UNDERSTANDING THE NUCLEAR EQUATION OF STATE FROM GRAVITATIONAL WAVE OBSERVATIONS

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Penn State and Cardiff University

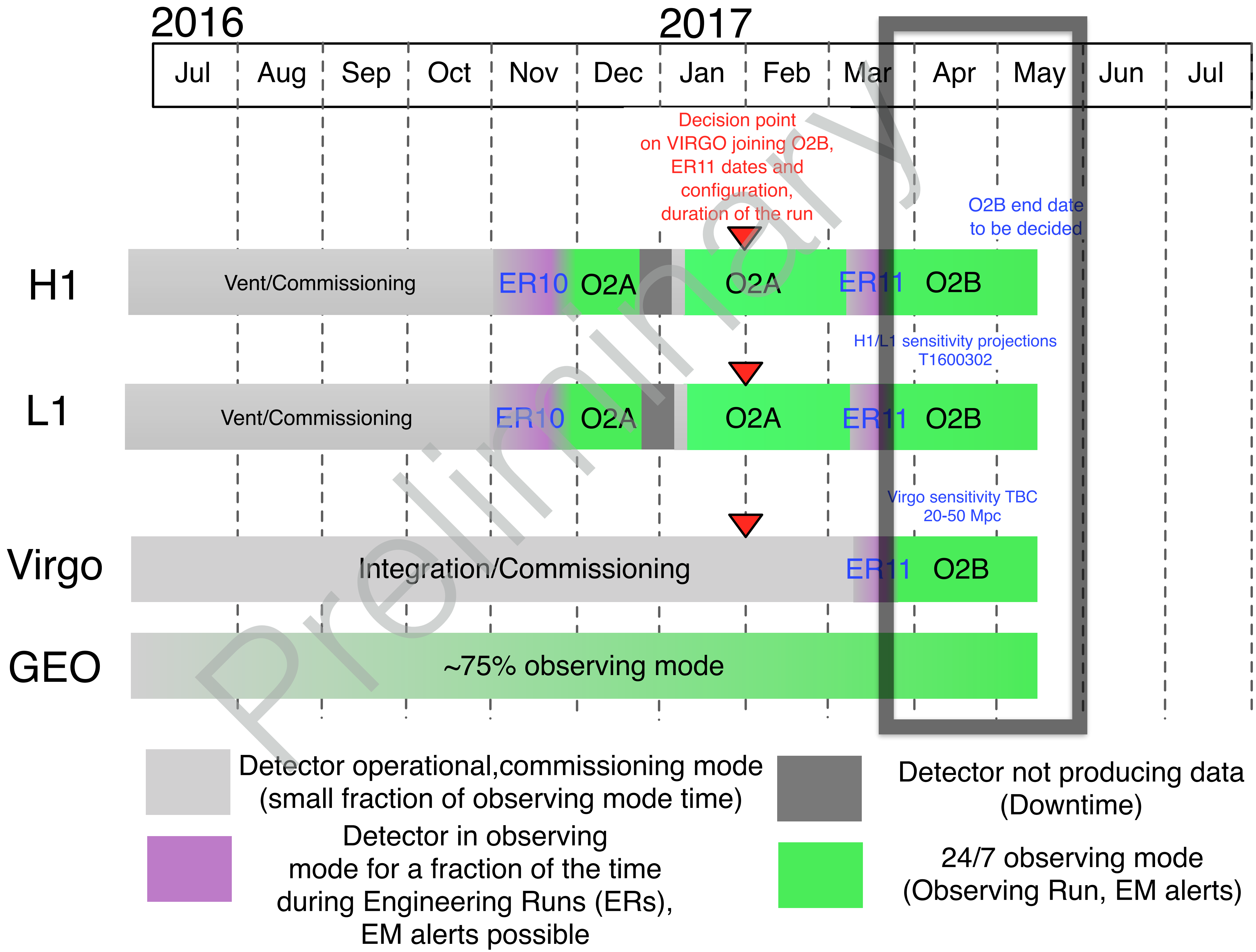
Hirschegg, Neutron star mergers: From gravitational waves to nucleosynthesis, January 16-20, 2017

GW DISCOVERY - TRULY COLLABORATIVE WORK

- The discovery is the work of ~1000 scientists and engineers across the globe
  - 3 different collaborations: LIGO, GEO600, Virgo
  - 15 countries, 80 institutions
  - 100's of graduate students and postdocs
- It is an engineering marvel, as much as it is a scientific discovery
- Many thanks to hundreds of colleagues from the three collaborations who have made this possible

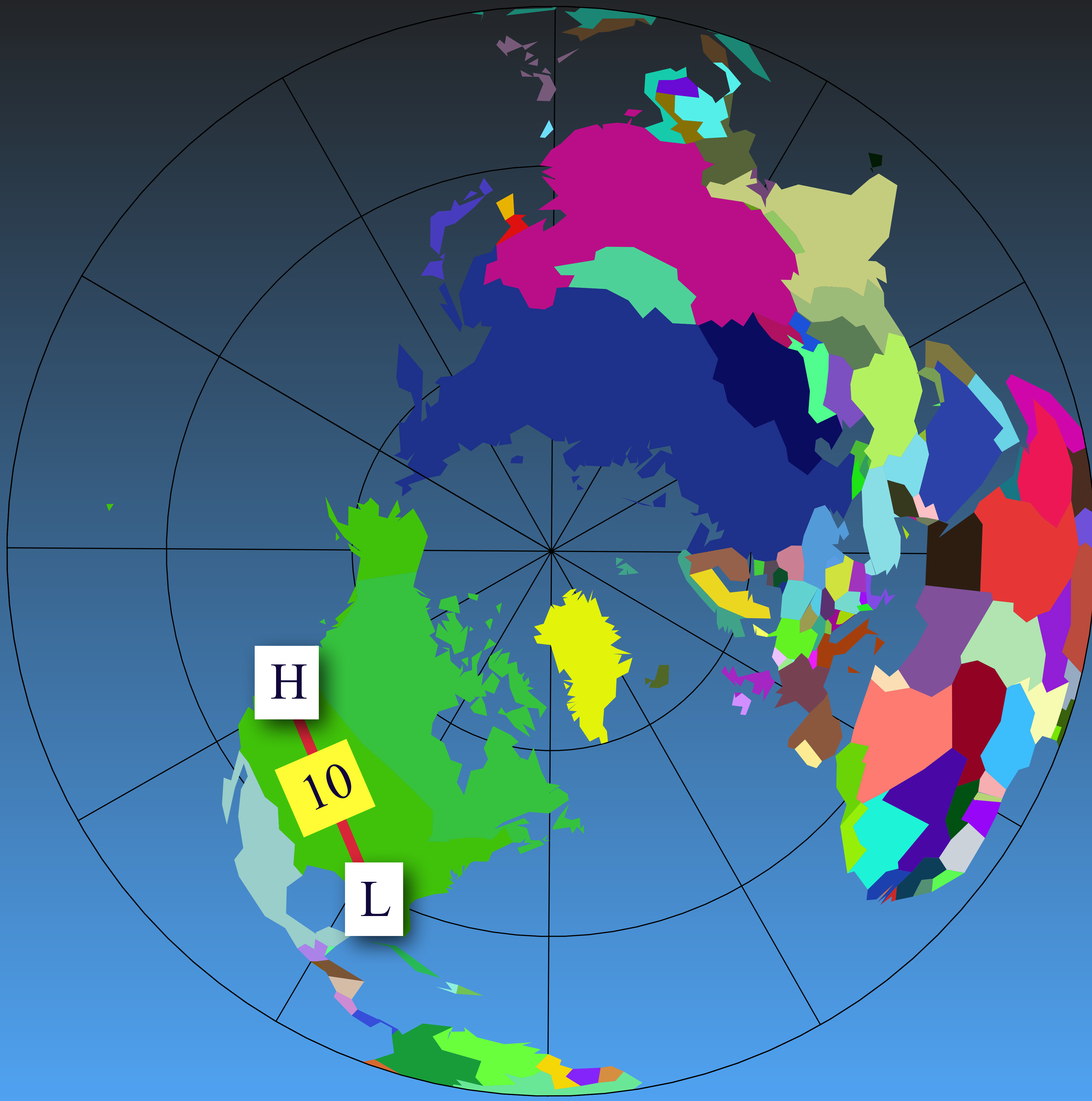
Joint Run Planning Committee

Working schedule for O2 (G1501561- v10)



DETECTORS

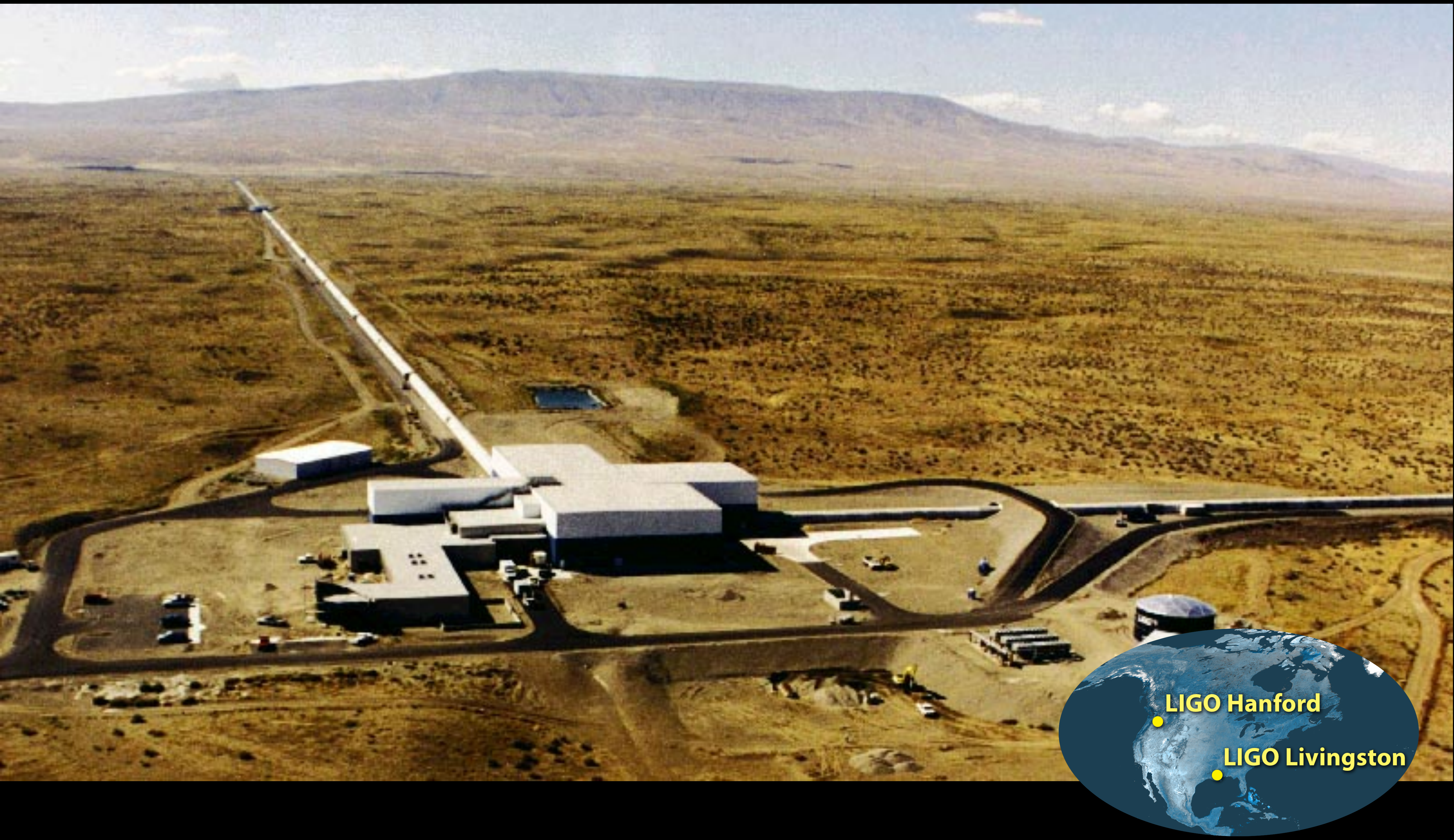
GW Detector Networks 2015+



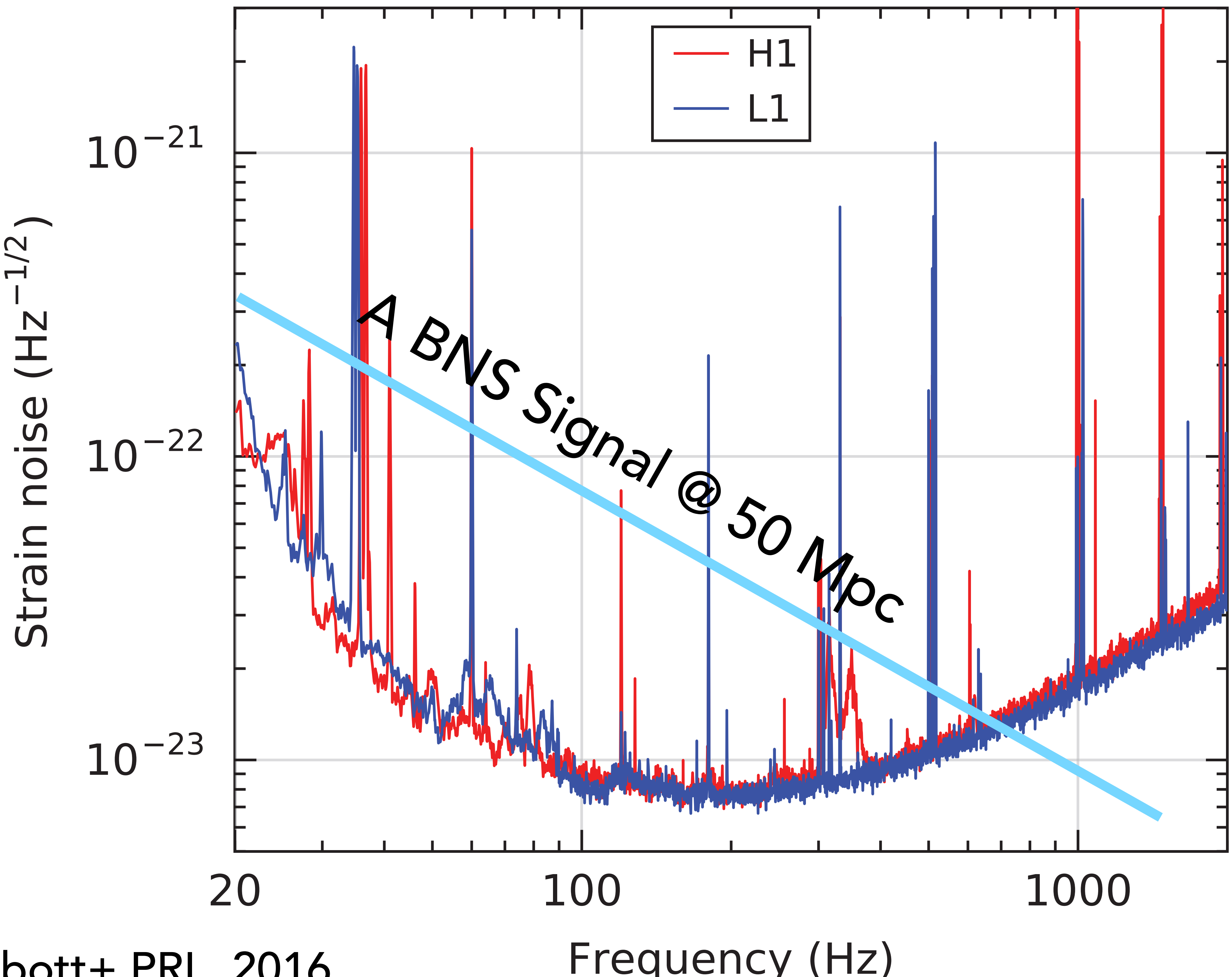
BASELINES IN LIGHT TRAVEL TIME (MS)

LIGO-LIVINGSTON

LIGO-HANFORD



LIGO SENSITIVITY DURING FIRST OBSERVING RUN (O1)



Abbott+ PRL, 2016

BINARY BLACK HOLES (BBH)

PROGRESS IN TWO-BODY PROBLEM

- Caltech group pointed out the importance of computing phasing beyond leading order, followed by very impressive progress in post-Newtonian computation of two-body dynamics
- construction of LIGO, Virgo, GEO600 and TAMA brought theory and observations closer
- effective one-body approach developed: bold prediction for the late inspiral, merger and ringdown
- first successful NR simulations broke conventional wisdom - a far simpler merger than anyone predicted
- remarkable interactions between GW data analysts, astrophysicists and theorists to open a new observational window

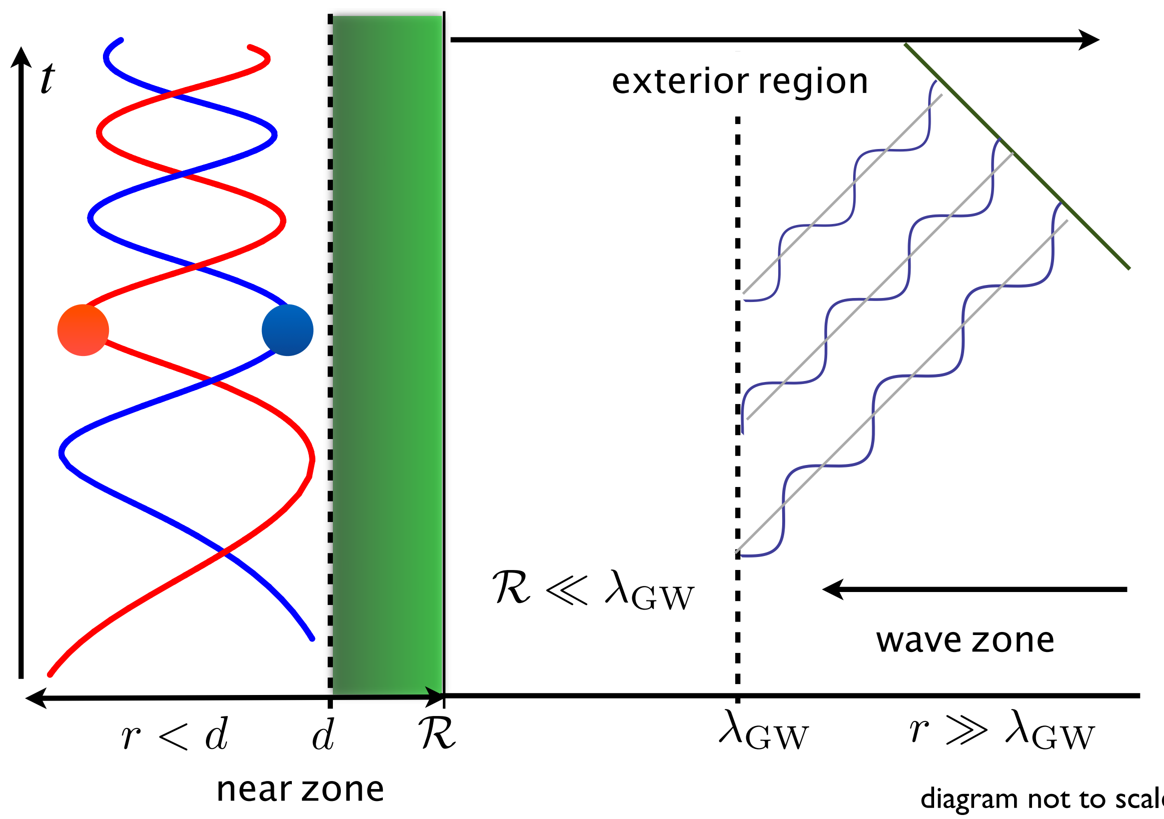
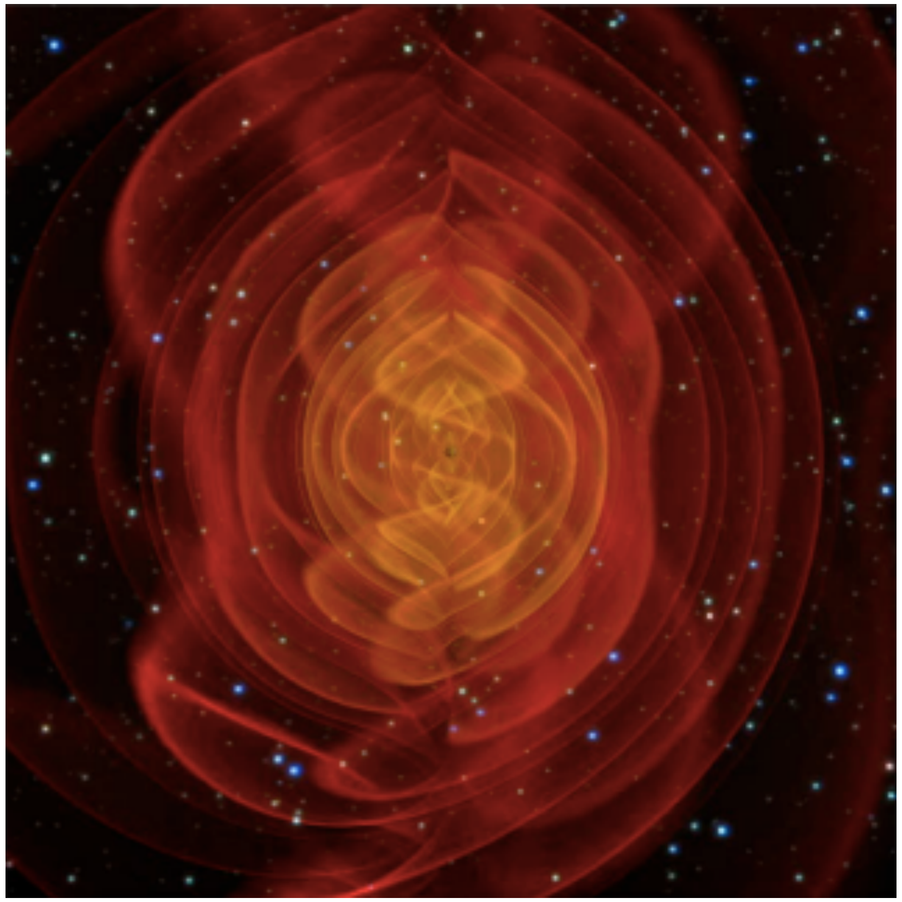
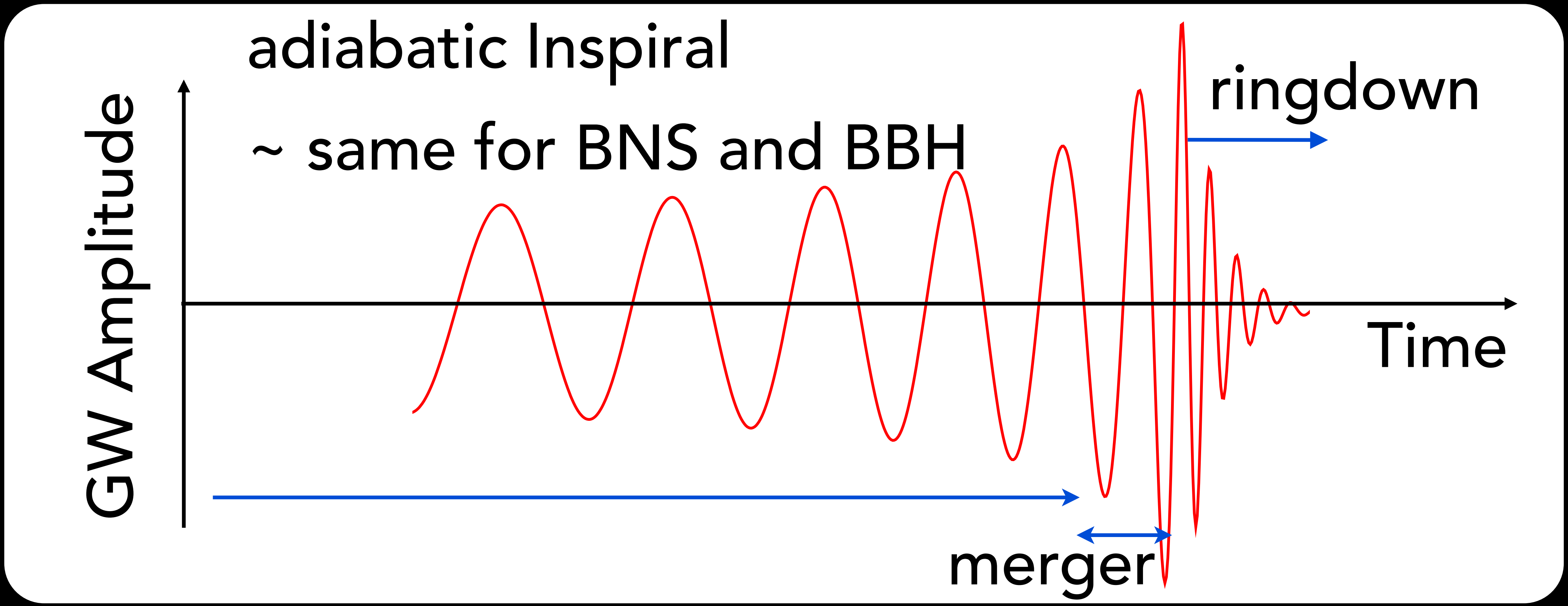


