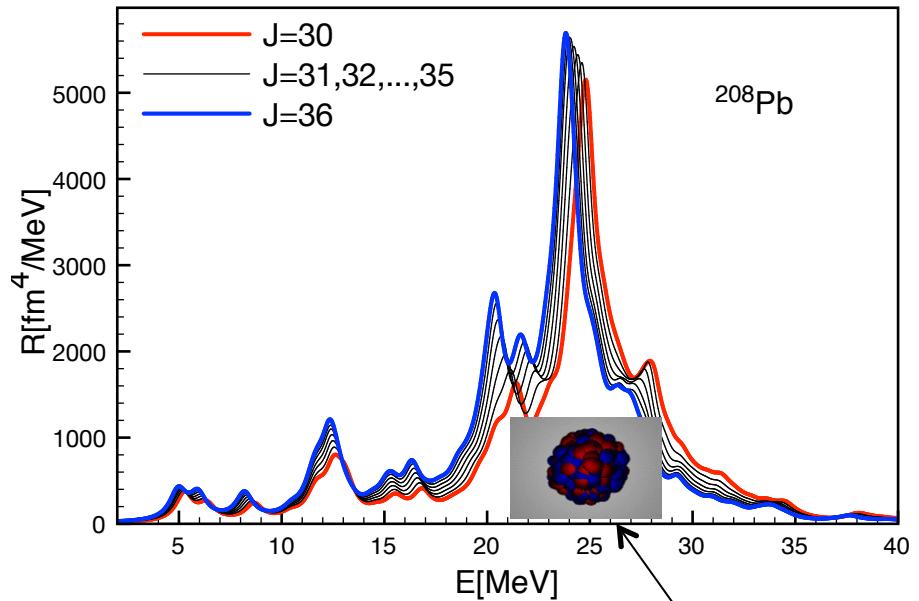
# DIPOLE POLARIZABILITY IN $^{208}\text{Pb}$ AND SYMMETRY ENERGY

- Using relativistic point coupling interactions and measurement of dipole polarizability in  $^{208}\text{Pb}$



# GIANT QUADRUPOLE RESONANCES IN $^{208}\text{Pb}$ AND SYMMETRY ENERGY

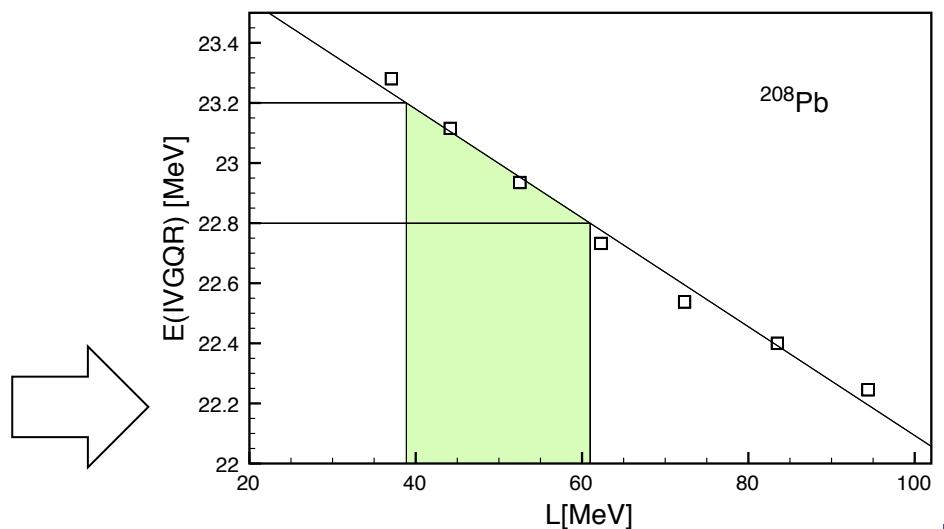
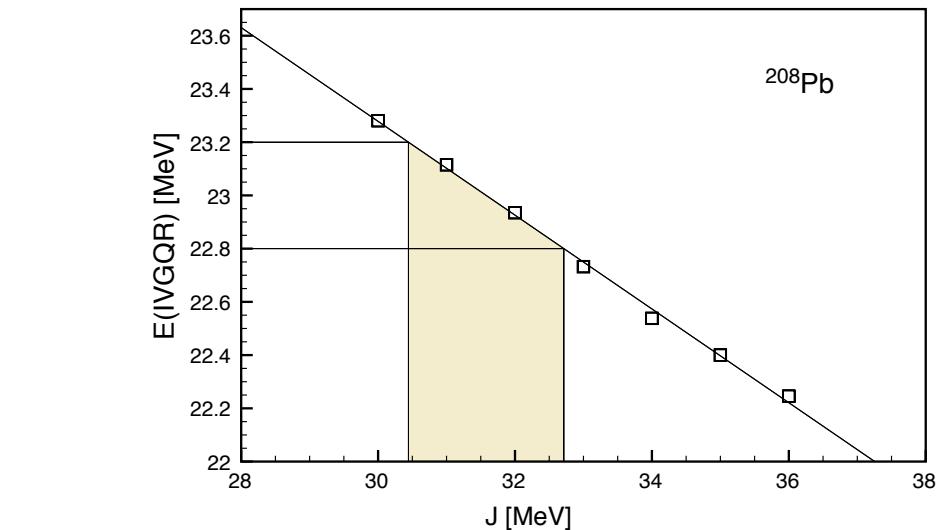
- Isovector quadrupole transition strength for  $^{208}\text{Pb}$  using a set of relativistic point coupling interactions which vary the symmetry energy properties ( $J=30, 31, \dots, 36$  MeV)



