Nuclear and Particle Physics Aspects of Neutron Star Mergers

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- Equation of state: tidal polarizability.
- Neutron star seismology.
- New particles: axions in mergers.
Neutron Star Merger Dynamics

(General) Relativistic (Very) Heavy-Ion Collisions at ~ 100 MeV/nucleon

- **Inspiral:**
  - Gravitational waves, Tidal Effects & Dense Matter EoS

- **Merger:**
  - Disruption, NS oscillations, ejecta and r-process nucleosynthesis

- **Post Merger:**
  - Ambient conditions power GRBs, Afterglows, and Kilo/Macro Nova

Simulations: Rezzola et al (2013)
Late Inspiral: \( R_{\text{orbit}} \lesssim 10 R_{\text{NS}} \)

Tidal forces deform neutron stars. Induces a quadrupole moment.

\[
Q_{ij} = \lambda \, E_{ij} \\
E_{ij} = -\frac{\partial^2 V(r)}{\partial x_i \partial x_j}
\]
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Quadrupole polarizability

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External field
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External field

\[ V(r) \approx -\frac{GM_a}{r} - \frac{GQ_a}{r^3} \approx -\frac{GM_a}{r} - \frac{G\lambda M_b}{r^6} \]

This advances the orbit and changes the rotational phase.
Tidal Love Number

Is a property of the unperturbed spherical neutron star.

Quadrupole polarizability: \( \lambda = k_2(\beta, \bar{y}) \, R_{NS}^5 \)

tidal love number

compactness: \( \beta = \frac{GM_{NS}}{R_{NS}} \approx 0.2 \left( \frac{M_{NS}}{1.4M_{\odot}} \right) \left( \frac{12 \text{ km}}{R_{NS}} \right) \)

For neutron stars:

\[
k_2(\beta, \bar{y}) = \frac{1 - 2\beta^2}{2} \left( \frac{2 - \bar{y}}{3 + \bar{y}} + \frac{\bar{y}^2 - 6\bar{y} - 6}{(\bar{y} + 3)^2} \right) \beta + O[\beta^2]
\]

\( \bar{y} = y(R_{NS}) \) is obtained by solving

\[
r \frac{dy(r)}{dr} + y(r)^2 + y(r)e^{\lambda(r)} \left( 1 + 4\pi r^2 (p(r) - \rho(r)) \right) + r^2 \Phi(r) = 0
\]
The dense matter EOS \( (P(\rho)) \) determines:

\[ p(r) \text{ and } \rho(r) \text{ for a given } M \]

\[ \lambda = k_2(\beta, \bar{y}) \: R_{NS}^5 \]

\[ \beta = \frac{GM}{R} \]
The dense matter EOS \( (P(\rho)) \) determines:

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\]
Combining the GW & EM Signatures

Short-gamma ray bursts argued to be NS-NS or NS-BH mergers. SGRBs show interesting temporal features.

Figure from Fernandez & Metzger (2015)
Pre-merger Neutron Star Dynamics

• What are the fundamental modes of excitation of cold neutron stars?

• Which of these are strongly excited during the merger?

• Can internal excitations couple to the (EM) emitting region to produce observable QPOs?

QPOs in SGR Giant Flares

SGR 0525-66 (1979)
SGR 1627-41 (1998)
Phases of Dense Matter in Neutron Stars

*Outer Crust:* (solid) nuclei, electrons

*Inner Crust:* (Solid-Superfluid) nuclei, electrons, neutrons

*Outer Core:* (Superfluid-Superconductor) neutrons, protons, electrons, muons

*Inner Core:* 

- Outer Crust: ~12 km
- Inner Crust: ~11.8 km
- Inner Core: ~11 km
- Outer Core: ~3 km

Density (g/cm³): 10⁴, 4×10¹¹, 10¹⁴, 6×10¹⁴, 10¹⁵

Radius (km): M ≈ 1.4 M☉
Low Energy Theory of Phonons

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitations are associated with the fluctuations of the phase of the neutron condensate.

$$\langle \psi^\uparrow(r, t) \psi^\downarrow(r, t) \rangle = |\Delta(r, t)| \exp(2i\phi(r, t))$$

Collective coordinates:

- Vector Field: $\xi_i(r, t)$
- Scalar Field: $\phi(r, t)$
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Scalar Field: \( \phi(r, t) \)
Symmetries & Derivative Expansion