image curtsey NASA/C. Henze

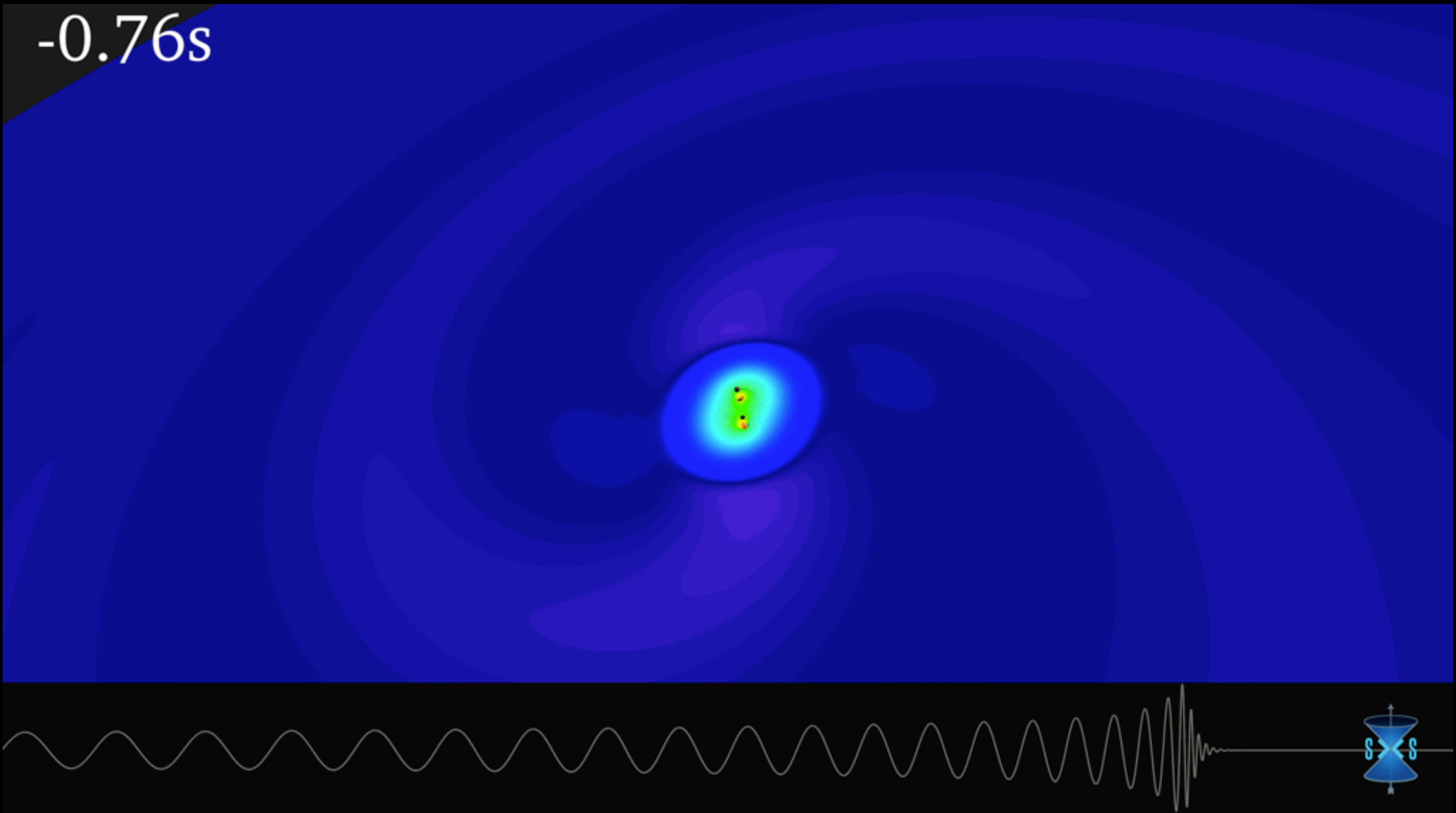


BINARY BLACK HOLE WAVEFORMS

- waveform characterized by
  - slow adiabatic inspiral, fast and luminous merger, rapid ringdown
- very large parameter space
  - mass ratio, large BH spins misaligned with orbit, eccentricity
- waveform shape can tell us about component masses, spins and eccentricity
- waveform amplitude (in a detector network) can tell us about source's orientation, sky position, polarization and distance



NUMERICAL SIMULATION OF BINARY BLACK HOLE MERGER: SXS COLLABORATION



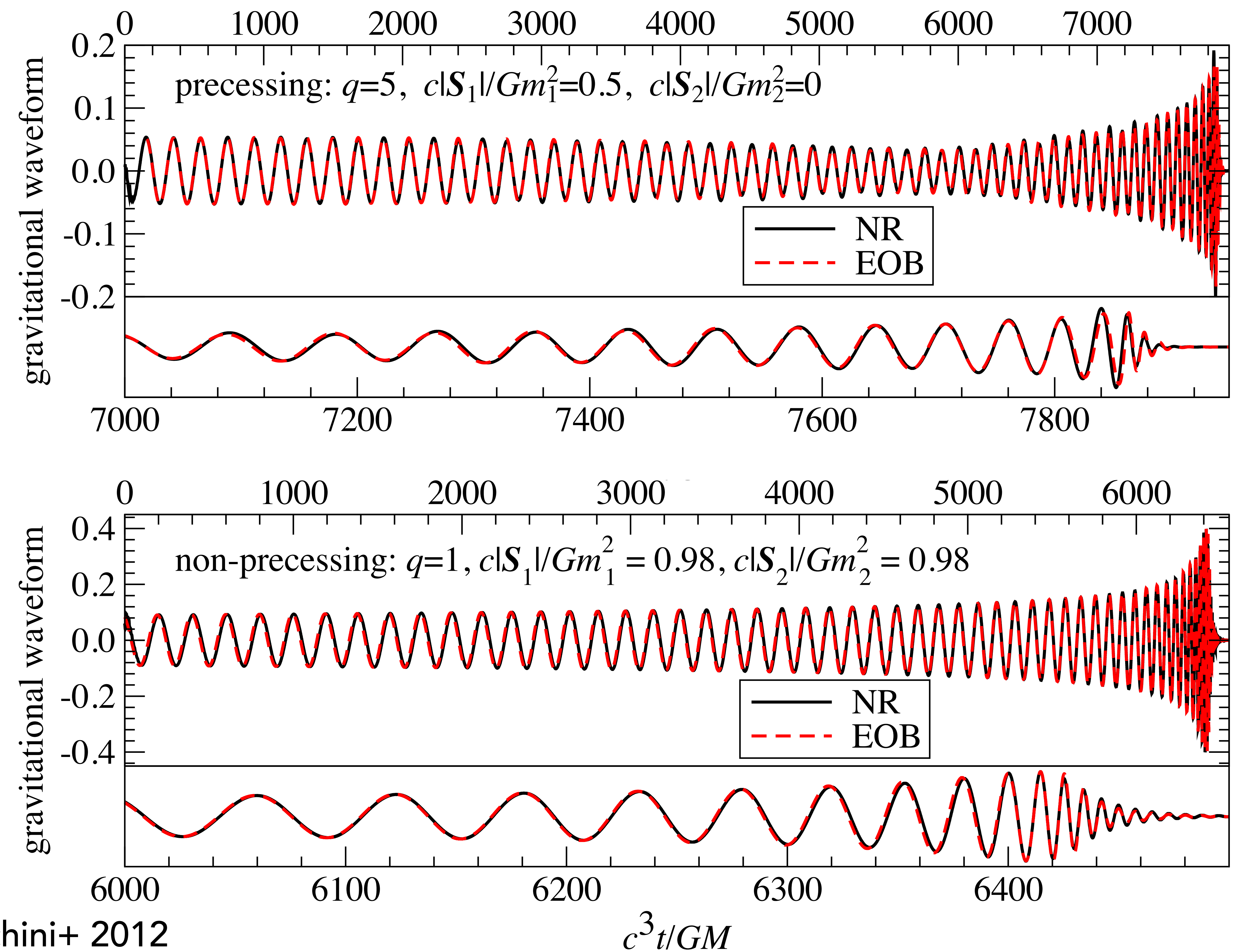
CURRENT STATUS OF PN CALCULATIONS

0.5 PN=v/c

	No Spin	Spin-Linear	Spin-Squared	Tidal
Conservative Dynamics	4PN	3.5PN	3PN	7PN
Energy Flux at Infinity	3.5PN	4PN	2PN	6PN
RR Force	4.5PN	4PN	4.5PN	6PN
Waveform Phase	3.5PN	4PN	2PN	6PN
Waveform Amplitude	3PN	2PN	2PN	6PN
BH Horizon Energy Flux	5PN	3.5PN	4PN	–

Table from Buonanno and BSS 2014

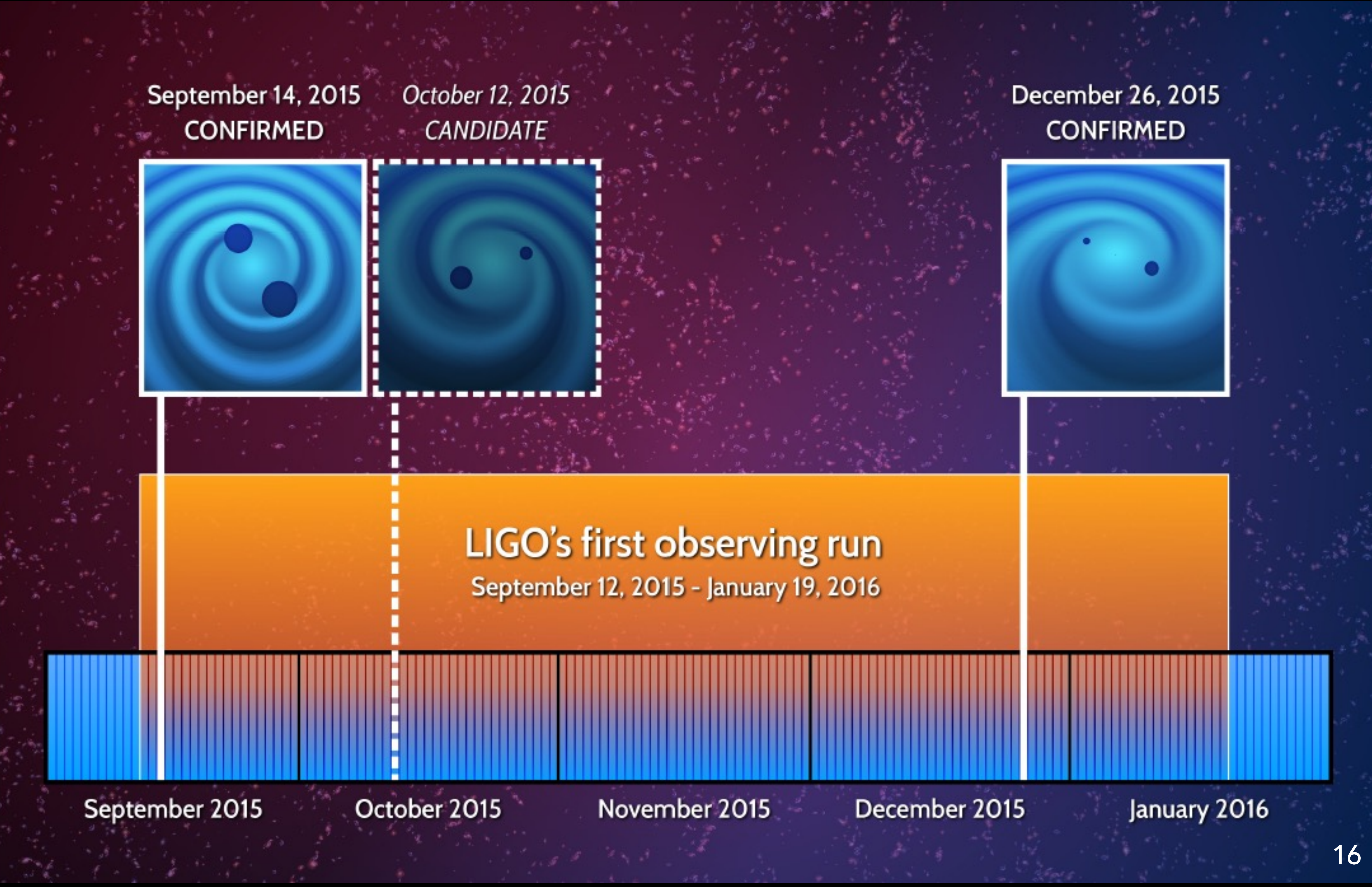
EOB VIS-A-VIS NR SIMULATIONS



Taracchini+ 2012

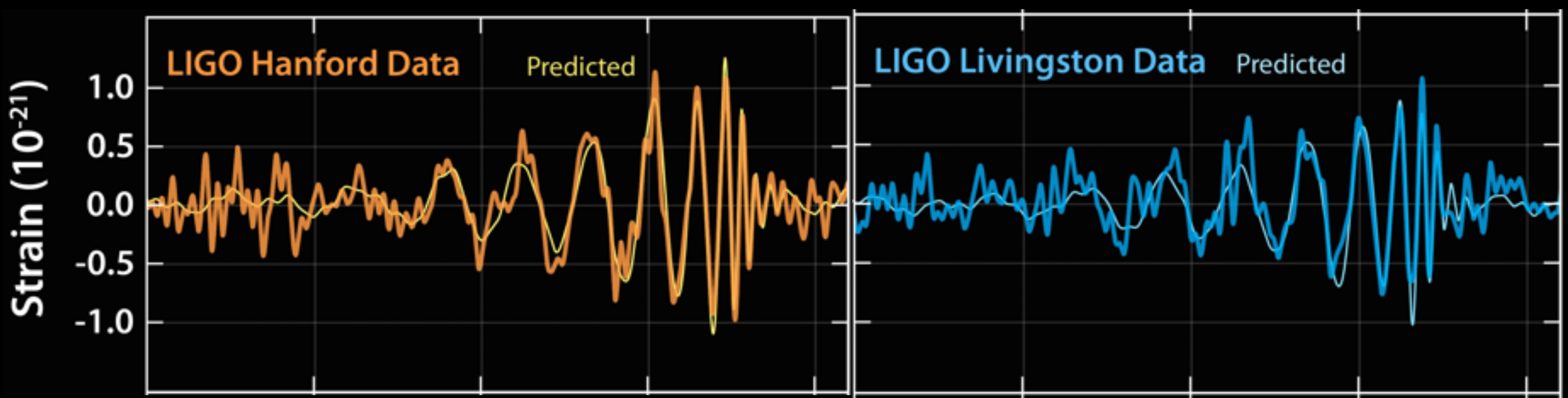
DISCOVERY

LIGO DETECTIONS SO FAR

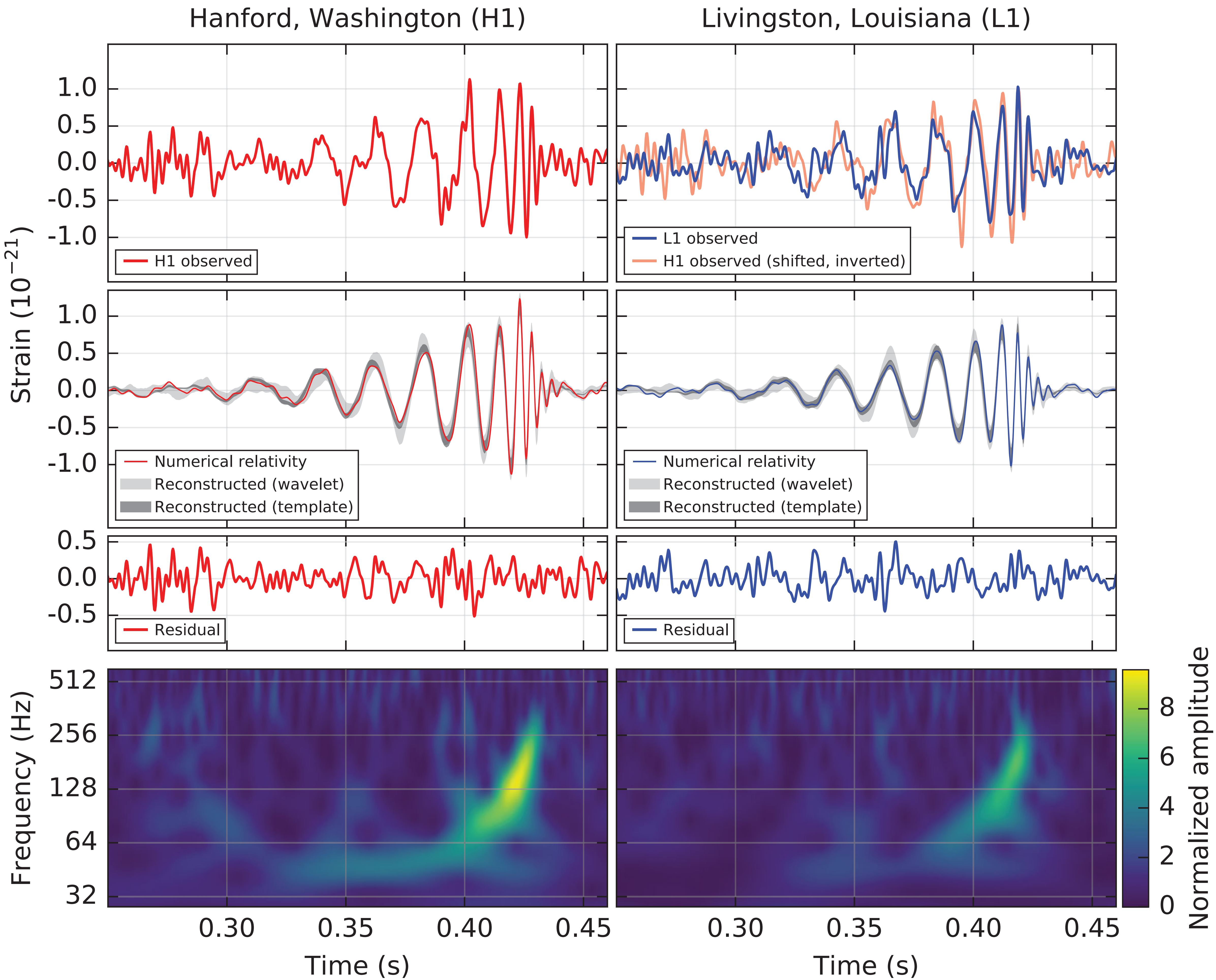


GW150914

Abbott+ PRL, 2016



Signal can be identified and reconstructed without the knowledge of the waveform, but waveform models are essential for interpretation