Isovector giant quadrupole resonance

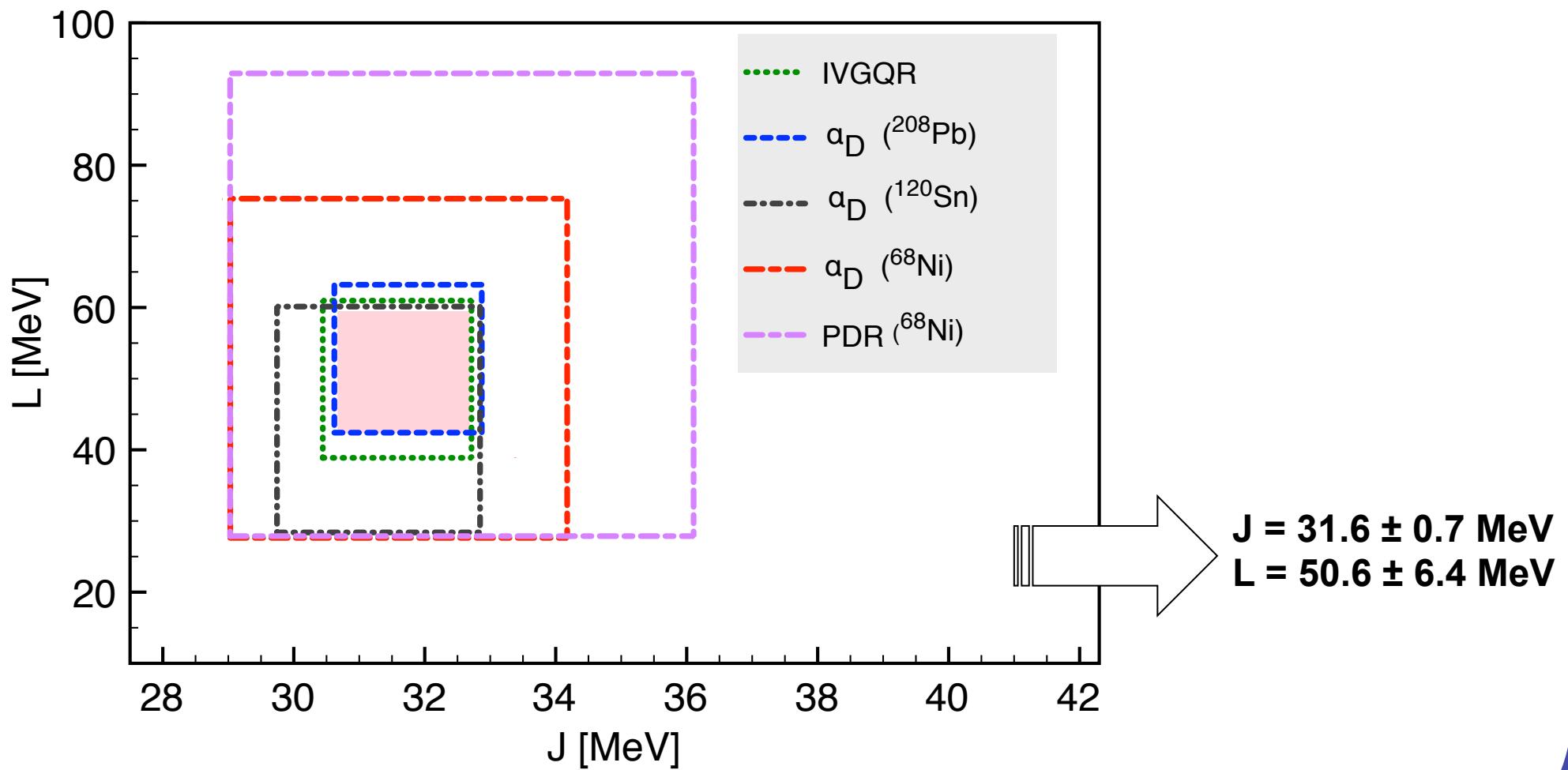
- Precise exp. determination of isovector giant quadrupole resonances

