Equation of state constraints from nuclear physics and observation

Kai Hebeler
Hirschegg, January 19, 2017

Neutron star mergers: from gravitational waves to nucleosynthesis
The nuclear landscape: New frontiers from rare isotope facilities

I. Supranuclear Density Matter

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A. Introduction

The nature of matter at such densities is one of the great uncertainties at nuclear densities is due to three-nucleon forces. Because neutron stars reach densities exceeding those in atomic nuclei, this makes them particularly sensitive to many-body forces (Akmal, Pandharipande, and Ravenhall, 1998). Measuring the neutron star mass–radius relation, which is a key test of many-body interactions at nuclear densities or the presence of deconfined quarks at high densities (Sec.I.B), is therefore of major importance.

Supernova explosions and their associated gravitational wave and gamma-ray bursts (Nakar, 2007) also depend sensitively on temperature corrections must be applied when describing either newborn neutron stars in the immediate aftermath of a supernova or the hot differentially rotating remnants that may survive for a short period of time following a neutron star and black hole binary inspiral and merger, prime laboratories for nuclear physics and quantum chromodynamics (QCD) under extreme conditions. Because neutron stars reach densities exceeding those in atomic nuclei, this makes them particularly sensitive to many-body forces (Akmal, Pandharipande, and Ravenhall, 1998).

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The nuclear landscape: New frontiers from rare isotope facilities
The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei

since 1980’s
‘Exact’ methods:
nore-core shell model, Greens function Monte Carlo
factorial scaling
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plus frameworks for few-body systems (A=3,4)
Faddeev, Faddeev-Yakubovski
The theoretical nuclear landscape:
Scope of ab initio methods for atomic nuclei

since year ~2000
closed-shell nuclei:
coupled cluster, in-medium SRG, self-consistent Greens function
polynomial scaling
The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei since year ~2010
open-shell nuclei: multi-reference IMSRG, Gorkov Greens function, Bogoliubov-coupled cluster
polynomial scaling
The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei since year ~2014
ab initio valence shell model based on nonperturbative calculation of effective interactions mixed scaling
The theoretical nuclear landscape: Scope of ab initio methods for atomic nuclei since year ~2014

- ab initio valence shell model based on nonperturbative calculation of effective interactions
- mixed scaling

Hagen et al., Nature Physics 12, 186 (2016)
Ab initio nuclear structure and reaction theory

nuclear structure and reaction observables

Quantum Chromodynamics
Lattice QCD

- requires extreme amounts of computational resources
- currently limited to 1- or 2-nucleon systems
- current accuracy insufficient for precision nuclear structure
Ab initio nuclear structure and reaction theory

nuclear structure and reaction observables

Chiral effective field theory
nuclear interactions and currents

Quantum Chromodynamics
Ab initio nuclear structure and reaction theory

nuclear structure and reaction observables

ab initio many-body frameworks
Faddeev, Quantum Monte Carlo, no-core shell model, coupled cluster ...

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Quantum Chromodynamics
Ab initio nuclear structure and reaction theory

- Nuclear structure and reaction observables
- Ab initio many-body frameworks
  - Faddeev, Quantum Monte Carlo, no-core shell model, coupled cluster ...
- Renormalization Group methods
- Chiral effective field theory
  - Nuclear interactions and currents
- Quantum Chromodynamics
“Traditional” NN interactions

- constructed to fit NN scattering data (long-wavelength information)
- long-range part dominated by one pion exchange interaction
- short range part strongly model dependent!
- traditional NN interactions contain strongly repulsive core at small distance
  - many-body problem hard to solve using basis expansion!
Systematic decoupling of high-momentum physics: The Similarity Renormalization Group

- generate unitary transformation which **decouples** low- and high momenta:
  \[ H_\lambda = U_\lambda H U_\lambda^\dagger \]
  with the resolution parameter \( \lambda \)

- change resolution systematically in small steps:
  \[ \frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda] \]

- generator \( \eta_\lambda \) can be chosen and **tailored** to different applications

- observables are **preserved** due to unitarity of transformation
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Diagram showing a contour plot with axes labeled `k (fm⁻¹)` and `k' (fm⁻¹)`.
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Resolution \( \lambda \)
Systematic decoupling of high-momentum physics: 
The Similarity Renormalization Group

\[ \overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r, r') \]

\[
\begin{align*}
\lambda &= 20 \text{ fm}^{-1} \\
\lambda &= 4 \text{ fm}^{-1} \\
\lambda &= 3 \text{ fm}^{-1} \\
\lambda &= 2 \text{ fm}^{-1} \\
\lambda &= 1.5 \text{ fm}^{-1}
\end{align*}
\]

Resolution \( \lambda \)
Nuclear effective degrees of freedom

- if a nucleus is probed at high energies, nucleon substructure is resolved
- at low energies, details are not resolved
Nuclear effective degrees of freedom

- if a nucleus is probed at high energies, nucleon substructure is resolved
- at low energies, details are not resolved
- replace fine structure by something simpler (compare multipole expansion)

Resolution

effective field theory
Chiral effective field theory for nuclear forces

- choose relevant degrees of freedom: here nucleons and pions
- operators constrained by symmetries of QCD
- short-range physics captured in short-range couplings
- separation of scales: $Q \ll \Lambda_b$, breakdown scale $\Lambda_b \sim 500$ MeV
- power-counting: expand in $Q/\Lambda_b$
- systematic, obtain error estimates
- many-body forces appear naturally
Aren’t 3N forces unnatural? Do we really need them?