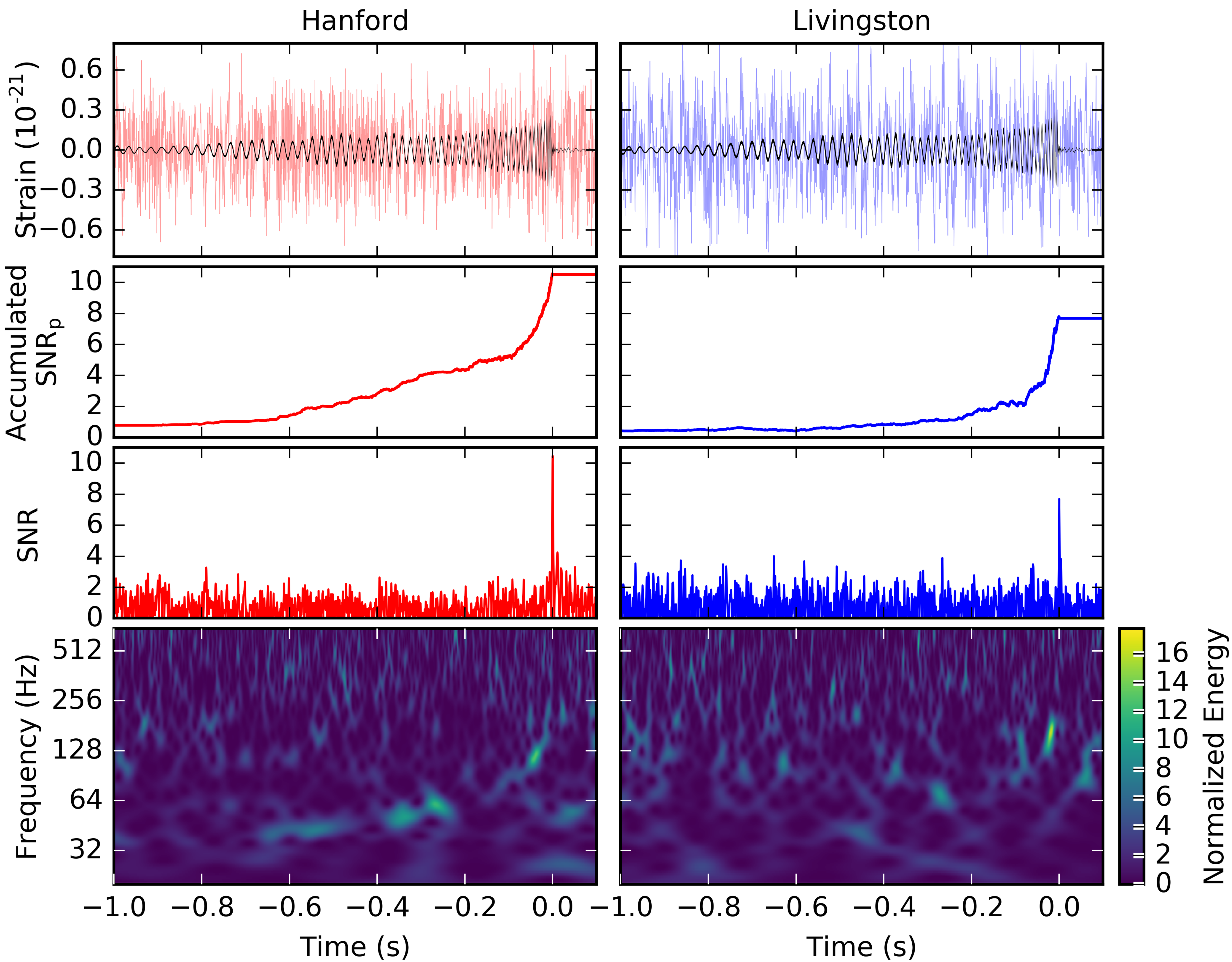


Abbott+ PRL, 2016

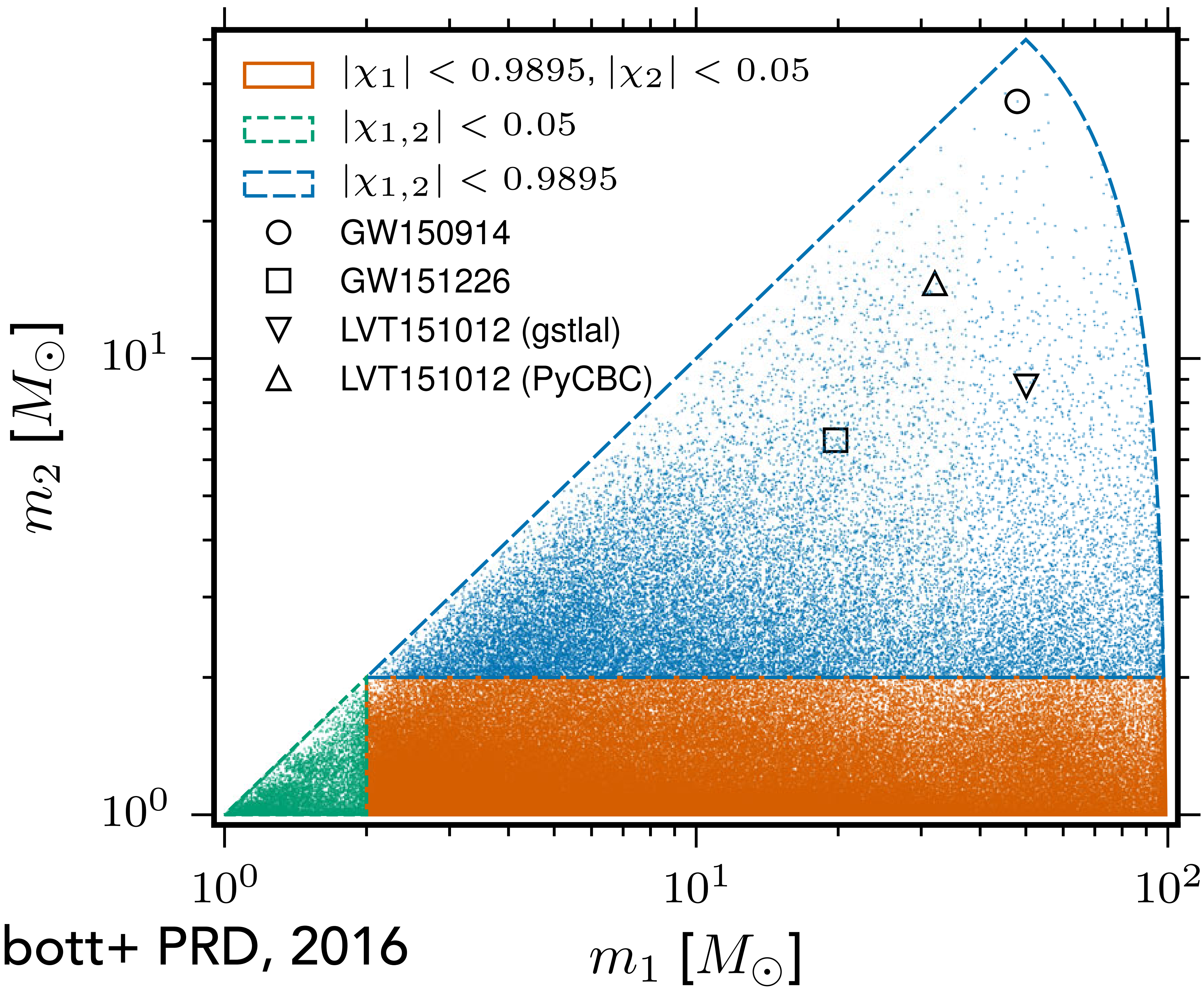
Matched filtering and waveform models were critical for detection and signal reconstruction

GW151226

Abbott+ PRL, 2016



TEMPLATE BANK USED IN O1 SEARCH FOR COMPACT BINARY COALESCENCES



Abbott+ PRD, 2016

PARAMETER ESTIMATION

PROBLEM OF PARAMETER ESTIMATION

- Bayesian analysis is used to infer the posterior probability density of parameters $\mu = \{\mu_1, \mu_2, \ldots, \mu_n\}$ given the data x:

$$P(\mu|x) = \frac{P(x|\mu)P(\mu)}{P(x)}$$

- in the case of binary black holes, signal parameters are masses (m_1, m_2), spins $(\mathbf{S}_1, \mathbf{S}_2)$, eccentricity (e), sky position $(\boldsymbol{\theta}, \boldsymbol{\varphi})$, distance (D), binary orientation angles $(\boldsymbol{\iota}, \boldsymbol{\delta})$, time of and phase at coalescence $(t_c, \boldsymbol{\varphi}_c)$

- the choice of prior could significantly influence the posterior when the likelihood is small (and there is no such thing as "uniform" or "uninformative" prior as this is a parameter-dependent statement; if the likelihood is large (or if we have a large number of observations) then prior doesn't matter

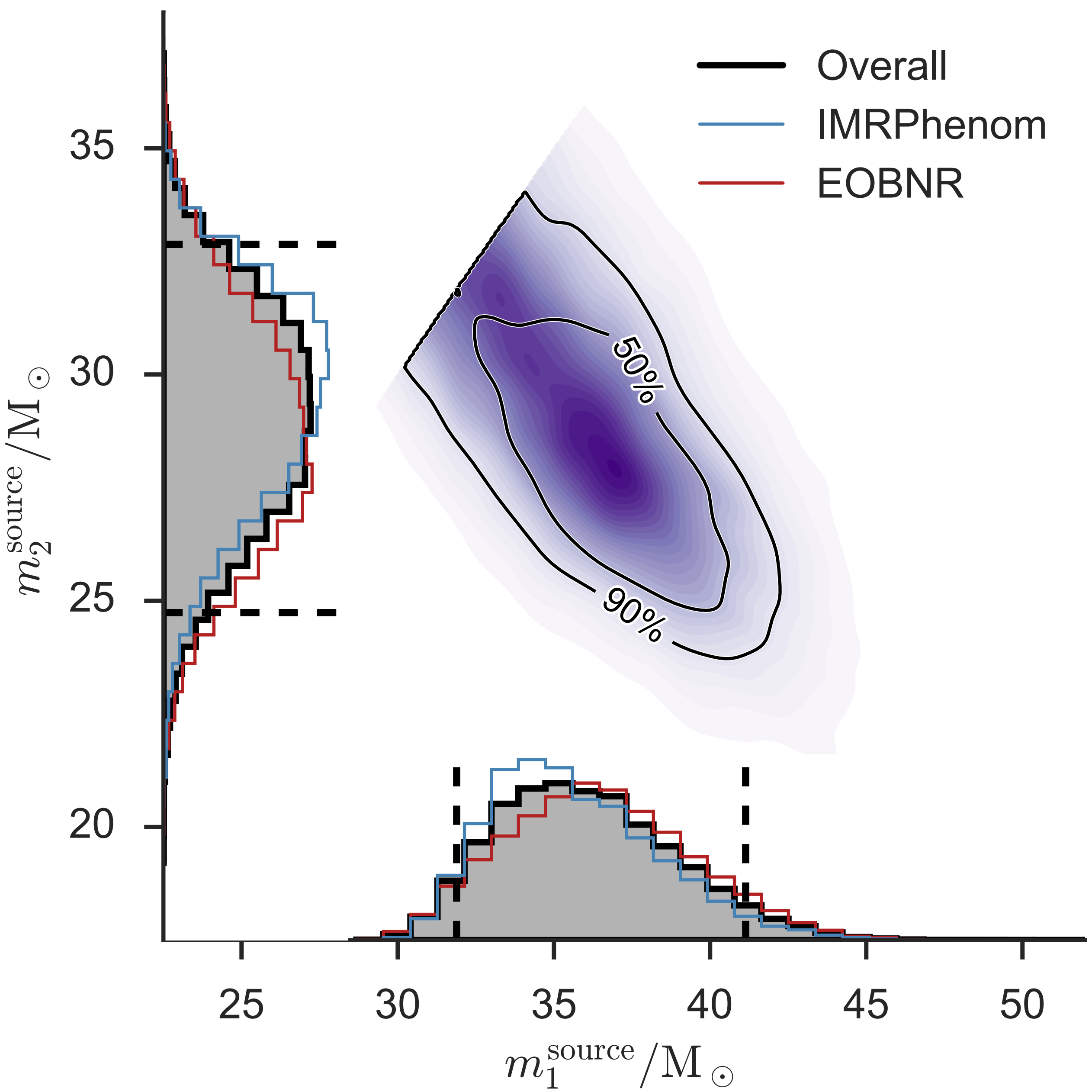
ONE-D AND TWO-D DISTRIBUTIONS

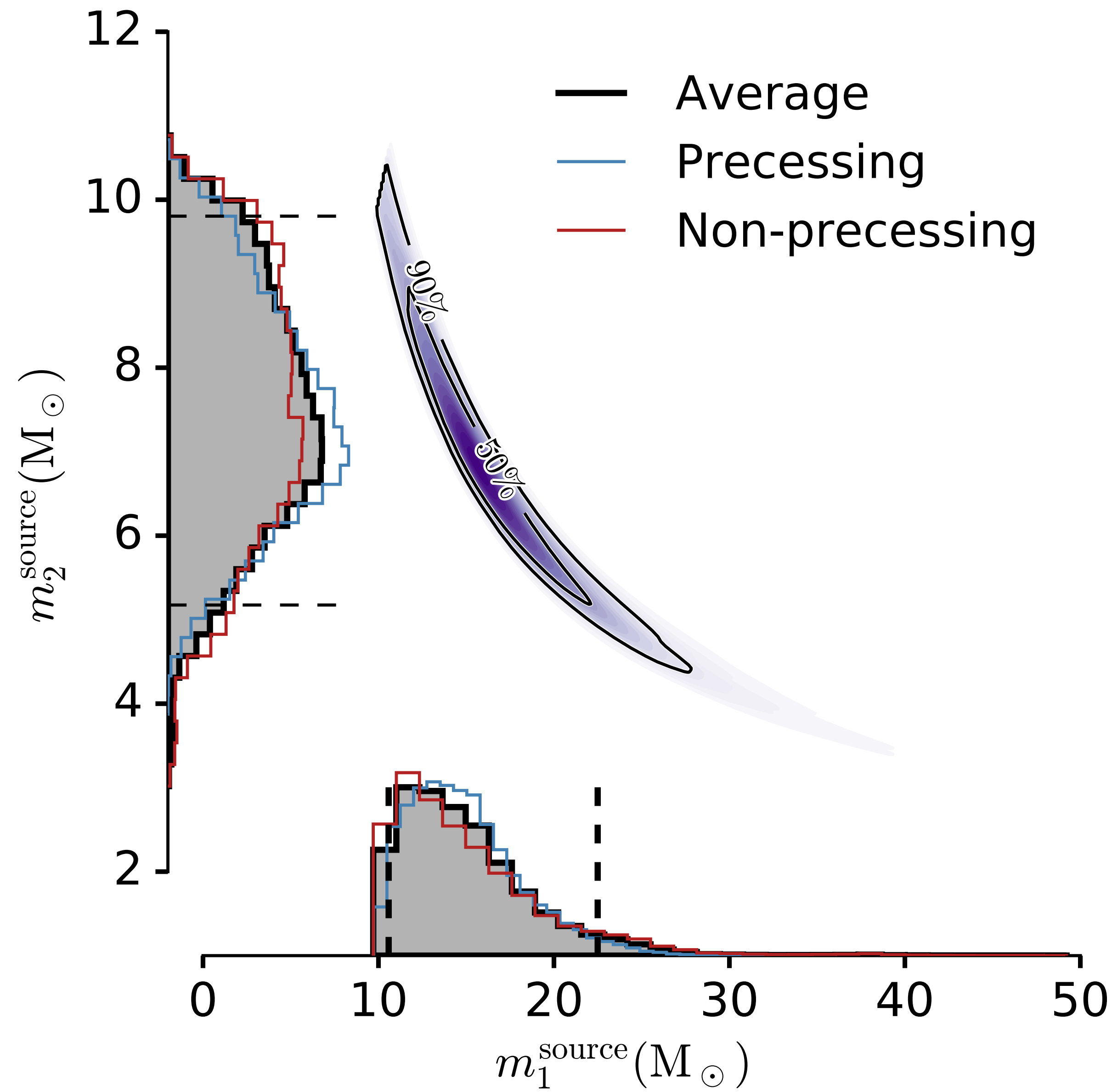
- one can integrate the multivariate posterior distributions to obtain one-d and two-d distributions to compute mean, median, mode, confidence interval, etc.

$$P(\mu_1) = \int P(\mu|x)\,\mathrm{d}\mu_2\,\mathrm{d}\mu_3, \ldots$$

$$P(\mu_1, \mu_2) = \int P(\mu|x)\,\mathrm{d}\mu_3\,\mathrm{d}\mu_4, \ldots$$

MASSES OF GW150914 AND GW151226 FROM BAYESIAN ESTIMATION

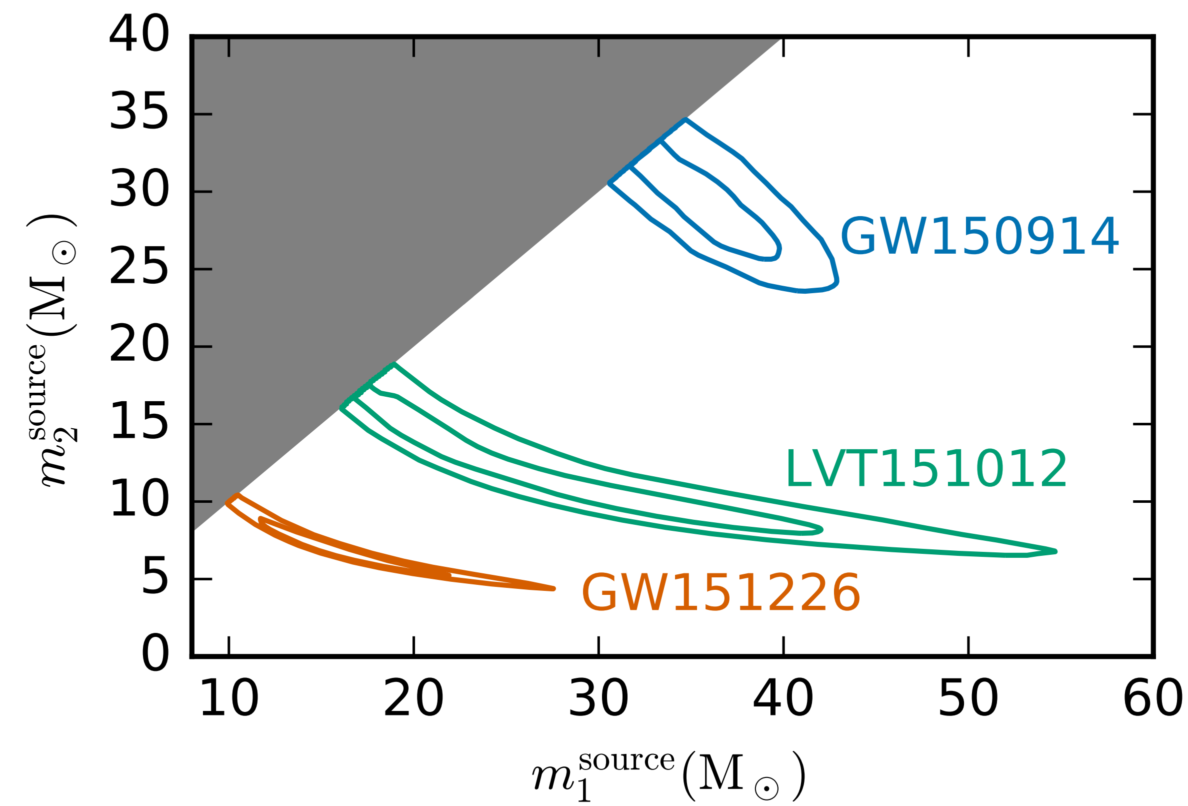


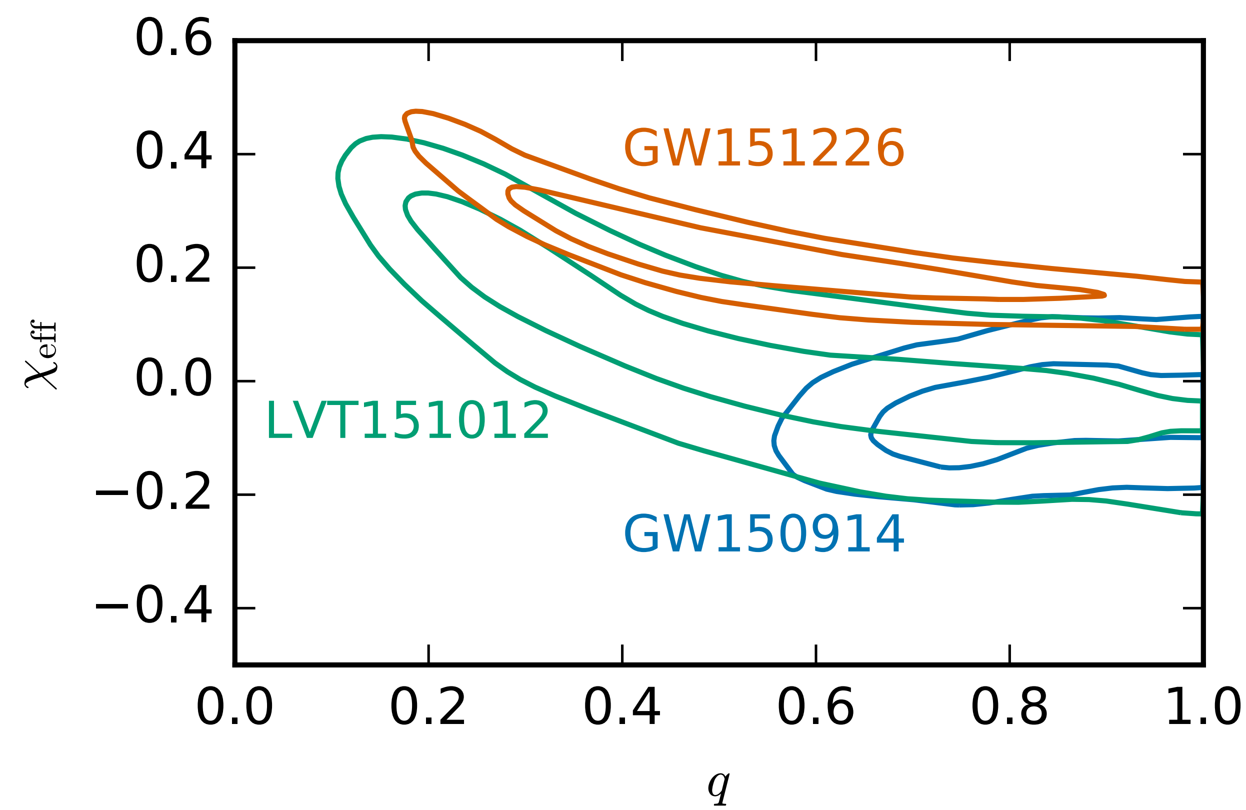


MASSES AND SPINS: ALL EVENTS

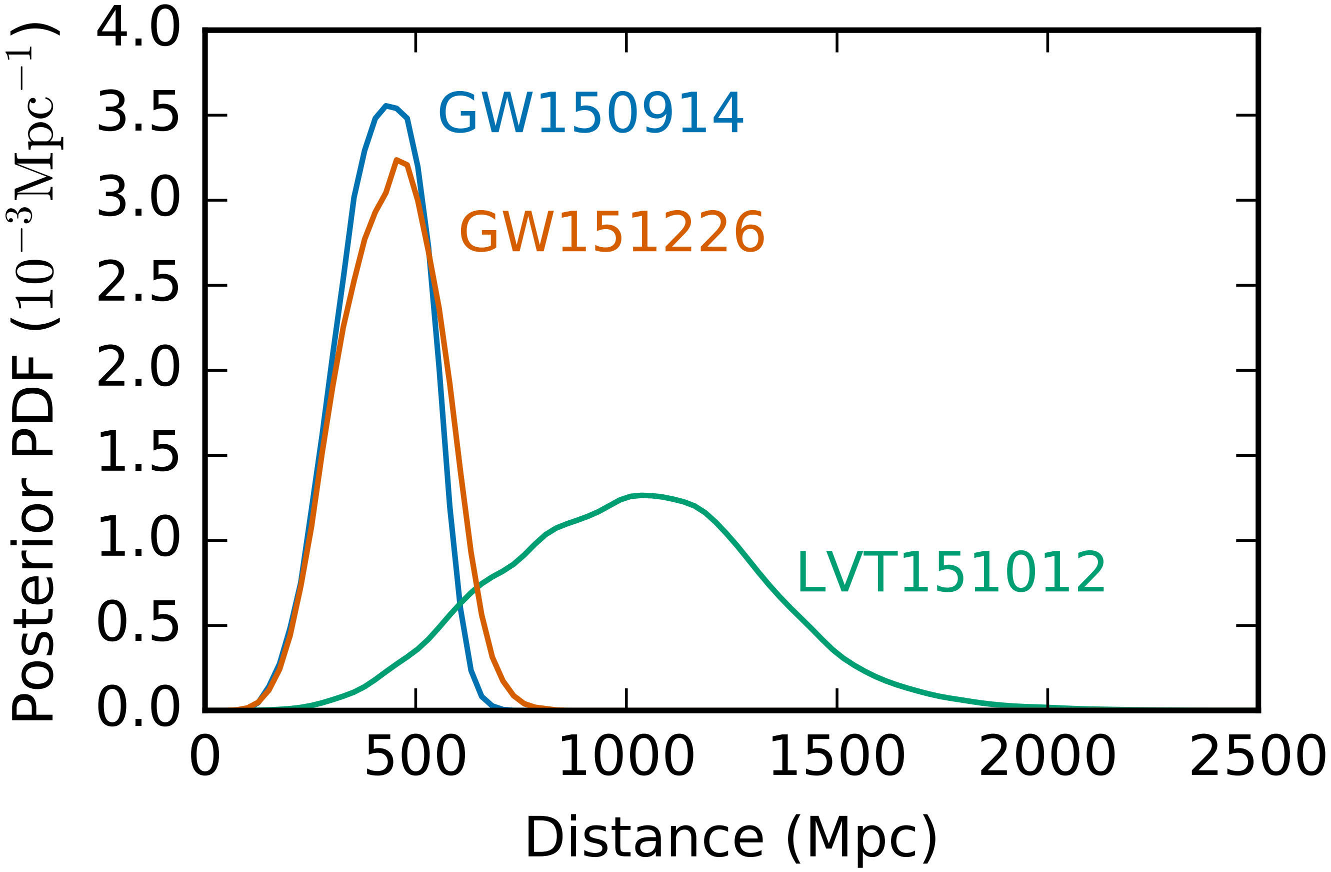
- GW150914
  - (36, 29) solar mass
- GW151226
  - (14, 8) solar mass
  - one of the BHs has nonzero spin
- LVT151012
  - (23, 13) solar mass