S.S. Henshaw, M.W. Ahmed, G. Feldman et al,  
PRL 107, 222501 (2011)

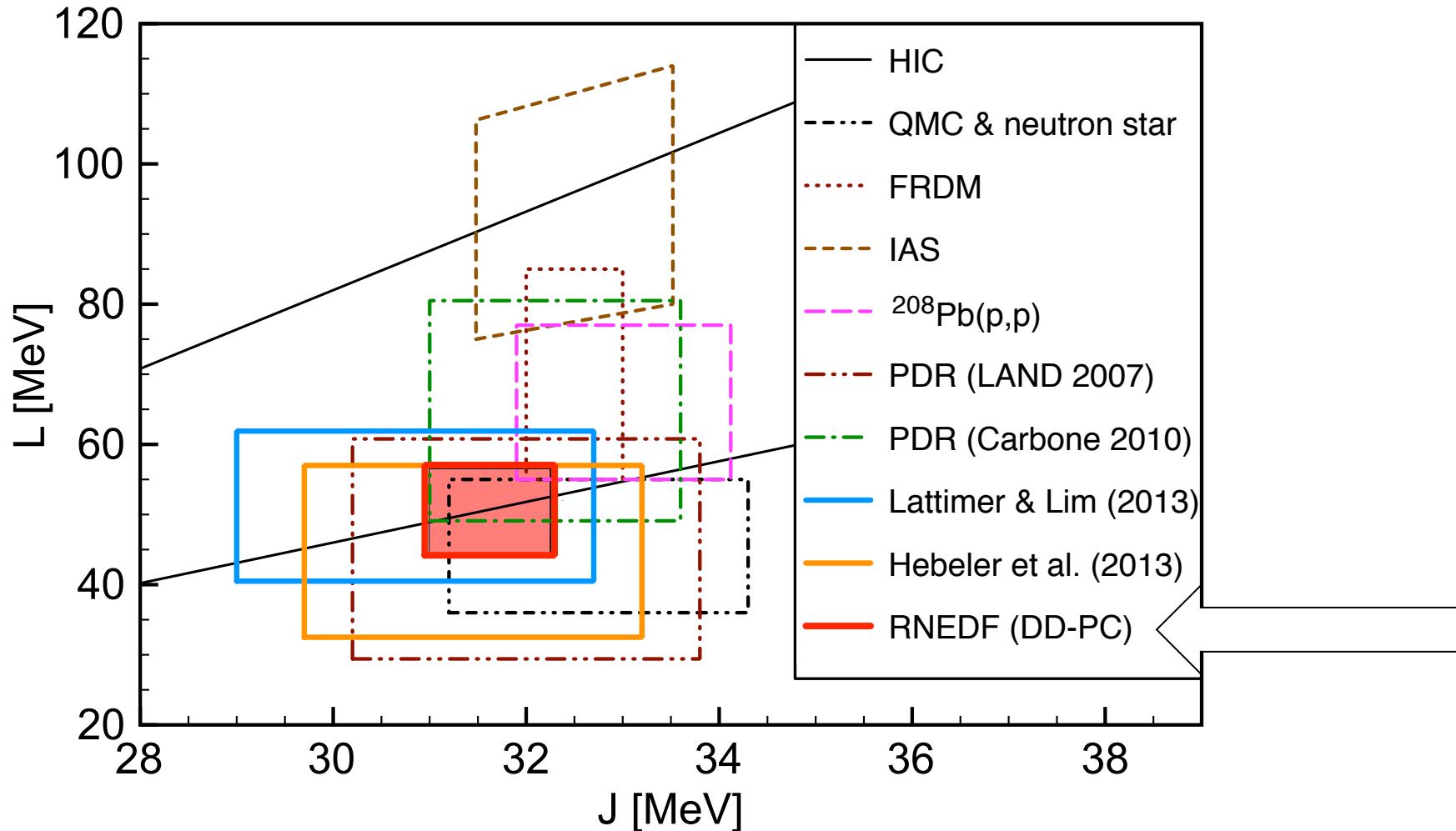


## CONSTRAINING THE SYMMETRY ENERGY (J-L)

- Based on the same set of relativistic density-dependent point coupling interactions



## CONSTRAINING THE SYMMETRY ENERGY (J-L)



- Lattimer & Lim, ApJ. 771, 51 (2013) – compilation from various approaches
- K. Hebeler et al., AJ 773, 11 (2013) – based on nuclear interactions derived from chiral EFT
- P. Danielewicz and J. Lee, Nucl. Phys. A 818, 36 (2009) – IAS
- A. Carbone, G. Colo, A. Bracco, et al., Phys. Rev. C 81, 041301 (2010) – PDR
- A. W. Steiner and S. Gandolfi, Phys. Rev. Lett. 108, 081102 (2012) – QMC (Av8') + neutron stars
- etc.

## CONSTRAINING THE SYMMETRY ENERGY: 2<sup>nd</sup> approach

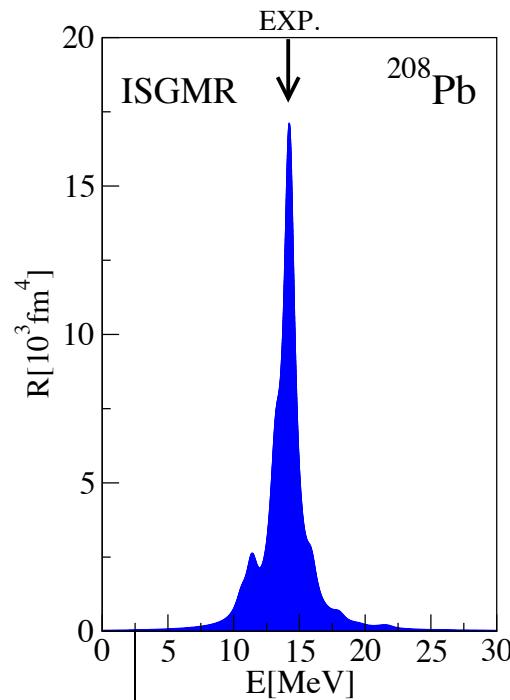
- use the experimental data on collective excitations to constrain the symmetry energy **within the fitting protocol** to determine the parameters of the functional (relativistic point coupling interaction; N.P., M. Hempel et al. in prep.)
- Adjust the properties of 72 spherical nuclei to exp. data (binding energies, charge radii, diffraction radii, surface thickness, pairing gaps)
- constrain the symmetry energy  $S_2(p_0)=J$  (2%) from exp. data on dipole polarizability ( $^{208}\text{Pb}$ ) within an iterative procedure [A. Tamii et al., PRL 107, 062502 \(2011\) + update \(2015\)](#).
- constrain the nuclear matter incompressibility  $K_{\text{nm}}$  (2%) from exp. data on ISGMR modes ( $^{208}\text{Pb}$ ); [D. Patel et al., PLB 726, 178 \(2013\)](#).
- constrain the maximal neutron star mass by solving the Tolman-Oppenheimer-Volkov (TOV) equations + observational data from [J. Antoniadis, et al. Science 340, 448 \(2013\)](#)