The low energy theory must respect symmetries of the underlying Hamiltonian

\[ \begin{align*}
\xi^{a=1..3}(r, t) & \rightarrow \xi^{a=1..3}(r, t) + a^{a=1..3} \\
\phi(r, t) & \rightarrow \phi(r, t) + \theta
\end{align*} \]

Only derivative terms are allowed. Lagrangian density for the phonon system with cubic symmetry:

\[ \mathcal{L}_{n+p} = \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} v_s^2 (\partial_i \phi)^2 + \frac{1}{2} (\partial_t \phi_i)^2 - \frac{1}{2} c_i^2 (\partial_i \phi_i)^2 \]

\[ + g \partial_t \phi \partial_i \xi_i + \gamma \partial_i \phi \partial_t \xi_i \]

Transverse lattice phonons:

\[ \mathcal{L}_t = \frac{1}{2} (\partial_t \xi_i)^2 - \frac{1}{2} c^2_t (\partial_i \xi_j + \partial_j \xi_i)^2 \]
Low Energy Constants

Are related to thermodynamics derivatives.

Velocities:

\[
\begin{align*}
v_s^2 &= \frac{n_f}{m \chi_n} \\
c_t^2 &= \frac{K + 4 \mu_s/3}{m(n_p + n_b)} \\
c_t^2 &= \frac{\mu_s}{m(n_p + n_b)}
\end{align*}
\]

Longitudinal lattice phonons and superfluid phonons are coupled:

\[
g = n_p \ E_{np} \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \\
\tilde{\gamma} = \frac{-n_b \ v_s}{\sqrt{(n_p + n_b)n_f}}
\]

Entrainment: protons drag neutrons.

\[
\begin{align*}
\text{Bound neutrons: } & \quad n_b = \gamma \ n_n \\
\text{Free neutrons: } & \quad n_f = n_n \ (1 - \gamma)
\end{align*}
\]
Low Energy Constants

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\[ v_s^2 = \frac{n_f}{m \chi_n} \]
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compressibility

shear modulus

number susceptibility

Longitudinal lattice phonons and superfluid phonons are coupled:

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\[ \tilde{\gamma} = \frac{-n_b v_s}{\sqrt{(n_p + n_b)n_f}} \]

Entrainment: protons drag neutrons.

\{ Bound neutrons: \[ n_b = \gamma n_n \] 
Free neutrons: \[ n_f = n_n (1 - \gamma) \] \}
Entrainment

\[ n_b \neq \text{number of “bound” neutrons.} \]

Bragg scattering off the lattice is important.

\[ n_f = \frac{m}{24\pi^3} \sum_{\alpha} \int_{F} |\nabla_k \epsilon_{\alpha,k}| \, dS^\alpha \]

\[ n_b = n_n - n_f \]

Complex interplay of nuclear and band structure effects.

The nuclear surface and disorder are likely to play a role.

Longitudinal lattice phonons and superfluid phonons are strongly coupled by entrainment.
Waves in the Crust

Longitudinal modes:
Oscillations of the neutron superfluid and ion solid become strongly mixed

Transverse (shear) modes:
Entrainment of neutrons lowers the velocity of shear waves in the solid.

Chamel, Page, Reddy (2013)
Pairing in the Core

- Proton pair due to s-wave interaction.
- For neutrons attraction is due to p-wave interaction in the spin-1 channel.

<table>
<thead>
<tr>
<th>Protons:</th>
<th>Spin–singlet pairs</th>
<th>$S = 0$</th>
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<tr>
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<td>$L = 0$</td>
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<td></td>
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<td>$L &gt; 0$</td>
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\[
\langle \psi_p^T \psi_P \rangle = \Delta_p^0 e^{i2\theta_p}
\]

Action is invariant under: $\theta_p \rightarrow \theta_p + \phi_p$

<table>
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</tr>
</tbody>
</table>

\[
\langle \psi_n^T \sigma_2 \sigma^i \gamma^j \psi_n \rangle = \Delta_n^{ij} e^{i2\theta_n}
\]

Action is invariant under: $\theta_n \rightarrow \theta_n + \phi_n$

\[
\Delta_n^{ij} \rightarrow \mathcal{R}(\beta) \Delta_n^{ij} \mathcal{R}^T(\beta)
\]
Low energy modes in the core


Superfluid Phonons:

\[
\mathcal{L}_0 = \frac{1}{2} (\partial_t \phi_n)^2 - \frac{v_n^2}{2} (\partial_i \phi_n)^2 + \frac{1}{2} (\partial_t \phi_p)^2 - \frac{v_p^2}{2} (\partial_i \phi_p)^2 + g_{pn} \partial_t \phi_n \partial_t \phi_p - v_{pn} \partial_i \phi_n \partial_i \phi_p
\]