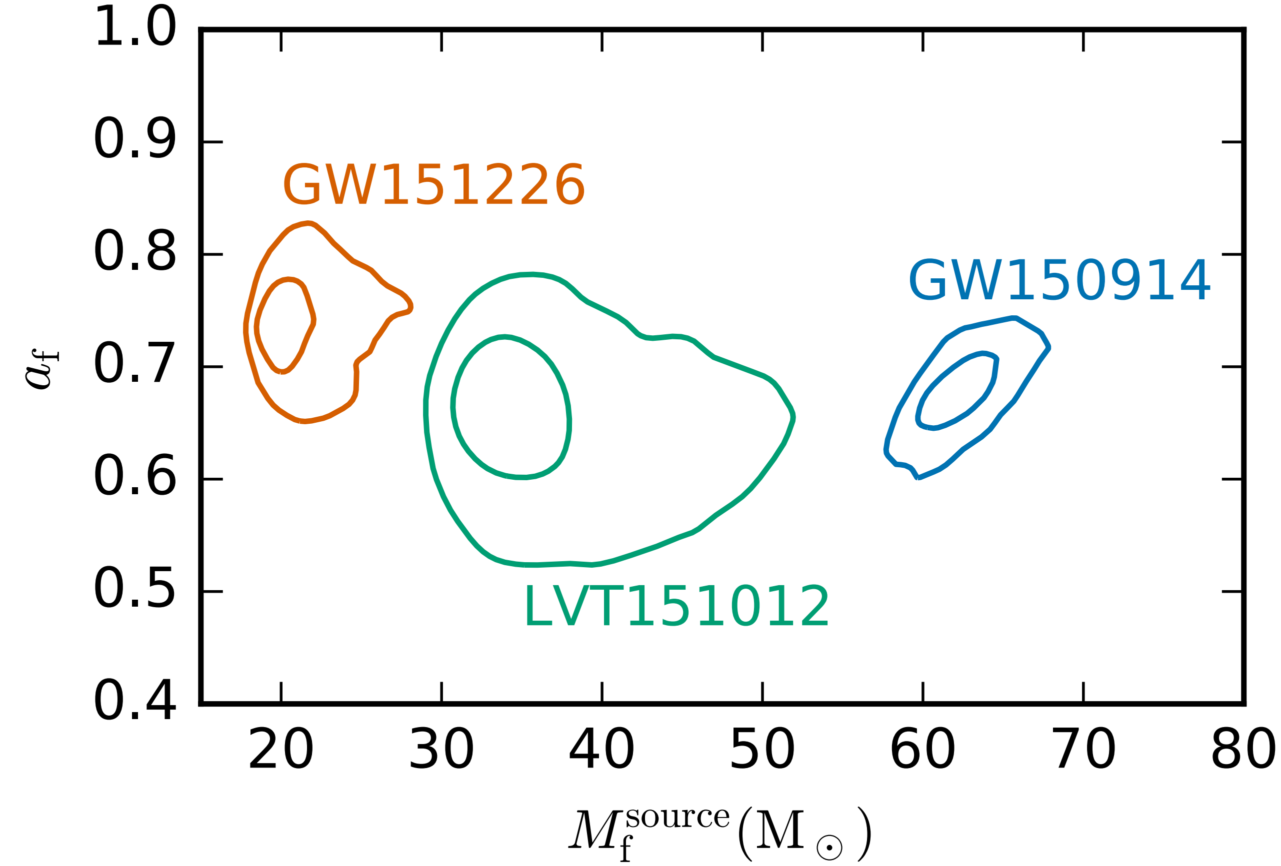
Abbott+ PRL, PRD, 2016





DISTANCE AND REMNANT MASSES & SPINS

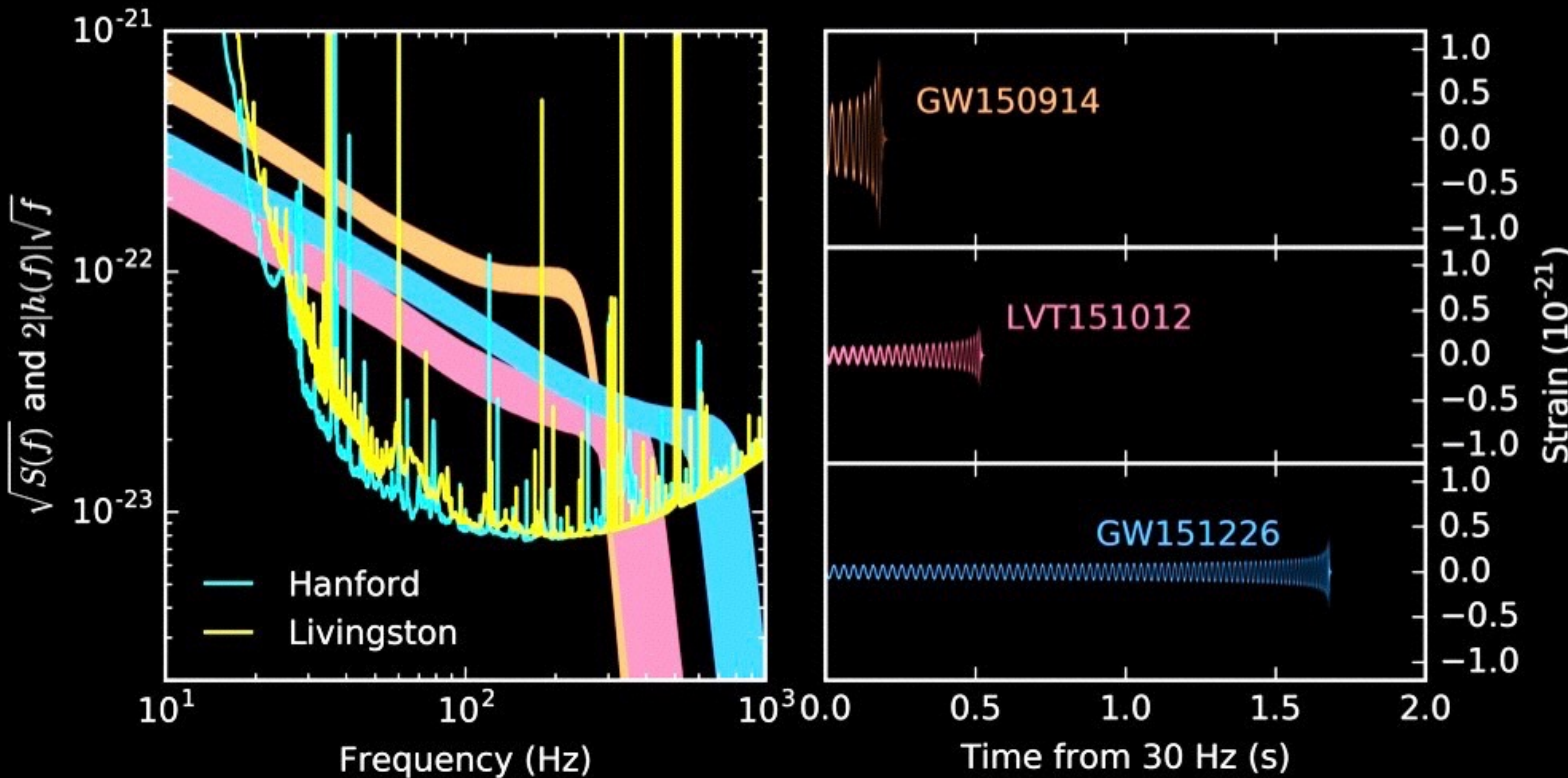




Abbott+ PRD, 2016

“RECONSTRUCTED” SIGNALS

Abbott+ PRD, 2016

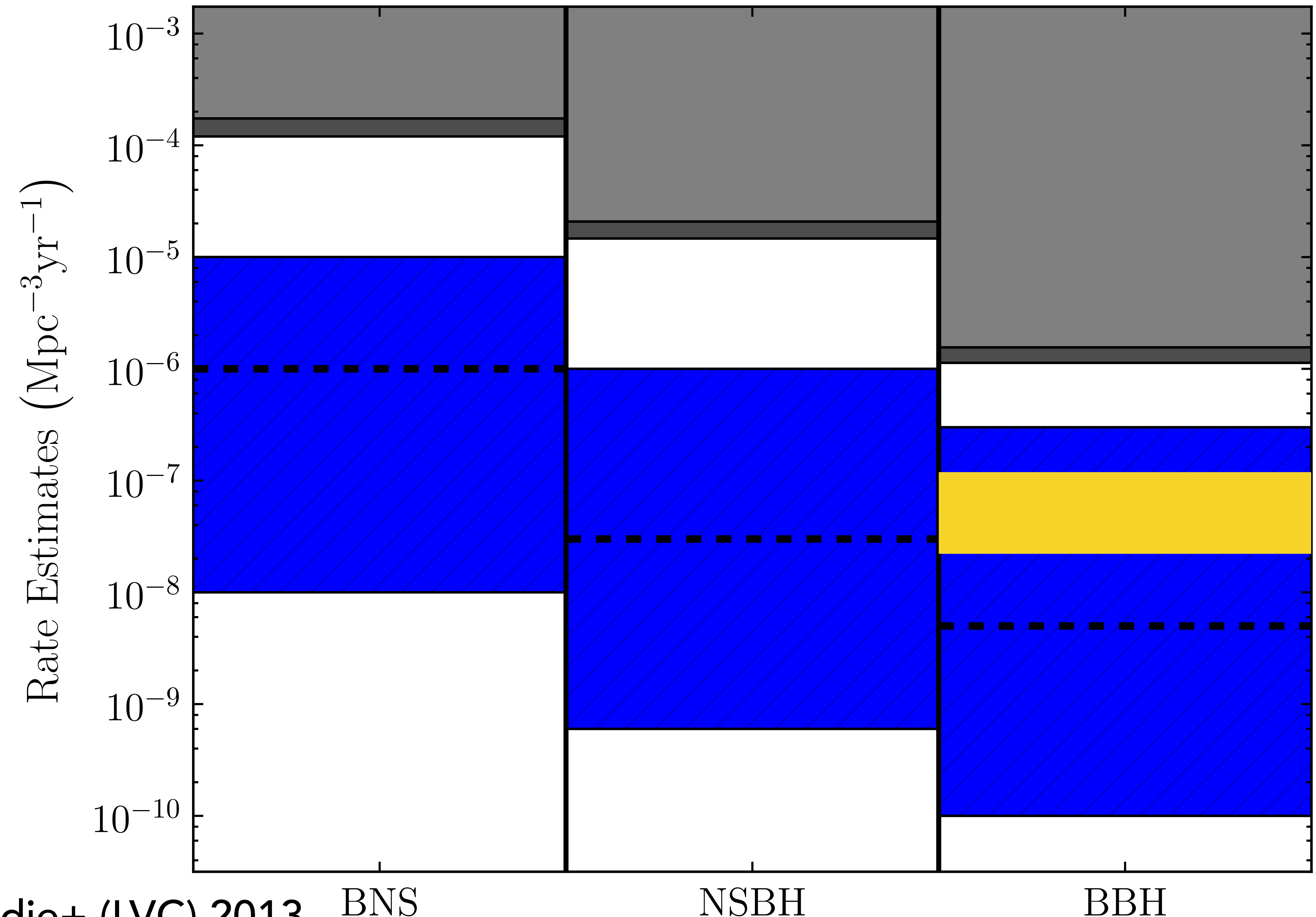


ASTROPHYSICAL IMPLICATIONS

- existence of heavy black holes
  - black holes ~30 $M_\odot$ can be in merging binaries, black holes of ~60 $M_\odot$ exist
  - questions on formation scenarios of black hole binaries; what does that imply for other types of binaries?
- rate of BH mergers: [9, 240] yr^{-1} Gpc^{-3}
  - at the higher end of the predicted rate
  - at design sensitivity we will observe ~1 merger a day

Abbott+ PRL and PRD 2016

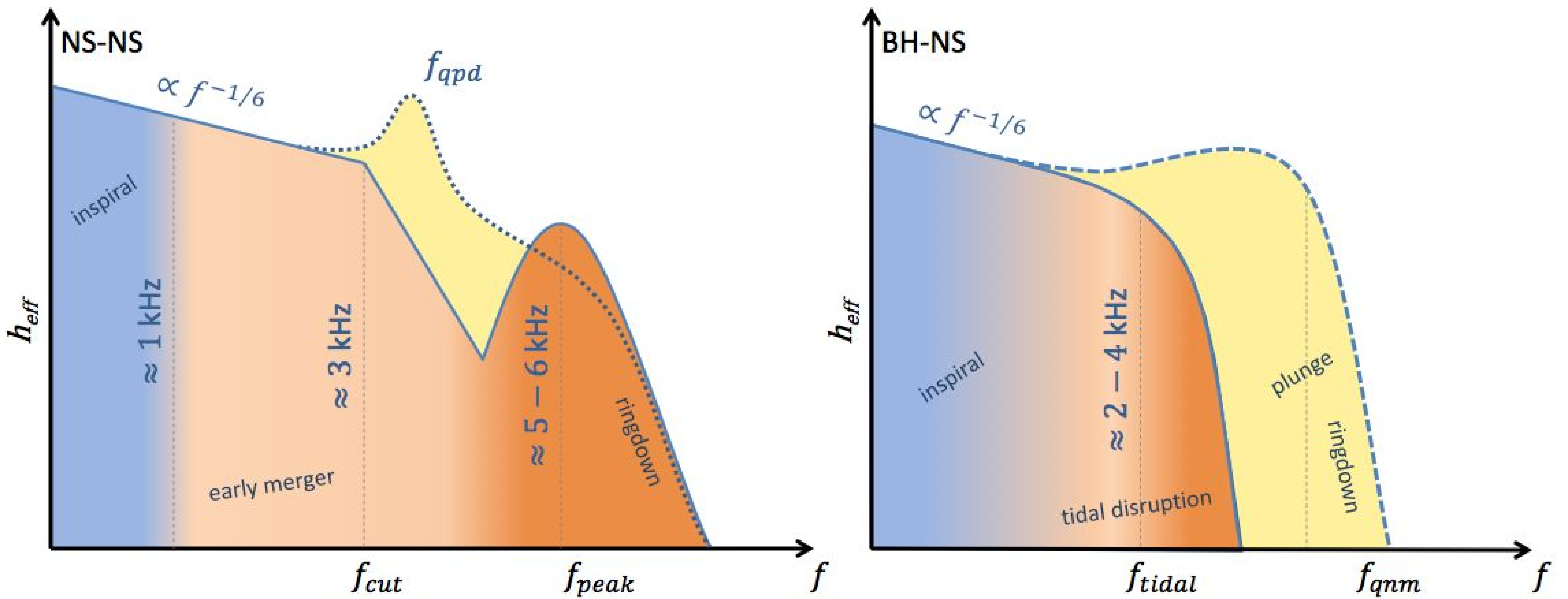
LIGO-VIRGO BEST UPPER LIMITS AND IMPLICATIONS FOR DETECTION



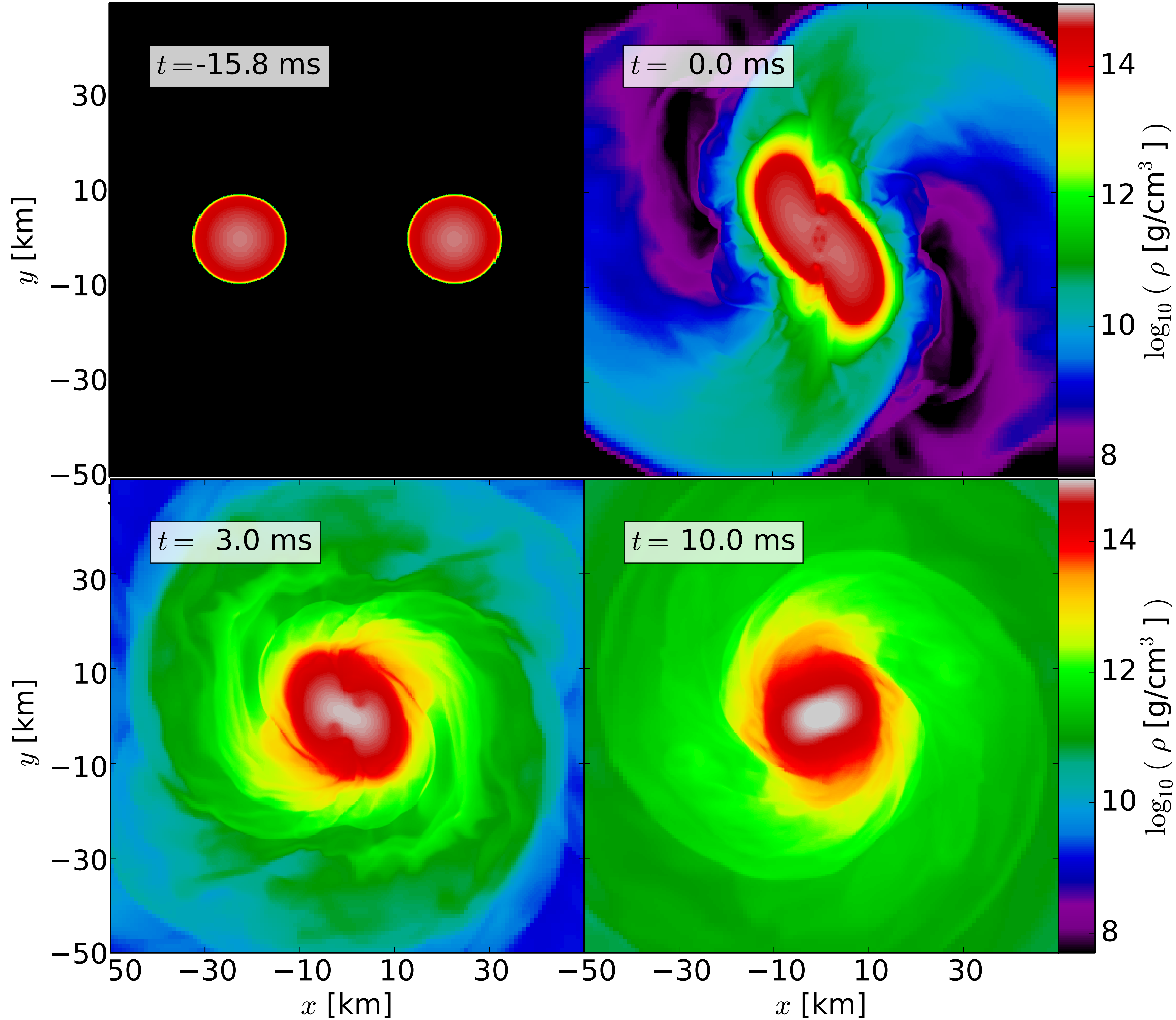
Abadie+ (LVC) 2013

BINARY NEUTRON STARS (BNS)

[Bartos, Brady, Marka, CQG **30**, 123001 (2013)]



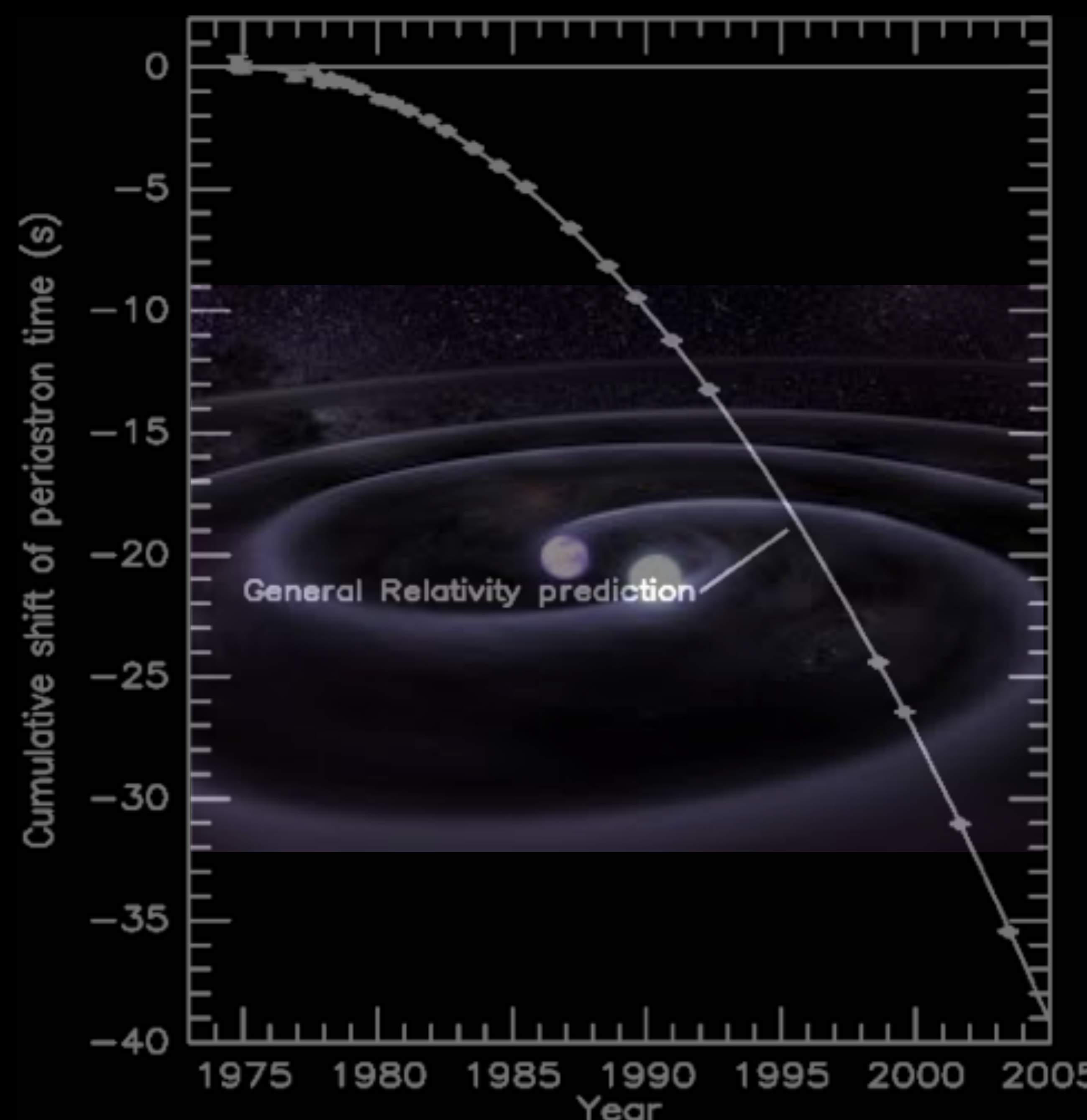
BINARY NEUTRON STAR MERGER



Takami+ 2014

BINARY NEUTRON STARS

- could be progenitors of some short gamma ray bursts
- observations should:
  - constrain models of formation and evolution of compact binaries
  - possibly equation of state of supra-nuclear matter
- rates highly uncertain