**“Correct symmetry energy” is the one obtained for the interaction that accurately reproduces the exp. data on dipole polarizability**

## CONSTRAINING THE SYMMETRY ENERGY: 2<sup>nd</sup> approach

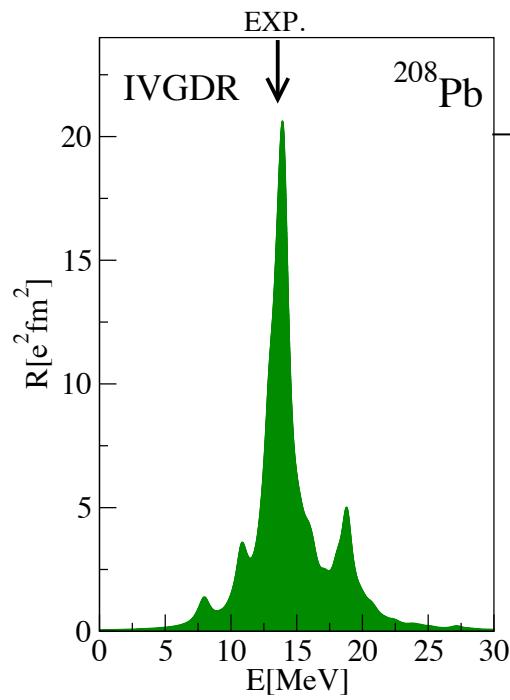
ISOSCALAR GIANT MONOPOLE RESONANCE



- ISGMR energy determines the nuclear matter incompressibility:  
 $K_{\text{nm}} = 232.4 \text{ MeV}$

$$\begin{aligned} E(\text{Exp.}) &= (13.91 \pm 0.11) \text{ MeV (TAMU)} \\ E(\text{Exp.}) &= (13.7 \pm 0.1) \text{ MeV (RCNP)} \end{aligned}$$

ISOVECTOR GIANT DIPOLE RESONANCE



Dipole polarizability:  
 $\alpha_D = (19.68 \pm 0.21) \text{ fm}^3$

Exp.  
 $\alpha_D = (19.6 \pm 0.6) \text{ fm}^3$

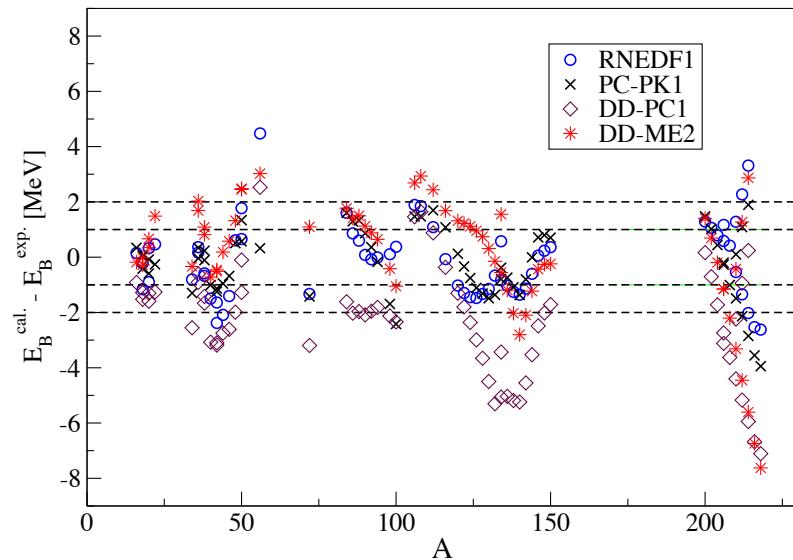
A.Tamii et al., PRL 107, 062502  
(2011). + update (2015).

