Constraining the Radius of Neutron Stars Through the Moment of Inertia

Neutron star mergers: From gravitational waves to nucleosynthesis
International Workshop XLV on Gross Properties of Nuclei and Nuclear Excitations
Hirschegg, Kleinwalsertal, Austria, January 15-21, 2017
Observation

Theoretical description of neutron stars

Equation of state and neutron star structure

Constraining the radius of neutron stars

Summary
• knowledge of EOS is restricted
• relationship between EOS and MR relation
• precise mass measurements are possible
• radius determination influenced by systematic uncertainties
• moment of inertia measurement seems feasible in the future
• goal: constraining radius/EOS by moment of inertia
Observation
Mass measurement

- pulsar observation $\rightarrow$ determine post–Keplerian parameters  

- post–Keplerian parameters as functions of the masses  

- intersection area yields masses for binary system

see also talk of Paulo Freire

Observation
Moment of inertia measurement

- relativistic spin-orbit (SO) coupling causes an additional contribution to advance of the periastron
- spin of PSR J0737–3039B is negligible
- contributions to advance of periastron

\[ \dot{\omega} = \dot{\omega}_{1\text{pN}} + \dot{\omega}_{2\text{pN}} + \dot{\omega}_{\text{SO}} \]

(pN: post-Newtonian)

Kramer & Wex, Class. Quant. Grav. (2009)
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- candidate for future measurement: PSR J0737–3039A
  - highly relativistic binary

Kramer & Wex, Class. Quantum Grav. (2009)
Theoretical description of neutron stars
Non-rotating neutron stars

- hydrostatic equilibrium
- Schwarzschild metric

\[ ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \]

- TOV equations

\[
\begin{align*}
\frac{dP}{dr} &= -\frac{1}{r^2} (\epsilon + P) \left( m + 4\pi r^3 P \right) \left( 1 - \frac{2m}{r} \right)^{-1} \\
\frac{dm}{dr} &= 4\pi \epsilon r^2 \\
\frac{d\nu}{dr} &= \frac{2 \left( m + 4\pi r^3 P \right)}{r^2} \left( 1 - \frac{2m}{r} \right)^{-1}
\end{align*}
\]
Theoretical description of neutron stars
Slowly rotating neutron stars

- hydrostatic equilibrium
- Schwarzschild metric $\rightarrow$ Hartle-Thorne metric \cite{Hartle:1967, HartleThorne:1968}

$$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta (d\phi - (\omega + O(\Omega^3)) \ dt)^2$$

- TOV equations remain

\[
\frac{dP}{dr} = -\frac{1}{r^2} (\epsilon + P) \left( m + 4\pi r^3 P \right) \left( 1 - \frac{2m}{r} \right)^{-1}
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\[
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\]
Theoretical description of neutron stars

Moment of inertia

Hartle, APJ (1967), Hartle & Thorne, APJ (1968)

► auxiliary function

\[ j = e^{-\frac{1}{2} \nu} \left( 1 - \frac{2m}{r} \right)^{\frac{1}{2}} \]

► in addition to TOV eqns. \((\ddot{\omega} = \Omega - \omega)\)

\[ \frac{d}{dr} \left( r^4 j \frac{d\ddot{\omega}}{dr} \right) = -4r^3 \frac{dj}{dr} \ddot{\omega} \]

► moment of inertia

\[ I = \frac{8\pi}{3} \int_0^R dr \ r^4 \ (\epsilon + P) \ j \left( 1 - \frac{2m}{r} \right)^{-1} \frac{\ddot{\omega}}{\Omega} \]
Equation of state and neutron star structure

Piecewise polytropes


see also talk of Kai Hebeler

- low density regime: knowledge of nuclear physics
  - BPS crust up to $\rho_{\text{sat}}/2$
  - chiral EFT expansion up to $\sim \rho_{\text{sat}}$
- high density regime: requirement of causality and constraints from 2.01 M$_{\odot}$ neutron stars

\[ P(\rho) = K \rho^\Gamma \]
Equation of state and neutron star structure

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Equation of state and neutron star structure

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**MR relation**


see also talk of Kai Hebeler

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- **high density regime**: requirement of causality and constraints from 2.01 $M_\odot$ neutron stars
  - polytropic expansion


\[ P(\rho) = K \rho^\Gamma \]

- **radius prediction for**
  - PSR J0737–3039A:
  - \( R \approx (9.9 - 13.6) \text{ km} \)

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Hebeler, Lattimer, Pethick, Schwenk, APJ (2013)
Constraining the radius of neutron stars
Universal relation

Several studies have investigated $I \left( M R^{-1} \right)$ (Bejger & Haensel, A&A (2002); Lattimer & Schutz, APJ (2005); Breu & Rezzolla, MNRAS (2016); ...)

→ Dimensionless moment of inertia is not sensitive to EOS

Find fit and determine radius constraints

Grey band holds for $0.07 M_\odot \text{ km}^{-1} \lesssim M R^{-1}$
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find fit and determine radius constraints

our approach: use the whole EOS uncertainty band created by polytropic expansion

grey band holds for
\[ 0.07 \, \text{M}_\odot \, \text{km}^{-1} \lesssim MR^{-1} \]
Constraining the radius of neutron stars

- use all EOS from uncertainty band
- consider different masses
- use $I(M)$ band in order to find reasonable moment of inertia values
Constraining the radius of neutron stars

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▶ assumption for PSR J0737–3039A: $I = (70 \pm 7) \, M_\odot \, \text{km}^2$

$$R \approx (9.9 - 13.6) \, \text{km}$$
Constraining the radius of neutron stars

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\[ \Downarrow \]

\[ R \approx (11.2 - 12.9) \, \text{km} \]
Summary

- derived constraints for the EOS over wide range of densities and resulting NS radii using
  - BPS crust EOS up to $\rho_{\text{sat}}/2$
  - results based on chiral EFT interactions up to $\sim \rho_{\text{sat}}$
  - causality at all densities
  - $M_{\text{max}} > 2.01 \, M_\odot$ + assumed fixed measured values for moment of inertia (including uncertainties)
- developed a framework to perform an extensive large scale sampling of all possible high-density extensions for EOS compatible with constraints
- find a reduction of radius uncertainty from moment of inertia measurements by about 50% ($\Delta I = \pm 10\%$)

In collaboration with K. Hebeler and A. Schwenk.
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Thanks for your attention!