Plot: Weisberg+, Image: NASA

EXPECTED NS-NS MERGER RATES

- observed short GRB rate ~ 10 yr^{-1} Gpc^{-3}
- we won't observe all GRBs because
  - most GRB satellites are not sensitive to the whole sky and gamma emission is not expected to be isotropic
- comoving volume rate depends on the beaming angle
  - smaller the beaming angle, less likely we will observe them and so greater the intrinsic rate
- half beaming angle of [5° ,90°] gives a comoving volume rate of [20, 2,000] yr^{-1} Gpc^{-3}
  - implies a detection rate of ~ 0.2-50 yr^{-1} at LIGO-Virgo design sensitivity; population synthesis models predict uncertain rates; radio observations are consistent with this range

ANATOMY OF A BINARY NEUTRON STAR COALESCENCE WAVEFORM

early inspiral modeled by post-Newtonian theory - $(v/c)^n$

late inspiral modeled by effective one-body approximation

merger and post-merger oscillations using numerical simulations

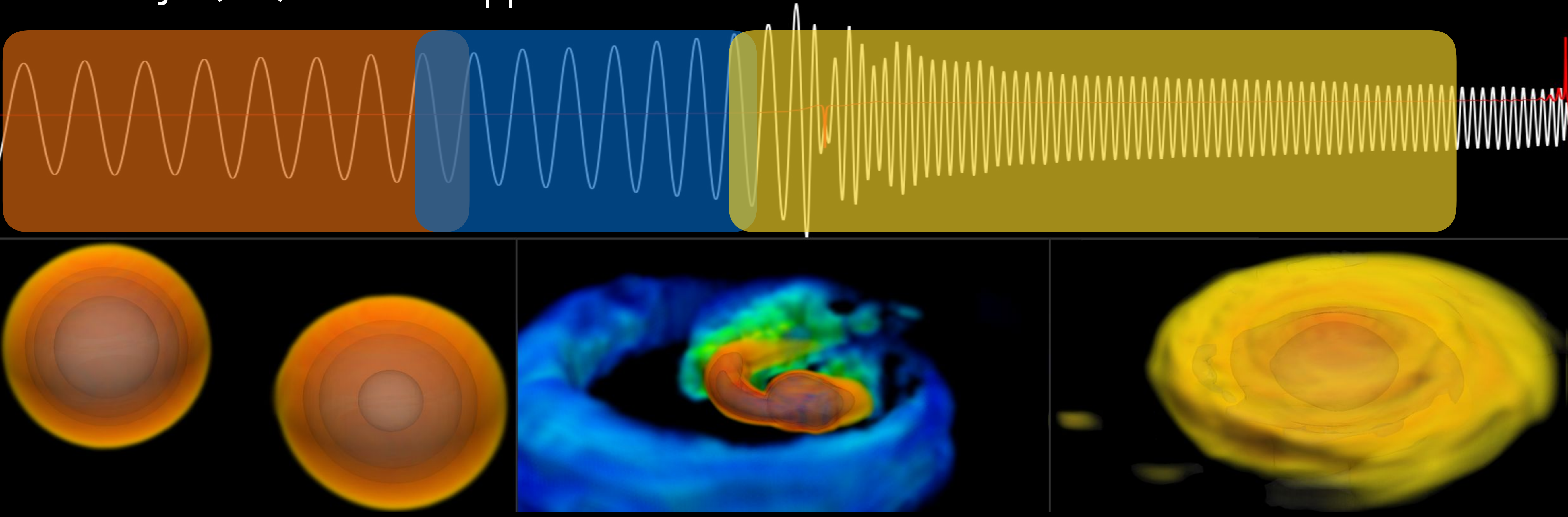


Image: Bernuzzi

PHYSICAL EFFECTS IN BINARY NEUTRON STAR COALESCENCE WAVEFORMS

dominated by gravitational radiation back reaction - masses and spins

tidal effects appear at high PN order, dynamical tides might be important

complex physics of the merger remnant, multi-messenger source, signature of neutron star EoS

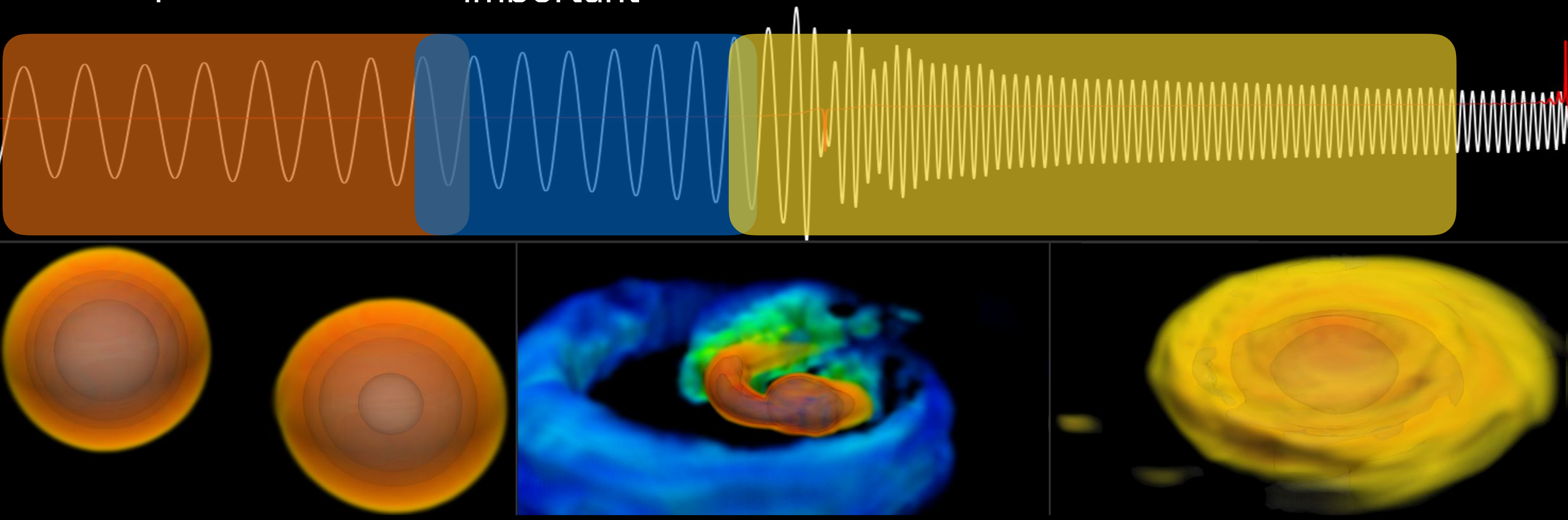
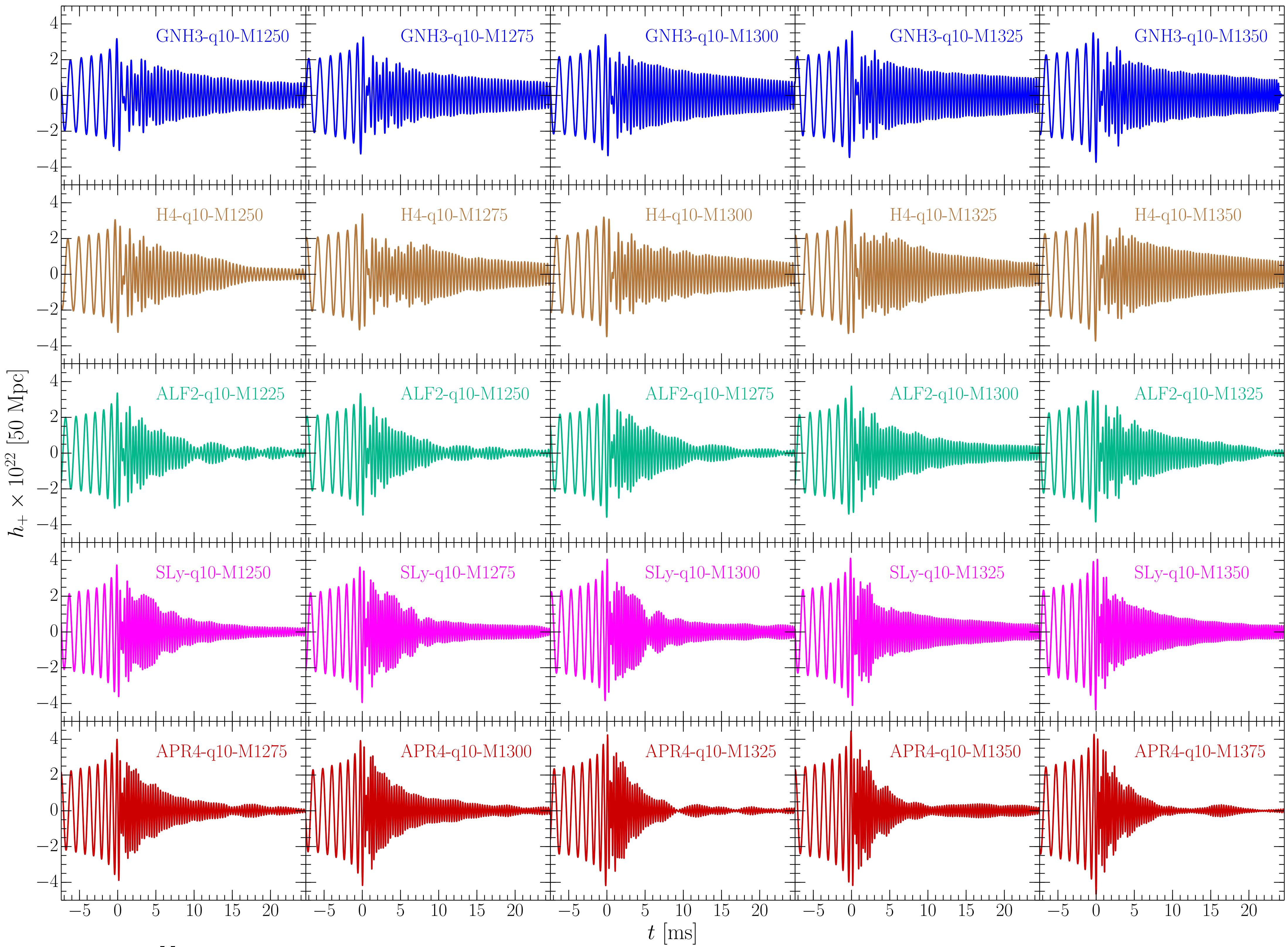


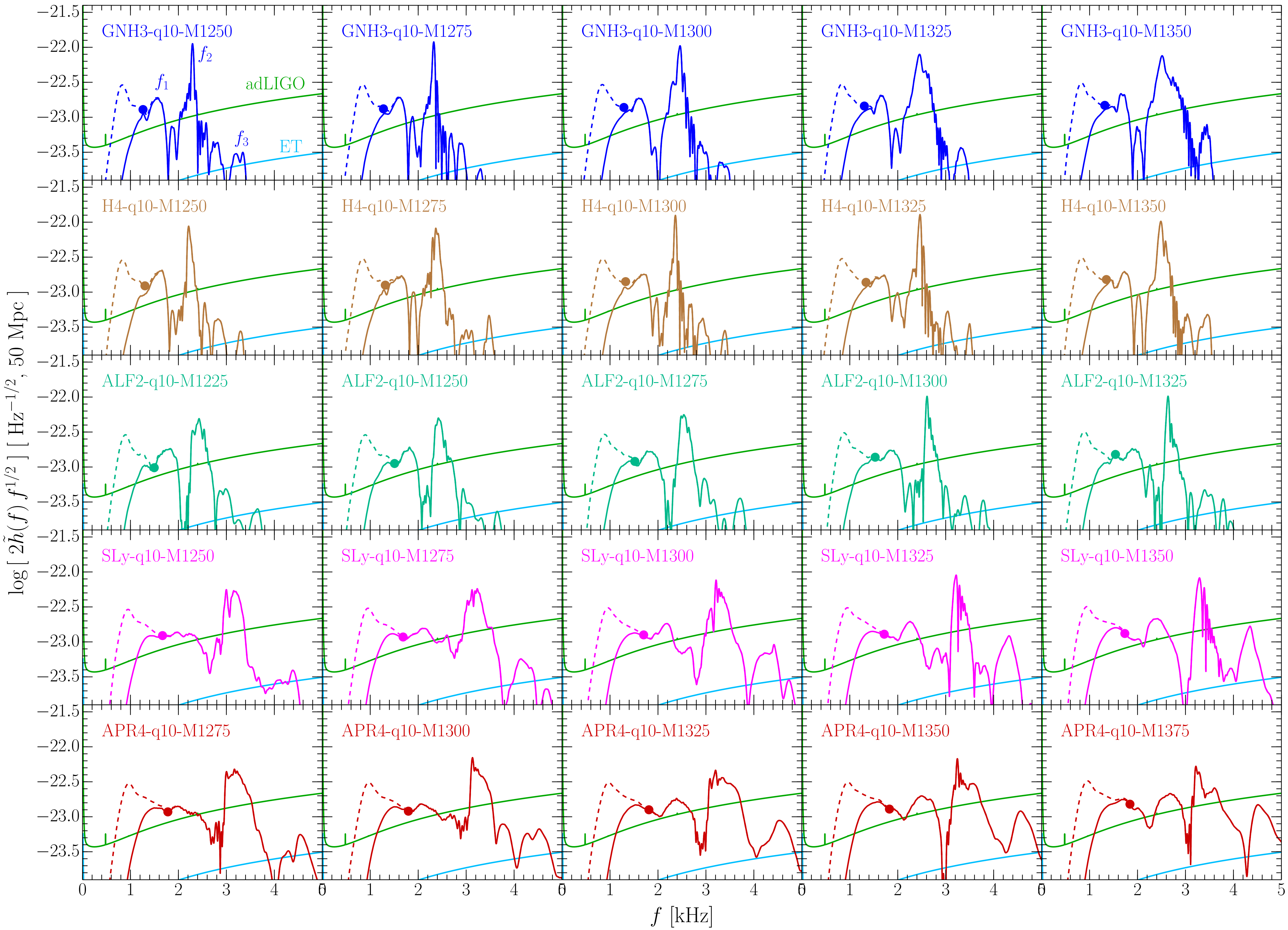
Image: Bernuzzi

BINARY NEUTRON STARS: POST-MERGER WAVEFORMS



Takami, Rezzolla, Baiotti, 2014

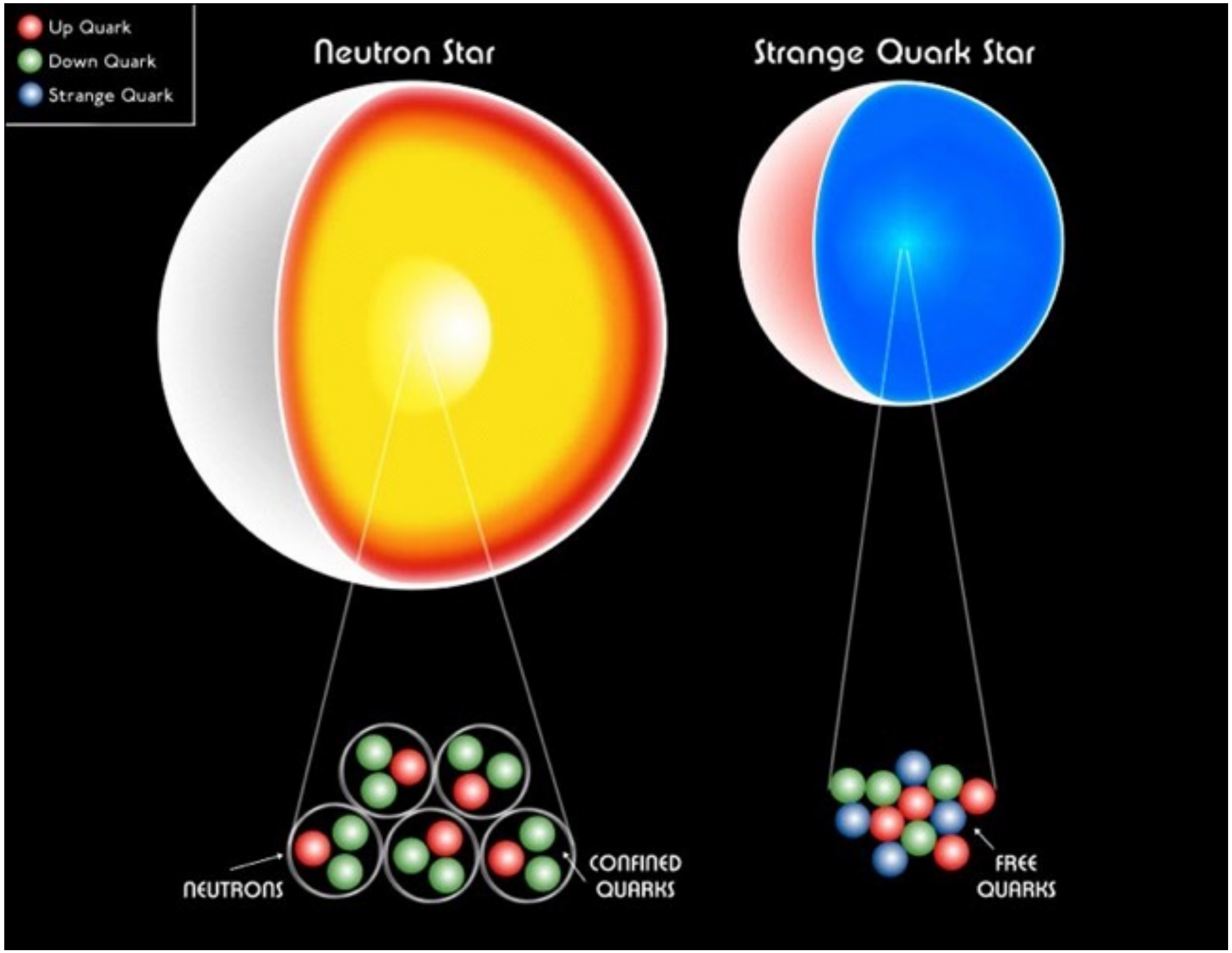
BINARY NEUTRON STARS: POST-MERGER SPECTRUM



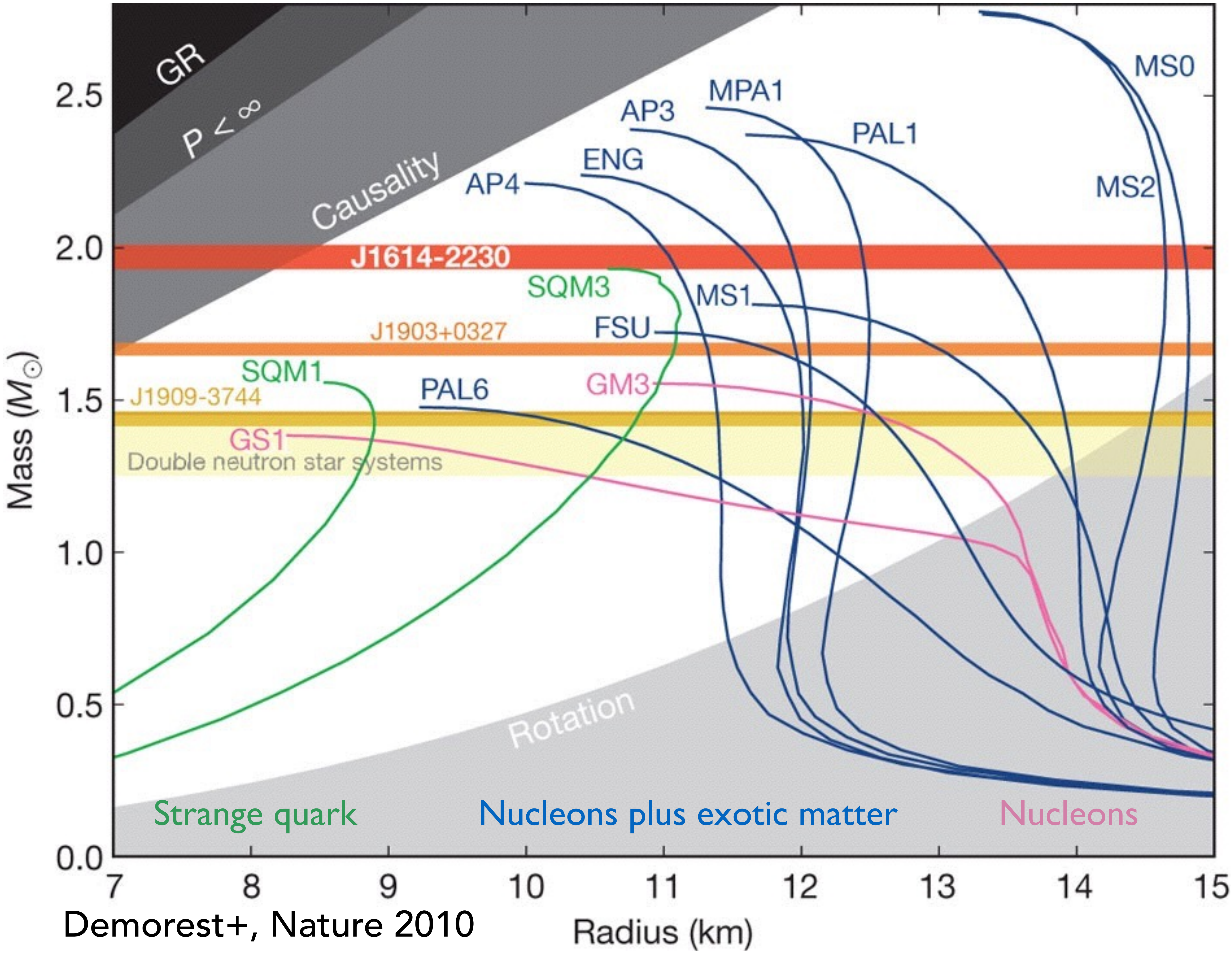
Takami, Rezzolla, Baiotti, 2014

SIZING UP NEUTRON STARS

Densities ~ 4×10^{17} kg/m^3



Hinderer 2008, Flanagan and Hinderer 2008, Hinderer+ 2010, Read+ 2009, 2013, Pannarale+ 2011, Damour+ 2012, Lackey+ 2012, Del Pozzo+ 2012, Lackey & Wade 2014