- IVGDR –  $\alpha_D$  determine the symmetry energy for the interaction

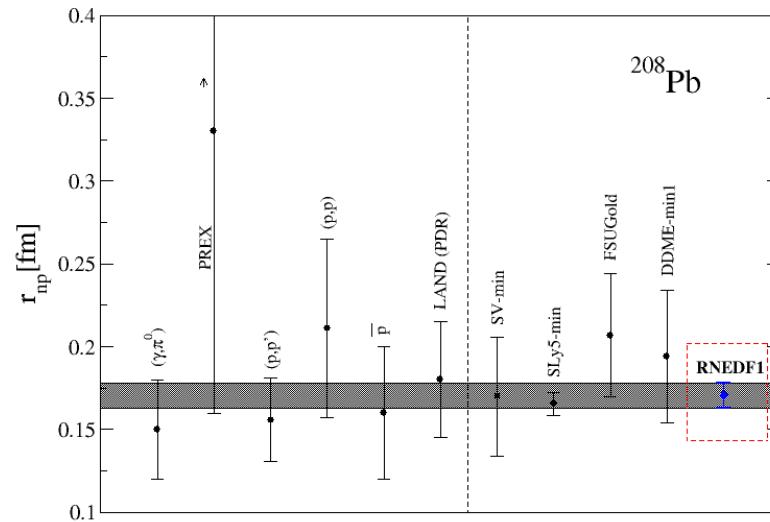
$J = 31.89 \text{ MeV}$   
 $L = 51.48 \text{ MeV}$

## SOME OTHER PROPERTIES...

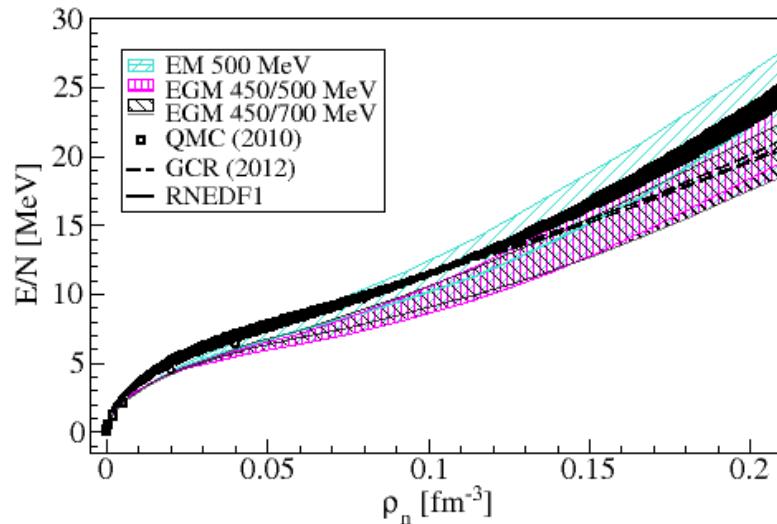
Nuclear binding energies (calc. – exp.)



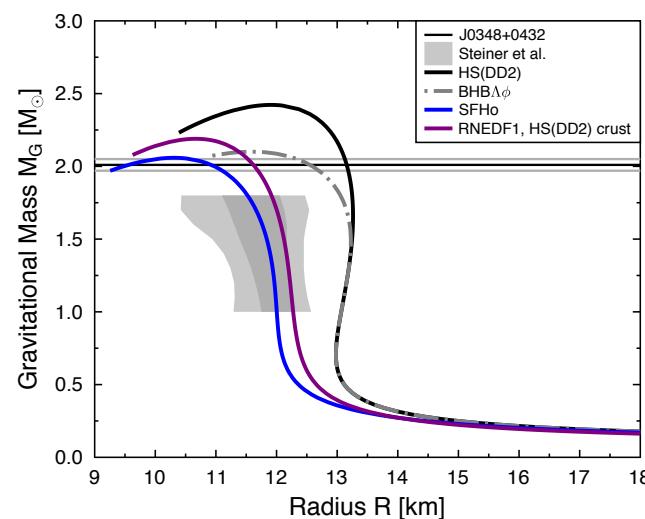
Neutron skin thickness



Neutron matter



Neutron star mass-radius relationship



# NEUTRON SKINS AND NEUTRON STAR PROPERTIES

- Neutron star properties
- The phase **transition density ( $n_t$ )** and **pressure ( $P_t$ )** at the inner edge separating the core from crust are sensitive to poorly constrained density dependence of the nuclear matter symmetry energy