Consider classical analog: tidal effects in earth-sun-moon system

- force between earth and moon depends on the position of sun
- tidal deformations represent internal excitations
- describe system using point particles → 3N forces inevitable!

- nucleons are composite particles, can also be excited
- change of resolution change excitations that can be described explicitly
  - existence of three-nucleon forces natural
  - crucial question: how important are their contributions?
### Many-body forces in chiral EFT

<table>
<thead>
<tr>
<th></th>
<th>NN</th>
<th>3N</th>
<th>4N</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO $\mathcal{O}(\frac{Q^0}{\Lambda^0})$</td>
<td>X</td>
<td>H</td>
<td>-</td>
</tr>
<tr>
<td>NLO $\mathcal{O}(\frac{Q^2}{\Lambda^2})$</td>
<td>X</td>
<td>X</td>
<td>-</td>
</tr>
<tr>
<td>$N^2$LO $\mathcal{O}(\frac{Q^3}{\Lambda^3})$</td>
<td>X</td>
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</tr>
<tr>
<td>$N^3$LO $\mathcal{O}(\frac{Q^4}{\Lambda^4})$</td>
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</table>

The table represents the fitting of many-body forces in chiral EFT to different systems:

- **NN**: Including short-range terms ($c_1, c_3, c_4$ terms)
- **3N**: Including intermediate-range terms ($c_D$ term)
- **4N**: Including long-range terms ($c_E$ term)

**Diagram Notes:**
- The diagram shows the progression of many-body forces from low to next-to-low to next-to-next-to-low orders.
- The colors and symbols indicate the range of forces.
- The years 1994, 2006, and 2011 mark significant advancements in the fitting procedures.

**Additional Text:**
- Need to be fit to three-body and/or higher-body systems.
Many-body forces in chiral EFT

values of LECs $c_1, c_3, c_4$ still contain large uncertainties!

first incorporation in calculations of neutron and nuclear matter

Tews, Krüger, KH, Schwenk, PRL 110, 032504 (2013)
Krüger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

all terms predicted
(no new low-energy couplings)
Many-body forces in chiral EFT

First incorporation in calculations of neutron and nuclear matter
Tews, Krüger, KH, Schwenk, PRL 110, 032504 (2013)
Krüger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

First calculation of matrix elements for ab initio studies of matter and nuclei
KH, Krebs, Epelbaum, Golak, Skibinski, PRC 91, 044001 (2015)
Equation of state of symmetric nuclear matter: nuclear saturation

\[ V_c(r) \text{[MeV]} \]

\[ r \text{[fm]} \]

\[ 1S_0 \text{ channel} \]

repulsive core

\[ 2\pi, \rho, \omega, \sigma \]

\[ \bar{\rho}_S \]

Bonn

Reid93

AV18

Batty et. al,
Karlsruhe (1987)
Equation of state of symmetric nuclear matter: nucleon
Equation of state of symmetric nuclear matter:

nuclear saturation

\[
\begin{align*}
V_c(r) \text{ [MeV]} & \quad \lambda = 20 \text{ fm}^{-1} \\
V(r) \text{ [MeV]} & \quad \lambda = 1.5 \text{ fm}^{-1}
\end{align*}
\]

repulsive core

Bonni
Reid93
AV18

\[\bar{l}_S\]

\[r \text{ [fm]}\]

\[\text{Energy/nucleon [MeV]}\]

\[k_F \text{ [fm}^{-1}]\]

\[0.8 \quad 1.0 \quad 1.2 \quad 1.4 \quad 1.6\]

\[0 \quad -10 \quad -20 \quad -30\]

\[\text{NN only}\]

3NF fit to \(E_{3H}\) and \(r_{4He}\) \(\Lambda_{3NF} = 2.0 \text{ fm}^{-1}\)

\[\text{NN + 3N}\]

3rd order pp+hh

\[c_D\text{ term}\]

\[c_E\text{ term}\]

\[c_1, c_3, c_4 \text{ terms}\]

\[c_D\text{ term}\]

\[c_E\text{ term}\]

intermediate \((c_D)\) and short-range \((c_E)\) 3NF couplings fitted to few-body systems at different resolution scales:

\[E_{3H} = -8.482 \text{ MeV} \quad r_{4He} = 1.464 \text{ fm}\]

KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)
Equation of state of symmetric nuclear matter: nuclear saturation

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\[ E_{3H} = -8.482 \text{ MeV} \quad r_{4\text{He}} = 1.464 \text{ fm} \]
Studies of neutron-rich nuclei: Neutron dripline and the oxygen anomaly

- remarkable agreement between different many-body frameworks
- very good agreement between theory and experiment for masses of oxygen and calcium isotopes based on specific chiral interactions
- contributions from 3N force play important role for drip line

Gallant et al.
PRL 109, 032506 (2012)

Wienholtz et al.
Nature 498, 346 (2013)
Ab initio calculations of heavier nuclei

**Coupled cluster (CC) framework**

Ab initio calculations of heavier nuclei

- Spectacular increase in range of applicability of ab initio many body frameworks
- Significant discrepancies to experimental data for heavy nuclei for (most of) presently used nuclear interactions
- Need to quantify theoretical uncertainties
The size of the atomic nucleus: challenges from novel high-precision measurements

The size of the atomic nucleus: challenges from novel high-precision measurements

The size of the atomic nucleus: challenges from novel high-precision measurements

Piekarewicz,
PRC 85, 041302 (2012)

PREX
Pb Radius Experiment

CREX
Ca Radius Experiment


Garcia Ruiz. et al.,
Nature Phys. 12, 594 (2016)
The size of the atomic nucleus: challenges from novel high-precision measurements


direct connections to astrophysics!
Microscopic calculations of the equation of state

- microscopic framework to calculate equation of state for general proton fractions
- uncertainty bands determined by set of 7 Hamiltonians
- many-body framework allows treatment of general 3N interaction

\[ x = \frac{n_p}{n_p + n_n} \]
Microscopic calculations of the equation of state

- microscopic framework to calculate equation of state for general proton fractions
- uncertainty bands determined by set of 7 Hamiltonians

Problem:
Calculation of neutron star properties require EOS up to high densities. Reliable calculations only possible up to $\sim 1-2 n_{sat}$.

Strategy:
Use observations to constrain the high-density part of the nuclear EOS.
Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter → neutron star matter

parametrize our ignorance via piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$ (results insensitive to particular form)
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)
Constraints on the nuclear equation of state

use the constraints:

recent NS observations

\[ M_{\text{max}} > 1.97 \, M_{\odot} \]

causality

\[ v_s(\rho) = \sqrt{dP/d\varepsilon} < c \]


constraints lead to significant reduction of EOS uncertainty band
• low-density part of EOS sets scale for allowed high-density extensions

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

→ see talk by Svenja Greif
Recent and current developments of novel nuclear interactions

1. local EFT interactions, suitable for Quantum Monte Carlo calculations

status: NN plus 3N up to N2LO, calculations of few-body systems and neutron matter

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- Gezerlis et al., PRL 111, 032501 (2013)
- Gezerlis et al., PRC 90, 054323 (2014)
- Lynn et al., PRL 116, 062501 (2016)

First Quantum Monte Carlo of neutron matter based on chiral EFT interactions

**perfect agreement** for soft interactions, first direct validation of calculations within many-body perturbation theory

Gezerlis et al., PRL 111, 032501 (2013)
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   Gezerlis et al., PRL 111, 032501 (2013)
   Gezerlis et al., PRC 90, 054323 (2014)
   Lynn et al., PRL 116, 062501 (2016)

2. simultaneous fit of NN and 3N forces to two- and few-body observables
   **status:** NN plus 3N up to N2LO, N3LO currently in development

   Carlsson et al., PRX 6, 011019 (2016)
Recent and current developments of novel nuclear interactions

3. Fits of NN plus 3N forces to two-, few- and many-body observables

**Status:** NN plus 3N up to N2LO, NN phase shifts only fitted up to $T_{\text{lab}} \sim 35$ MeV

Ekström et al., PRC91, 051301 (2015)
Recent and current developments of novel nuclear interactions

3. fits of NN plus 3N forces to two-, few- and many-body observables

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4. semilocal NN forces, development of improved method to estimate uncertainties

**status:** NN up to N4LO, 3N interactions in development (almost finished :-))

Epelbaum, Krebs, Meißner, PRL 115, 122301 (2015)

Binder et al., PRC 93, 044002 (2016)
Recent and current developments of novel nuclear interactions

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**Epelbaum, Krebs, Meißner, PRL 115, 122301 (2015)**

**Binder et al., PRC 93, 044002 (2016)**

**KH et al., PRC 91, 044001 (2015)**
Status and achievements

- significant increase in scope of ab initio many-body frameworks
- remarkable agreement between different ab initio many-body methods
- discrepancies to experiment dominated by deficiencies of present nuclear interactions

Current developments and open questions

- presently active efforts to develop improved nucleon interactions (fits of LECs, power counting, regularization...)

Key goals

- unified study of atomic nuclei, nuclear matter and reactions based on novel interactions
- systematic estimates of theoretical uncertainties