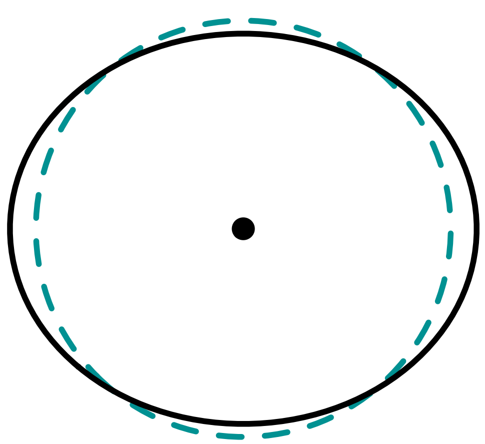
Demorest+, Nature 2010

SIGNATURE OF EOS IN BNS WAVEFORMS

- Tidal tensors $\mathcal{E}_{ij}$ of one of the component of the binary induces quadrupole moment Q_{ij} in the other
- variation in the quadrupole moment causes GW emission
- in the adiabatic approximation

$$Q_{ij} = -\lambda(m)\,\mathcal{E}_{ij}, \qquad \lambda(m) = (2/3)\,k_2(m)\,R^5(m)$$

- where λ(m) is EoS dependent tidal deformability, k_2(m) is the Love number and R is the NS radius
- Just from the scaling this is a 5-PN effect $(v/c)^{10}$



$$\lambda = \frac{Q}{\mathcal{E}} = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}}$$

Tidal deformability

Love number k_2

Radius R

$$\lambda = \frac{2}{3} k_2 R^5 \quad (G = c = 1)$$

$$\Lambda \equiv G\lambda(Gm_{\rm NS}/c^2)^{-5}$$

$$\Lambda \in [300, 600]$$

image: J. Read

COSMOLOGY FROM MICROPHYSICS

- Post-Newtonian phasing formula has binary M and freq. f together

$$\Psi(f) = 2\pi f t_C - \phi_C + \sum_{k=0}^{7} \alpha_k \left(\pi M f\right)^{(k-5)/3}$$

- So it is possible to scale away cosmological frequency redshift:
  $f \rightarrow f / (1+z)$ and $M \rightarrow M\,(1+z)$

- The tidal term, on the other hand, cannot be scaled away

$$\Psi_{\mathrm{Tide}}(f) = -\frac{1250 k_2 \alpha_0}{3} \left(\pi M f\right)^{(k-5)/3} \left(\frac{R}{M}\right)^5$$

- This helps measure neutron star radius and cosmological redshift directly from GW observations

Messenger+Read PRL 2012; Messenger+ PRX 2014

TIDAL TERMS IN THE INSPIRAL REGIME

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v),$$

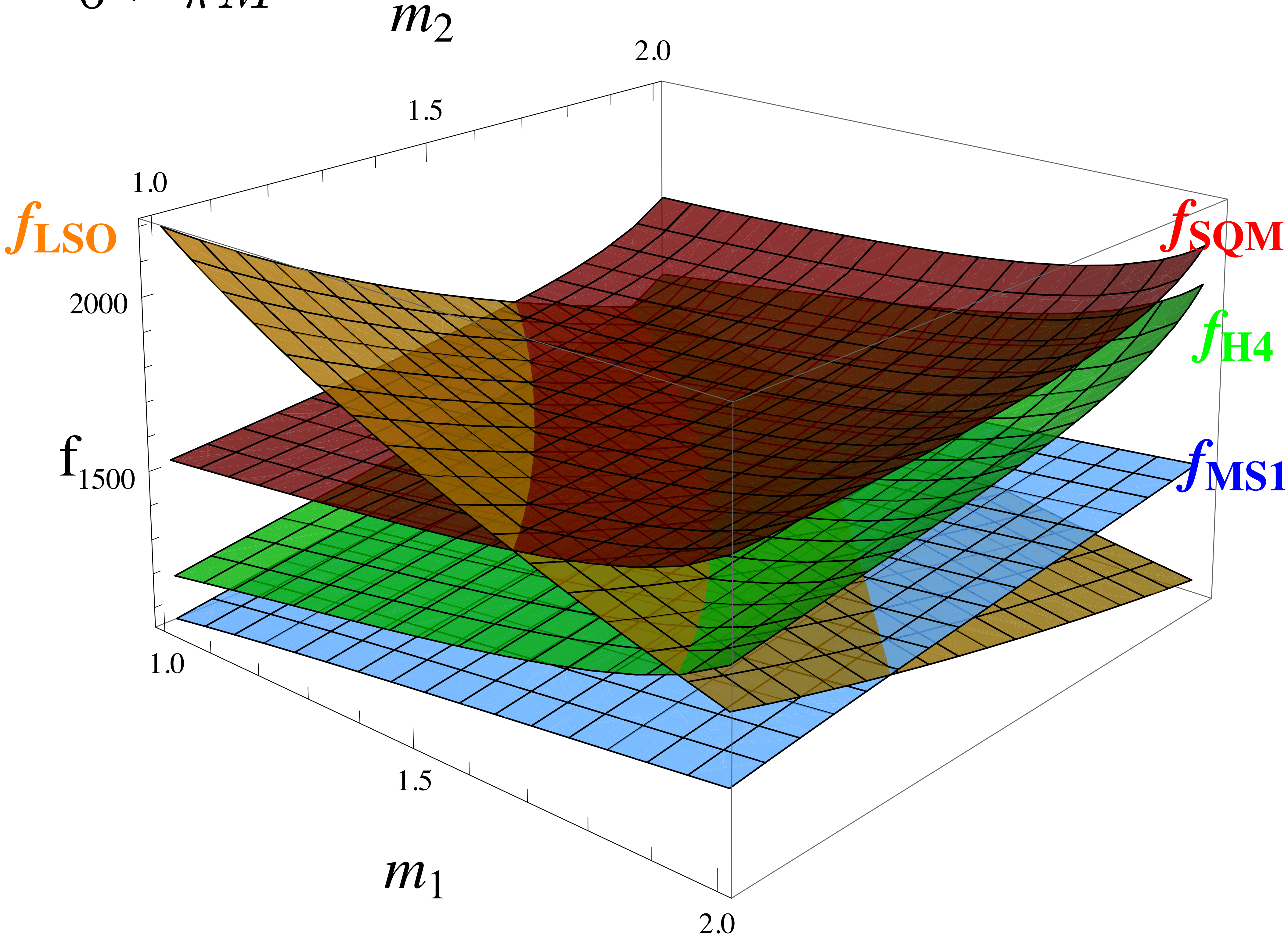
$$\begin{aligned}
\Psi_{\rm tidal}(v) \;=\; & \frac{3}{128\eta} v^{-5} \sum_{A=1}^{2} \frac{\lambda_A}{M^5 X_A} \Bigg[-24\,(12-11X_A)\; v^{10} \\
& + \frac{5}{28}\left(3179 - 919 X_A - 2286 X_A^2 + 260 X_A^3\right) v^{12} \\
& + 24\pi(12 - 11X_A) v^{13} \\
& - 24\left(\frac{39927845}{508032} - \frac{480043345}{9144576} X_A + \frac{9860575}{127008} X_A^2 \right. \\
& \left. - \frac{421821905}{2286144} X_A^3 + \frac{4359700}{35721} X_A^4 - \frac{10578445}{285768} X_A^5 \right) v^{14} \\
& + \frac{\pi}{28}\left(27719 - 22127 X_A + 7022 X_A^2 - 10232 X_A^3\right) v^{15} \Bigg]
\end{aligned}$$

$$X_A = m_A/M,\; A = 1, 2,\; \text{and}\; \lambda_A = \lambda(m_A)$$

plus the quadrupole-monopole interaction

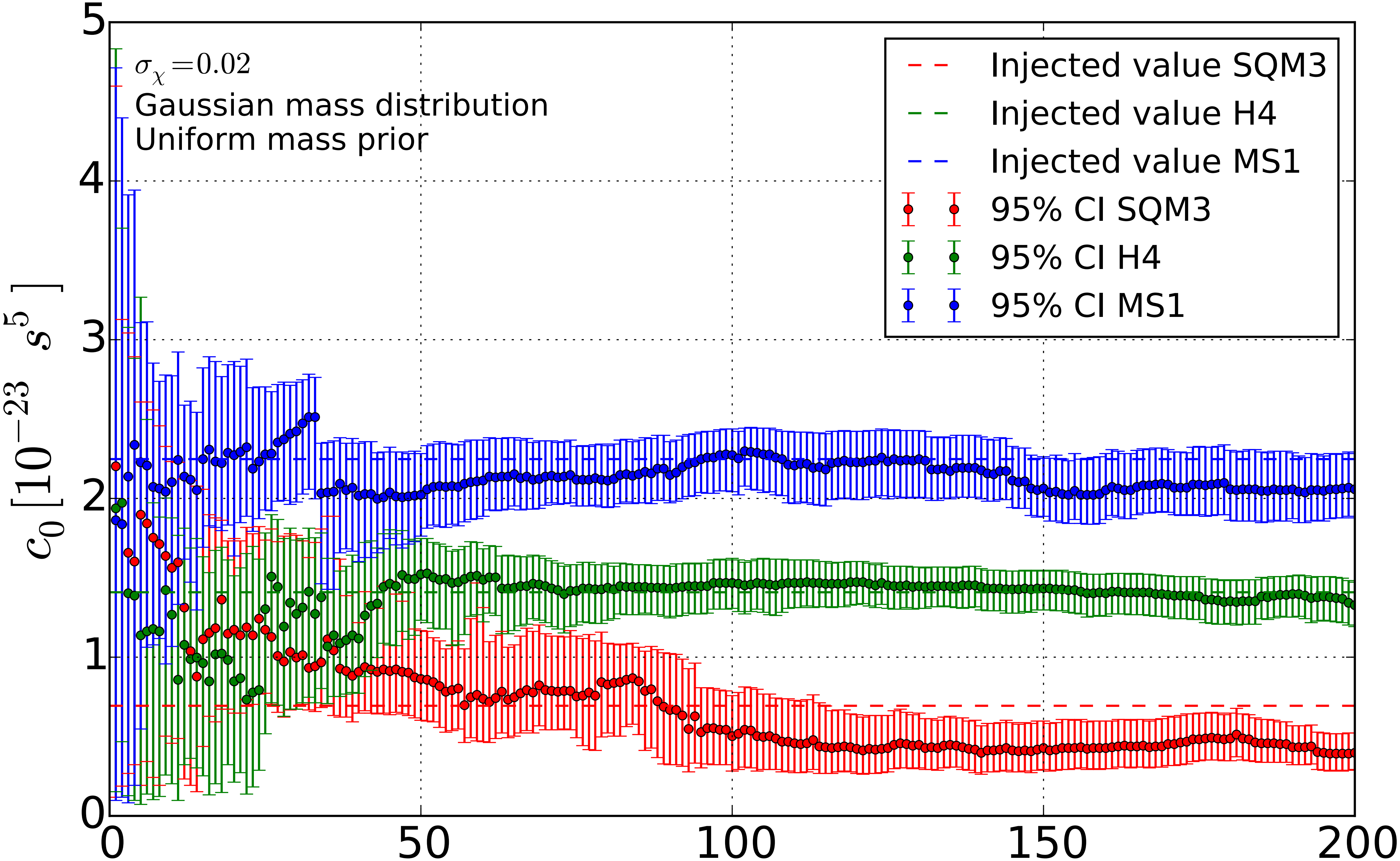
WILL THERE BE A PLUNGE PHASE?

$$f_{\mathrm{LSO}} = \frac{1}{6^{3/2}\pi M}.$$



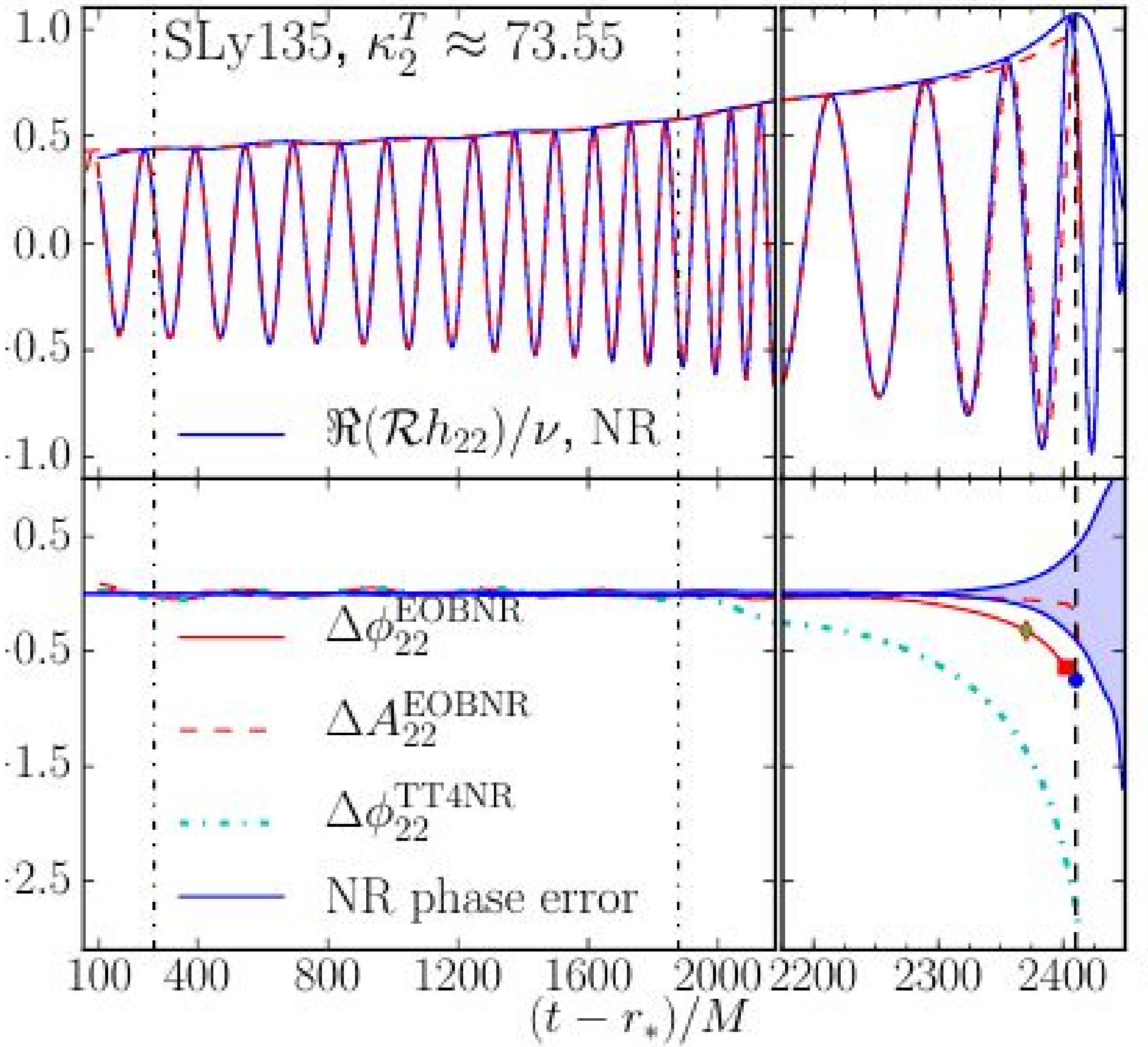
Agathos+, 2015

STATISTICAL AND SYSTEMATIC ERRORS ON C0

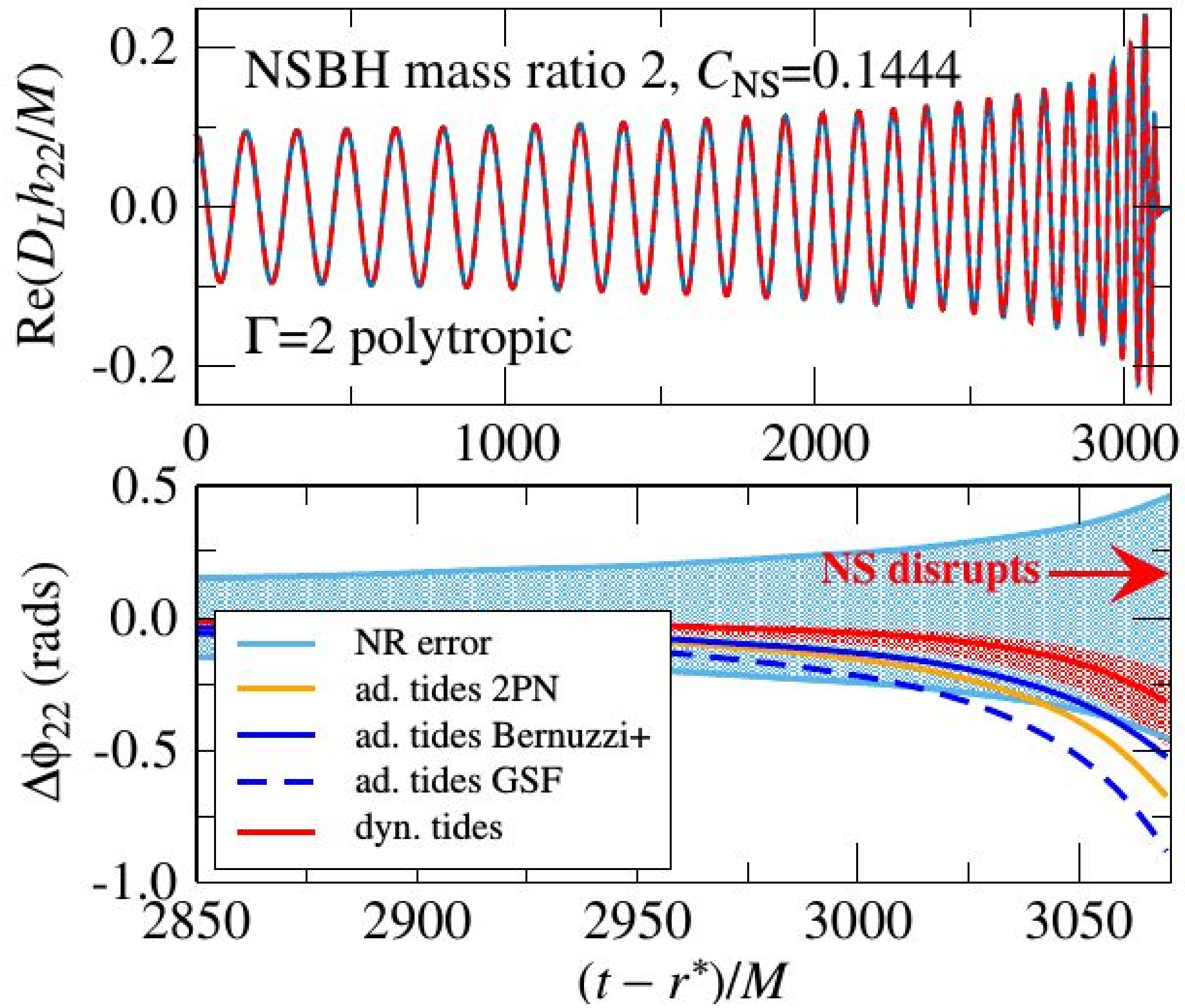


Agathos+, 2015

ACCURATE WAVEFORM MODELS IS KEY TO GW MEASUREMENT OF NS RADIUS



[Bernuzzi+, PRL **115**, 091101 (2015)]



[Hinderer+, PRL **116**, 181101 (2016)]

NS EQUATION OF STATE

INCLUDING THE POST-MERGER PHASE

with

Sukanta Bose, Kabir Chakravarti (IUCAA), Luciano Rezzolla (U. Frankfurt), Kentaro Takami (Kobe)

Goal of this work

- Develop analytical time-domain fits of post-merger (Takami, Rezzolla, Baiotti, PRL 2014) waveforms and combine them with those of pre-merger waveforms.

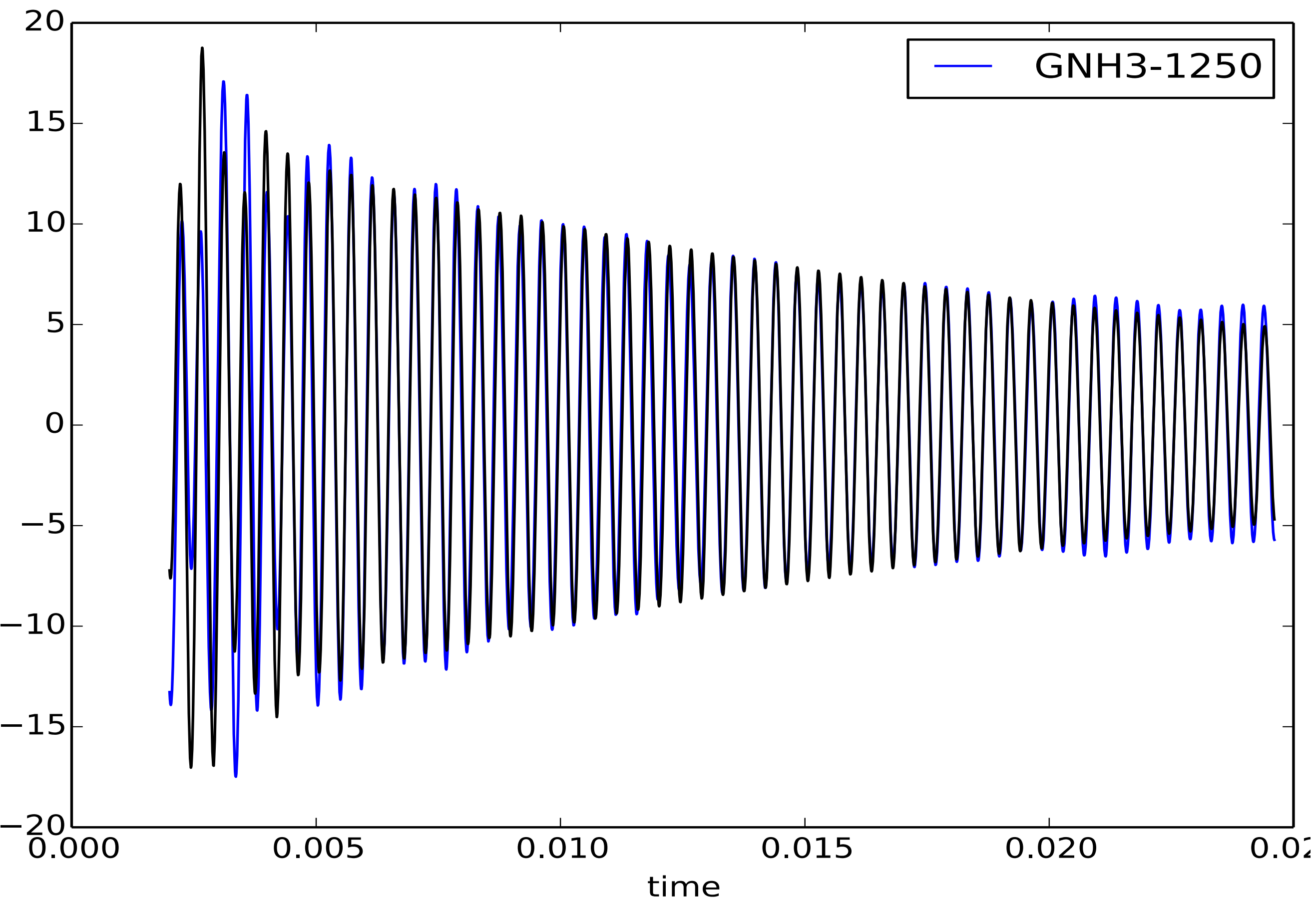
- Use these waveforms to estimate errors in BNS parameters, including NS EOS parameters.