C.J. Horowitz and J. Piekarewicz, PRL 86, 5647 (2001)

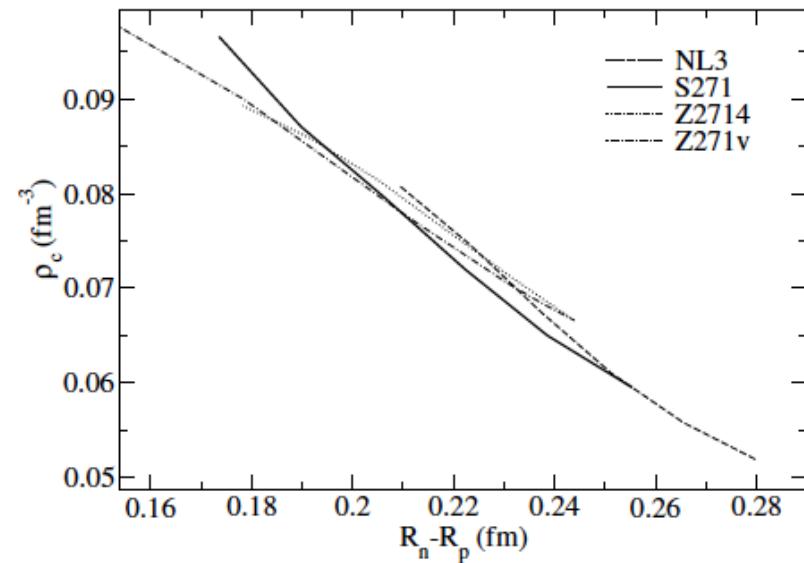
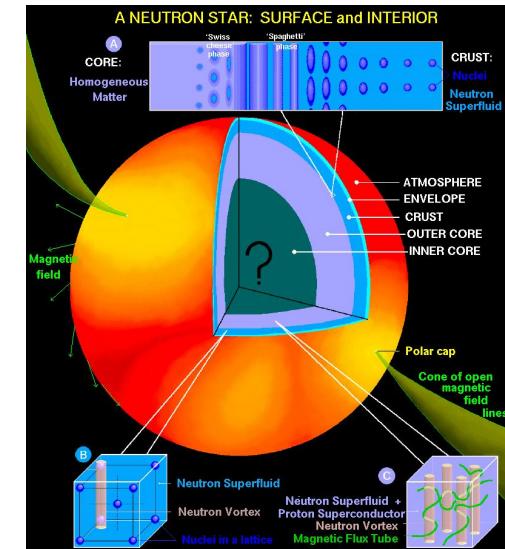


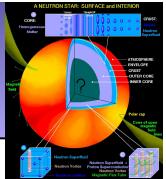
FIG. 1. Estimate of the transition density from nonuniform to uniform neutron-rich matter versus neutron-minus-proton radius in  $^{208}\text{Pb}$ . The curves are for the four parameter sets described in the text.