Angulons:

\[
\mathcal{L}_{\text{ang}} = \sum_{i=1,2} \left[ \frac{1}{2} (\partial_0 \beta_i)^2 - \frac{1}{2} v_\perp^2 ((\partial_x \beta_i)^2 + (\partial_y \beta_i)^2) + v_\parallel^2 (\partial_z \beta_i)^2 \right] + \frac{eg_n f_\beta}{2M \sqrt{-\nabla_\perp^2}} \left[ B_1 \partial_0 (\partial_y \beta_1 + \partial_x \beta_2) + B_2 \partial_0 (\partial_x \beta_1 - \partial_y \beta_2) \right]
\]

Bedaque and Reddy (2013),
Mixing and Damping of Goldstone Bosons

Modes decay due to the coupling to the large density of electron-hole states. At long-wavelength the damping timescales is related to the electron shear viscosity.

Bedaque and Reddy (2013)
Low energy modes and EOS

\[ \mathcal{L}_0 = \frac{1}{2} (\partial_t \phi_n)^2 - \frac{v_n^2}{2} (\partial_i \phi_n)^2 + \frac{1}{2} (\partial_t \phi_p)^2 - \frac{v_p^2}{2} (\partial_i \phi_p)^2 + g_{pn} \partial_t \phi_n \partial_t \phi_p - v_{pn} \partial_i \phi_n \partial_i \phi_p \]

The low energy constants are:

\[
\begin{align*}
v_p^2 &= \frac{n_p}{\mu_p \chi_{pp}} \\
v_n^2 &= \frac{n_n}{\mu_n \chi_{nn}} \\
g_{pn} &= \frac{\chi_{pn}}{\sqrt{\chi_{pp} \chi_{nn}}} \\
v_{pn}^2 &= \frac{n_{pn}}{\sqrt{\mu_p \mu_n} \sqrt{\chi_{pp} \chi_{nn}}} 
\end{align*}
\]

where 
\[ n_i = \frac{\partial P}{\partial \mu_i} \quad \text{and} \quad \chi_{ij} = \frac{\partial^2 P}{\partial \mu_i \partial \mu_j} \]

For a given EOS the LECs are specified and the velocity and damping timescales of the eigen-modes are easily calculated.

At \( n_B=0.16 \text{ fm}^{-3} \)

<table>
<thead>
<tr>
<th></th>
<th>( v_n/c )</th>
<th>( v_p/c )</th>
<th>( v_+/c )</th>
<th>( v_-/c )</th>
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</thead>
<tbody>
<tr>
<td>“bare”</td>
<td>0.21</td>
<td>0.23</td>
<td>0.28</td>
<td>0.20</td>
</tr>
<tr>
<td>“mixed”</td>
<td></td>
<td></td>
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</tbody>
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New Particles in Mergers

• The merged object is very hot and very dense. Copious production of any weakly particles with masses less than about 200 MeV. Axions and other dark matter candidates.

• The vicinity is optically thin and may contain large magnetic fields. Some features of the EM emission suggest the existence of a magnetar.

Axions are particularly interesting because they couple to nucleons and photons

\[ \mathcal{L}_{aN} = -\frac{C_{aN}}{f_a} \partial_\mu a \bar{N} \gamma^\mu \gamma_5 N \]

\[ C_{aN} \approx 1 \]

\[ \mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} a \ F \tilde{F} = g_{a\gamma\gamma} \ a \ \vec{E} \cdot \vec{B} \]

\[ g_{a\gamma\gamma} \approx \frac{\alpha_{em}}{2\pi f_a} \]
Axion energy deposition in the magnetosphere

\[
\frac{dE}{dt} \approx L_a \times \frac{\langle \lambda_a \rangle}{R}
\]
Axion energy deposition in the magnetosphere:

\[
\frac{dE}{dt} \approx L_\alpha \times \frac{\langle \lambda_\alpha \rangle}{R}
\]

- Resonant conversion
- Pair production

Reddy & Rrapaj (2017 in prep)
Summary

The SGRB-NS merger association has implications for nuclear and particle physics.

Combining the EM and GW signals can help identify EM precursors. Potentially sensitive to the seismology of cold neutron stars.

Seismology of superfluid and superconducting neutron stars is quite distinct.

Post merger emission can be sensitive to new particle physics. Axions in the window are especially of interest.