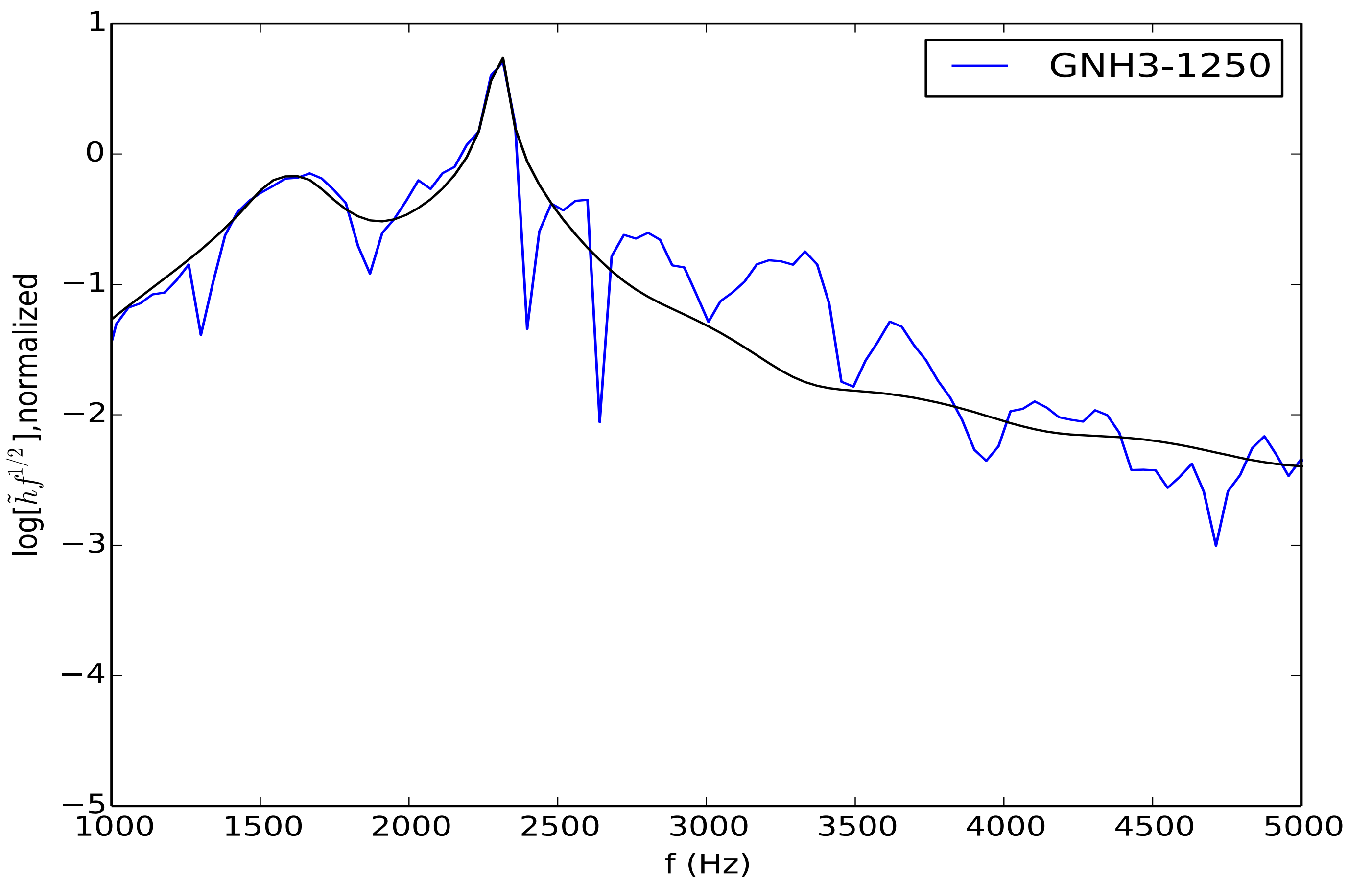
- Future work: combine M_{total} estimate from pre-merger phase with post-merger spectral features to break M_{total} - redshift degeneracy (more relevant for future detectors). [Cf. Messenger et al. arXiv:1312.1862v2.]

PRELIMINARY RESULTS-I

- We have analytically modelled (in black below) NR post-merger waveforms (in blue, with emphasis on frequencies f_1 and f_2 (also called f_{peak}).
- This has been done for a set of EOS, e.g., GNH3, H4, ALF2, SLy.

EOS: *GNH3*, $m_1 = m_2 = 1.25\ M_{sun}$.





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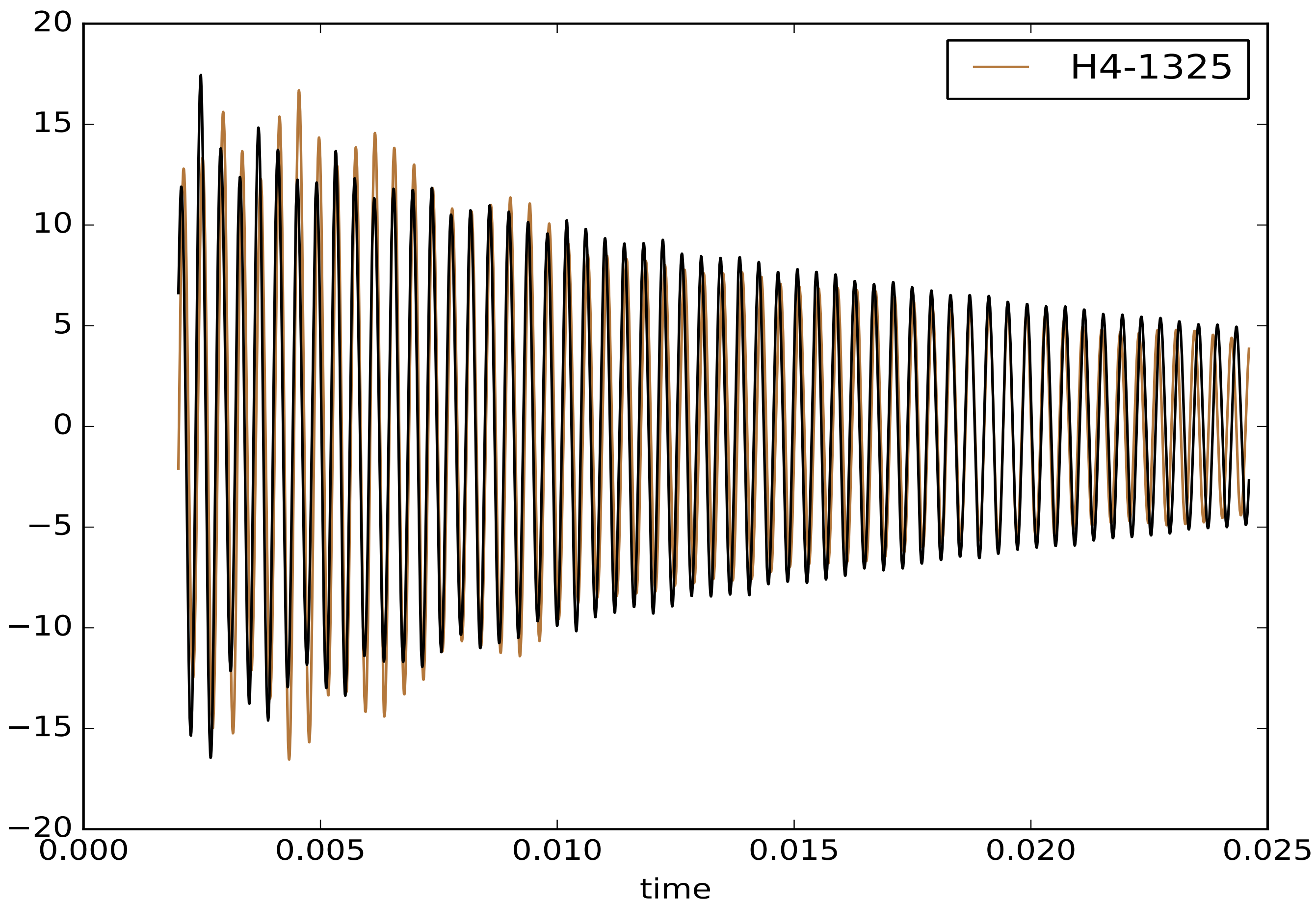
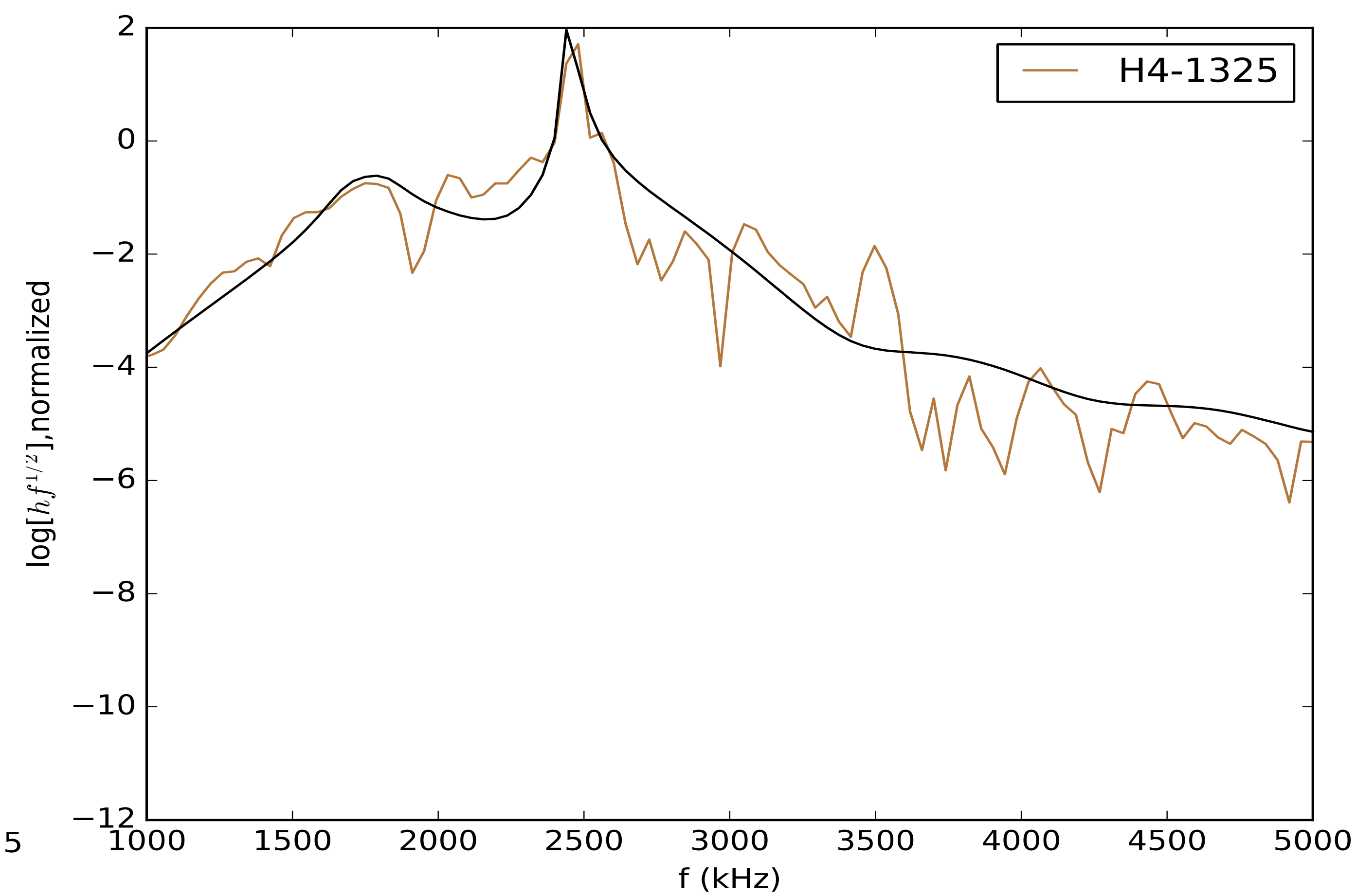
EOS: *H4, $m_1 = m_2 = 1.325\ M_{sun}$.*



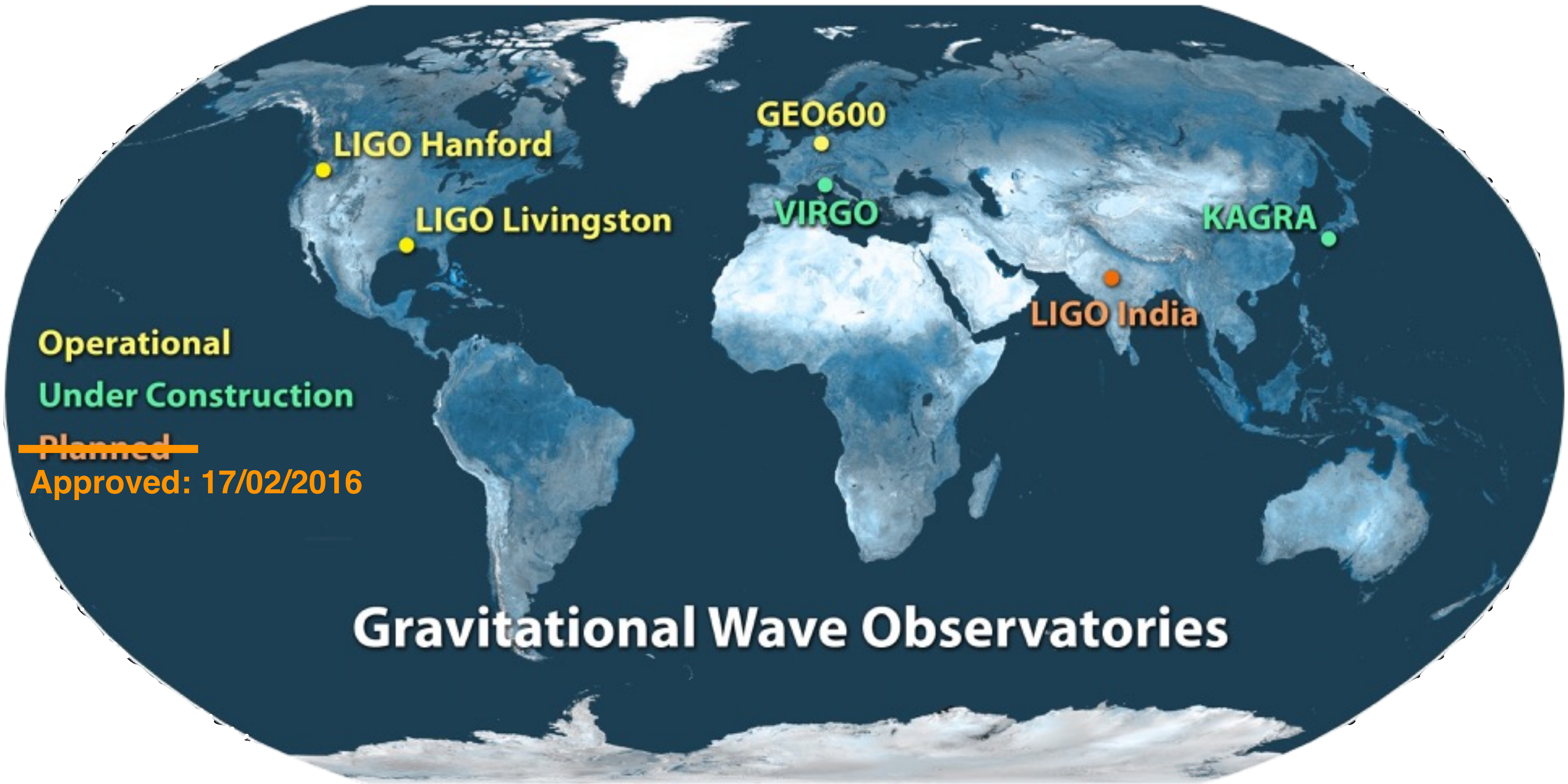


ACCURACY OF MEASUREMENT OF COMPACTNESS

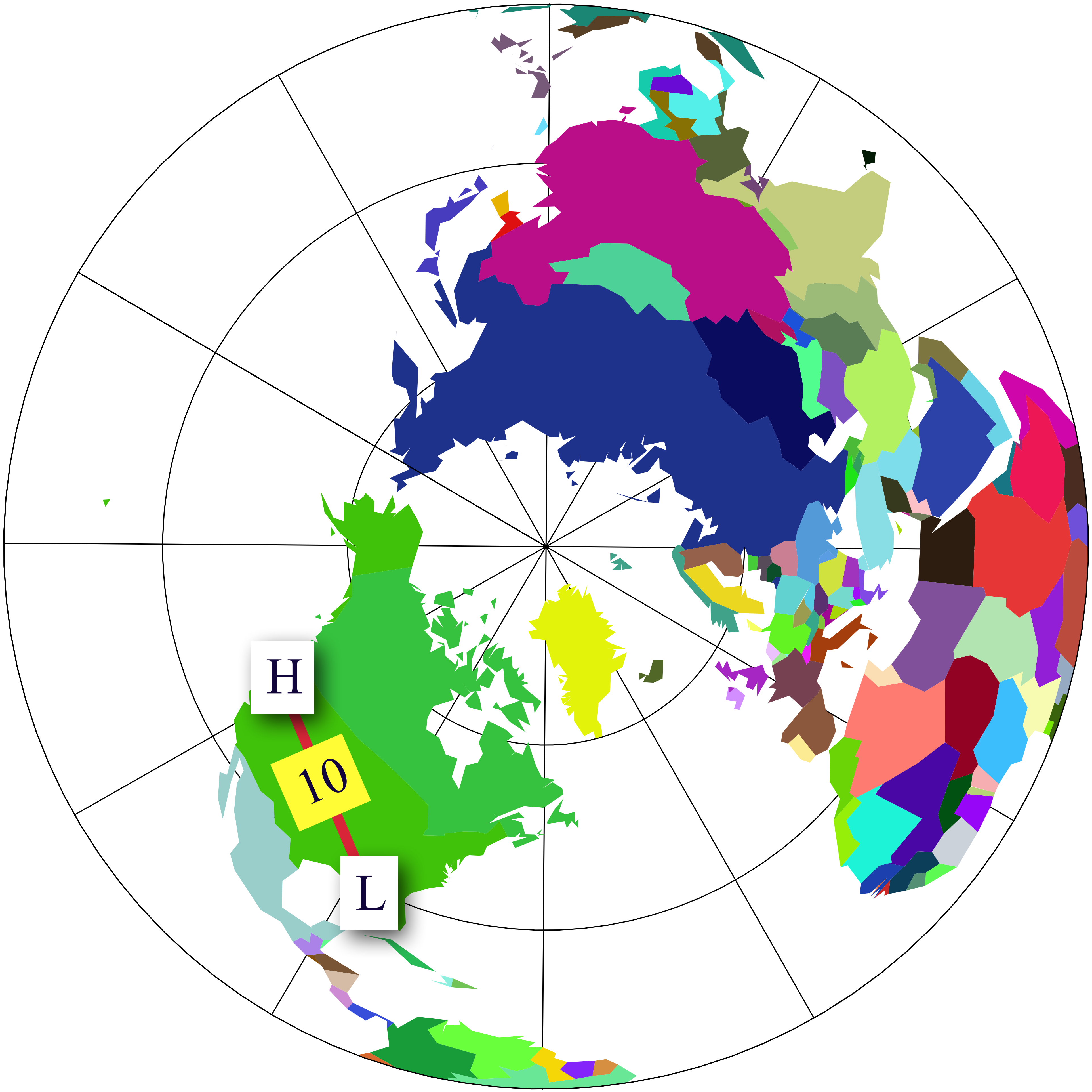
EOS	NS mass ($M_\odot$)	Δf_2 (Hz)	$\Delta C/C$
GNH3	1.250	29	1.0%
H4	1.250	43	1.2%
ALF2	1.250	133	3.4%
GNH3	1.325	40	1.0%
H4	1.325	27	1.0%
ALF2	1.325	60	1.6%

- Above: Statistical error estimates of f_2, and the compaction C deduced from it, for 100 post-merger systems distributed uniformly in aLIGO volume, with an average distance of 200Mpc and SNR of 8.
- If component masses can be determined to an accuracy of 10 - 20% from the inspiral phase, then the above compaction errors imply that the radius will be measured to an accuracy of ~10-20%. (But this is a loose statement since masses and radii will vary among the 100 sources.)
- CAVEAT: At the moment systematic errors between post-merger waveforms from different NR groups can be as high as ~10% in estimating the compaction. (Compare this to a few percent statistical error listed in the table above, arising from detector noise.)