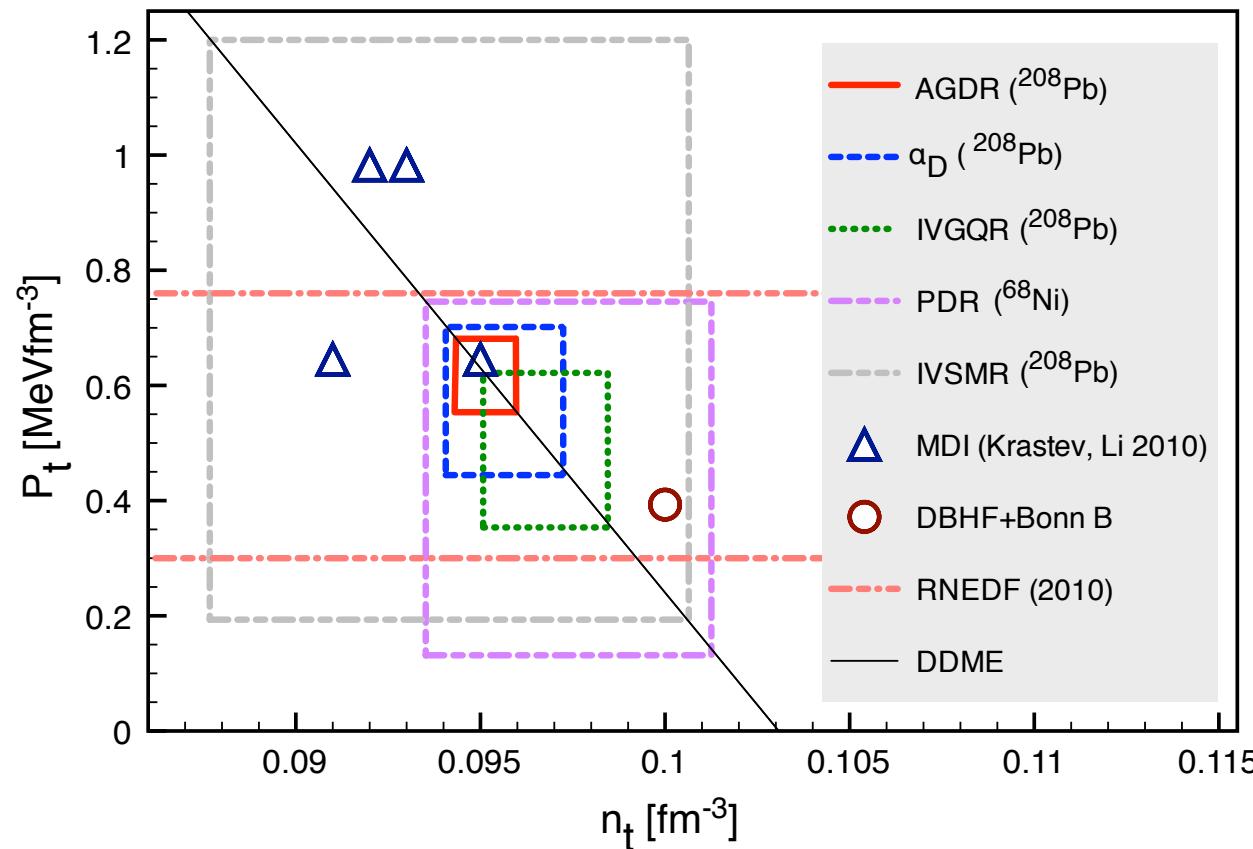
(D. Page)

- Relationship between the neutron-rich skin of  $^{208}\text{Pb}$  and liquid-to-solid transition density for neutron rich matter.

# Neutron star structure and excitations in finite nuclei



- Constraints on the neutron star core-to-crust transition density  $n_t$  and pressure  $P_t$
- RNEDF and thermodynamic method [Ch. C. Moustakidis et al., PRC 81, 065803 \(2010\)](#)
- Exp. data for AGDR, IVGQR and IVSMR excitation energies ( $^{208}\text{Pb}$ ), dipole polarizability ( $^{208}\text{Pb}$ ) and PDR energy weighted strength ( $^{68}\text{Ni}$ ).



$n_t$ [ $\text{fm}^{-3}$ ]	
RNEDF (EXC.)	0.0955
A18+ $\delta v$ +UIX* (A. Akmal, 1998)	0.087
EOS Friedman Pandharipande (C.P. Lorenz, 1993)	0.096
Chiral EFT (NN+3N) (K. Hebeler, 2013)	0.076-0.088
HIC (B.A.Li, 2005)	0.040-0.065

## CONCLUDING REMARKS

- Dipole excitations in nuclei (PDR,  $\alpha_D$ , IVGDR, AGDR) and other modes (IVGQR,...) provide valuable constraints for the nuclear matter symmetry energy
- Accurate measurements have important implications to reduce uncertainties in the symmetry energy – motivation for the future experiments
- Small uncertainty in the calculated symmetry energy ( $J,L$ ) limits the choice of currently available equations of state used in modeling neutron stars and supernova matter.
- Neutron star core-to-crust transition density and pressure can be assessed using information on collective nuclear excitations / symmetry energy

Final conclusion:

**The EDF / EOS with the correct symmetry energy is the one that reproduces the exp. data on isovector constraints from nuclear experiments**