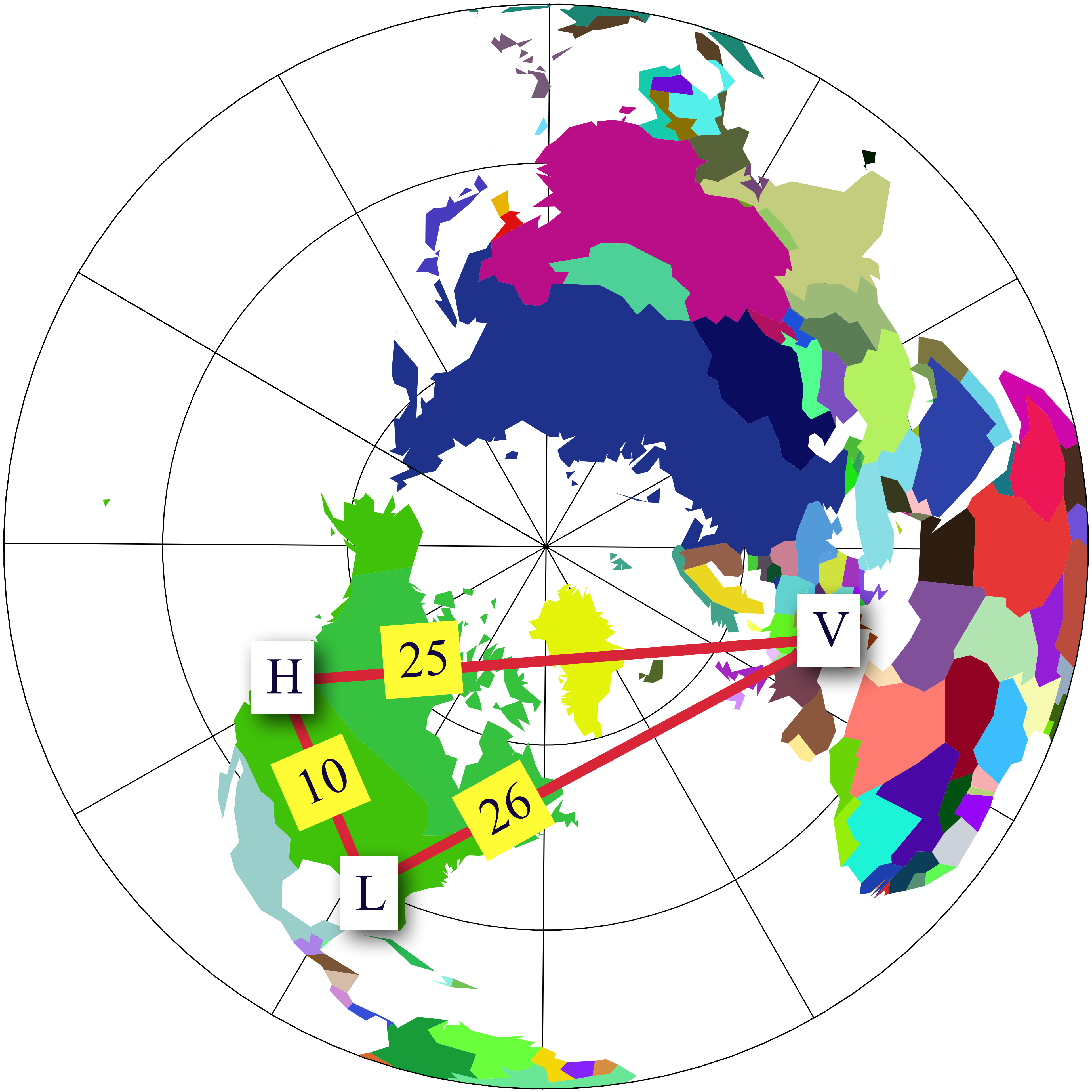
A GLOBAL NETWORK OF GRAVITATIONAL WAVE DETECTORS



Detector Networks 2015+



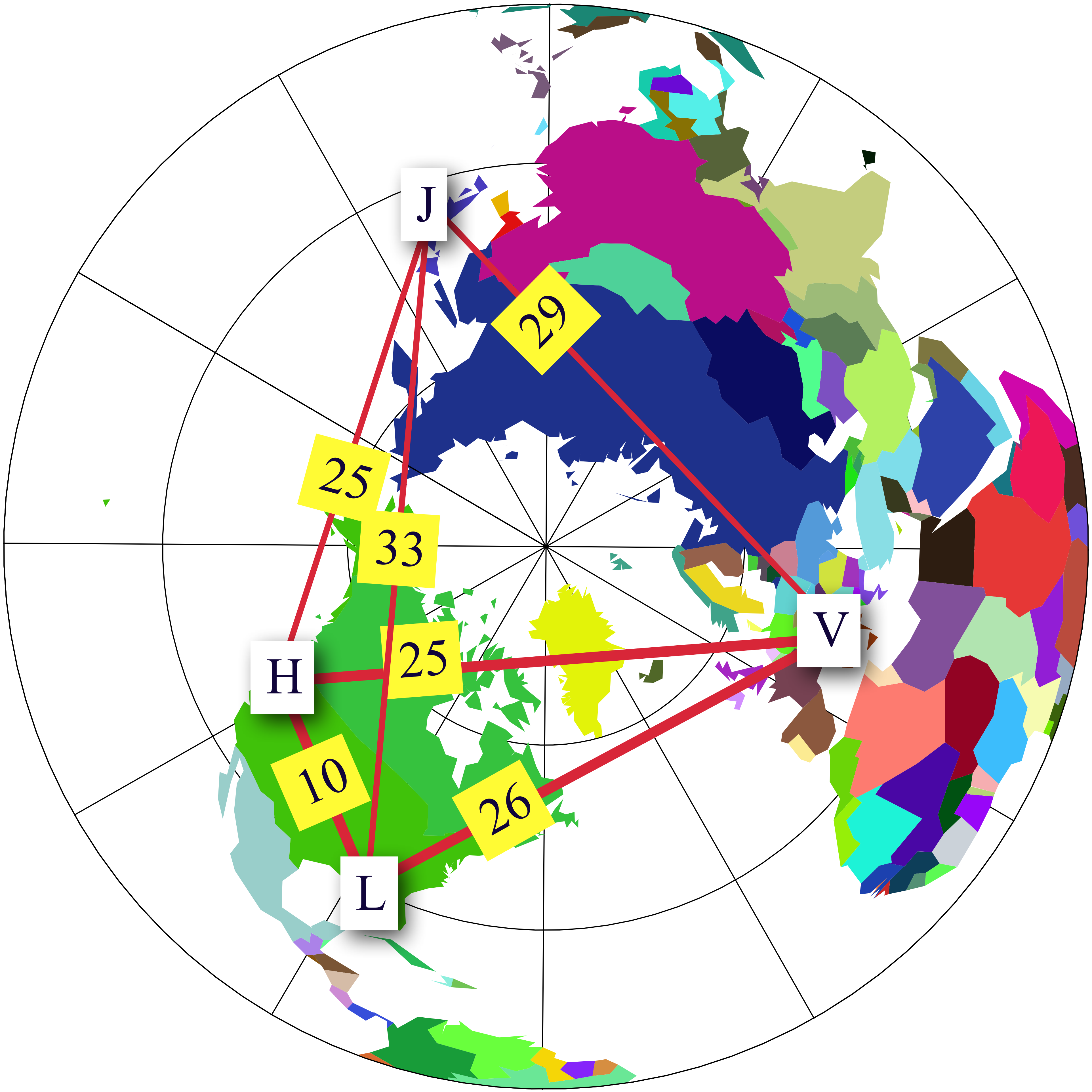
BASELINES IN LIGHT TRAVEL TIME (MS)



Detector Networks 2016+

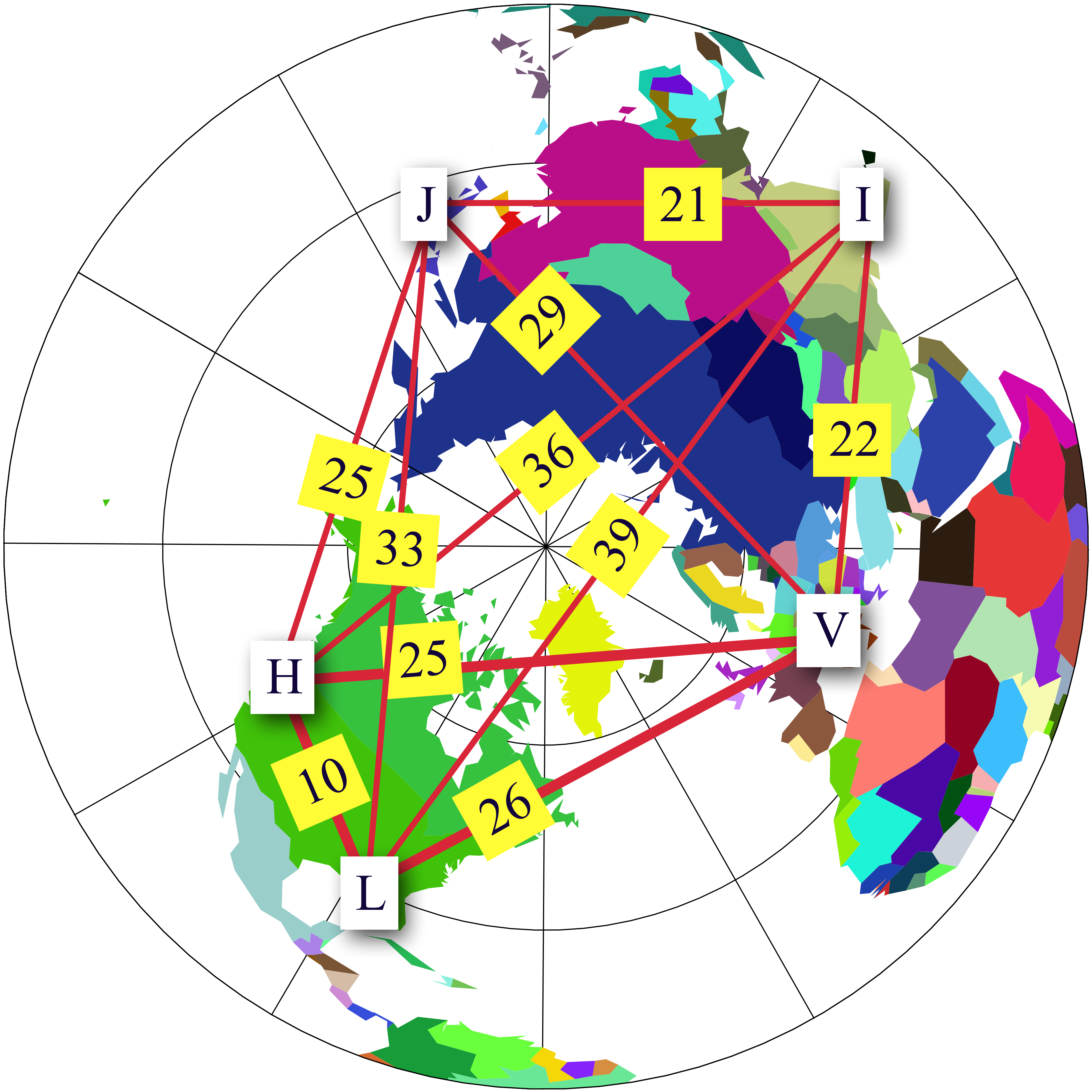
BASELINES IN LIGHT TRAVEL TIME (MS)

Detector Networks 2018+



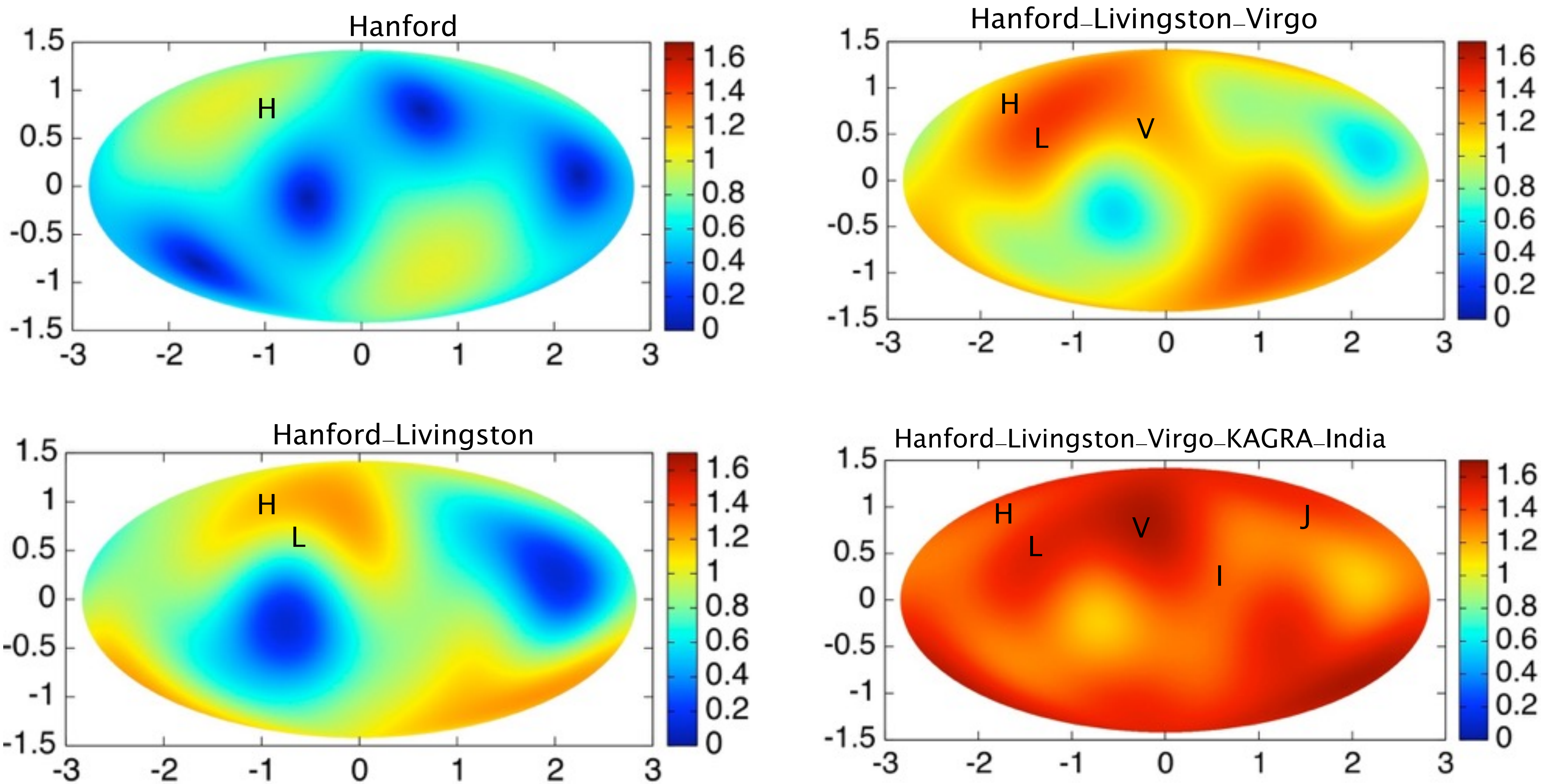
BASELINES IN LIGHT TRAVEL TIME (MS)

Detector Networks 2024+

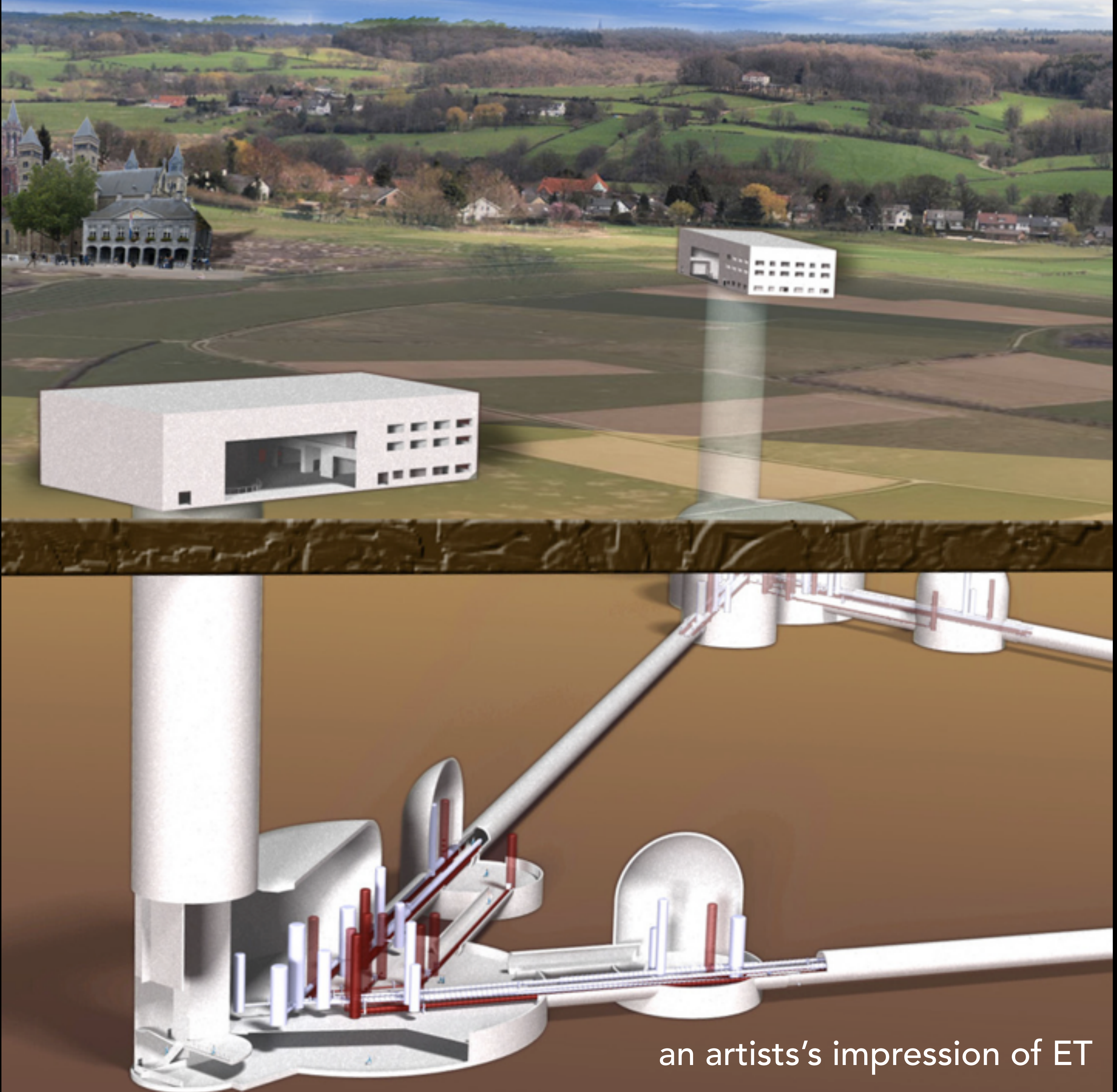


BASELINES IN LIGHT TRAVEL TIME (MS)

BEAM PATTERNS OF NETWORKS



BEYOND ADVANCED DETECTORS



an artists's impression of ET

VOYAGER:
x 3 improvement in aLIGO strain sensitivity

EINSTEIN TELESCOPE:
Triangular, 10 km arm length, underground, cryogenic detectors

COSMIC EXPLORER:
40 km arm length, cryogenic, overground interferometer

3G SCIENCE DRIVERS

Extreme gravity

Extreme matter

Cosmic history

EXTREME GRAVITY

Quasi-normal modes and the no-hair theorem

Dynamical spacetime: Higher modes, precessing orbits, Extremal spins…

GR violations and alternative gravity theories

Bursts and stochastic background from cosmic strings

Gravitational collapse, supernova

EXTREME MATTER

What are the most compact object in Nature

Equation of state of neutron star cores

GRB physics from Binary neutron star observations

Dynamics of neutron star interiors, tidal instabilities

Nature of Low-mass x-ray binaries

COSMIC HISTORY

Mapping the history of black hole formation

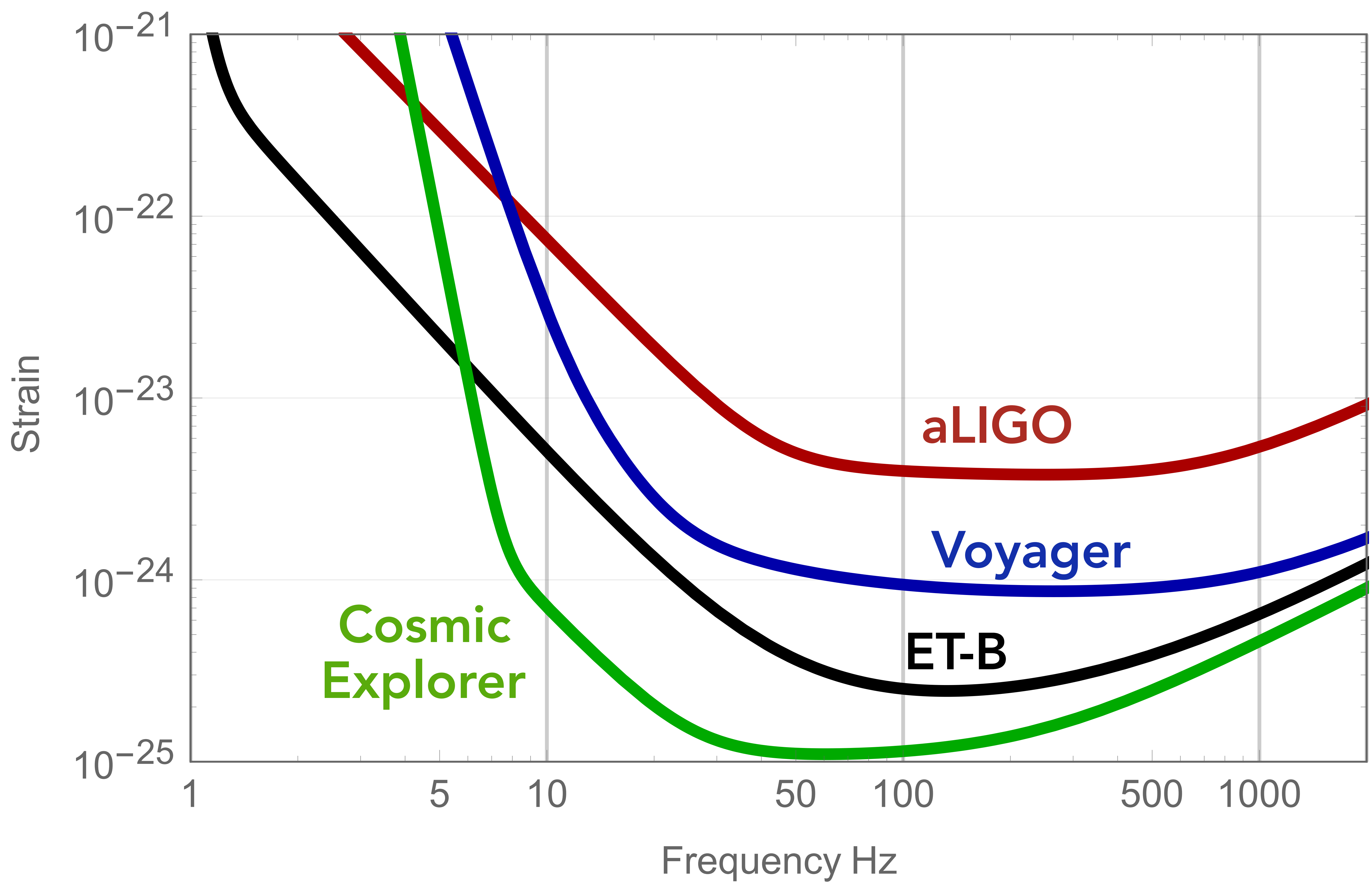
Do gravitational waves see the same universe as light

Formation and evolution of compact objects throughout the Universe

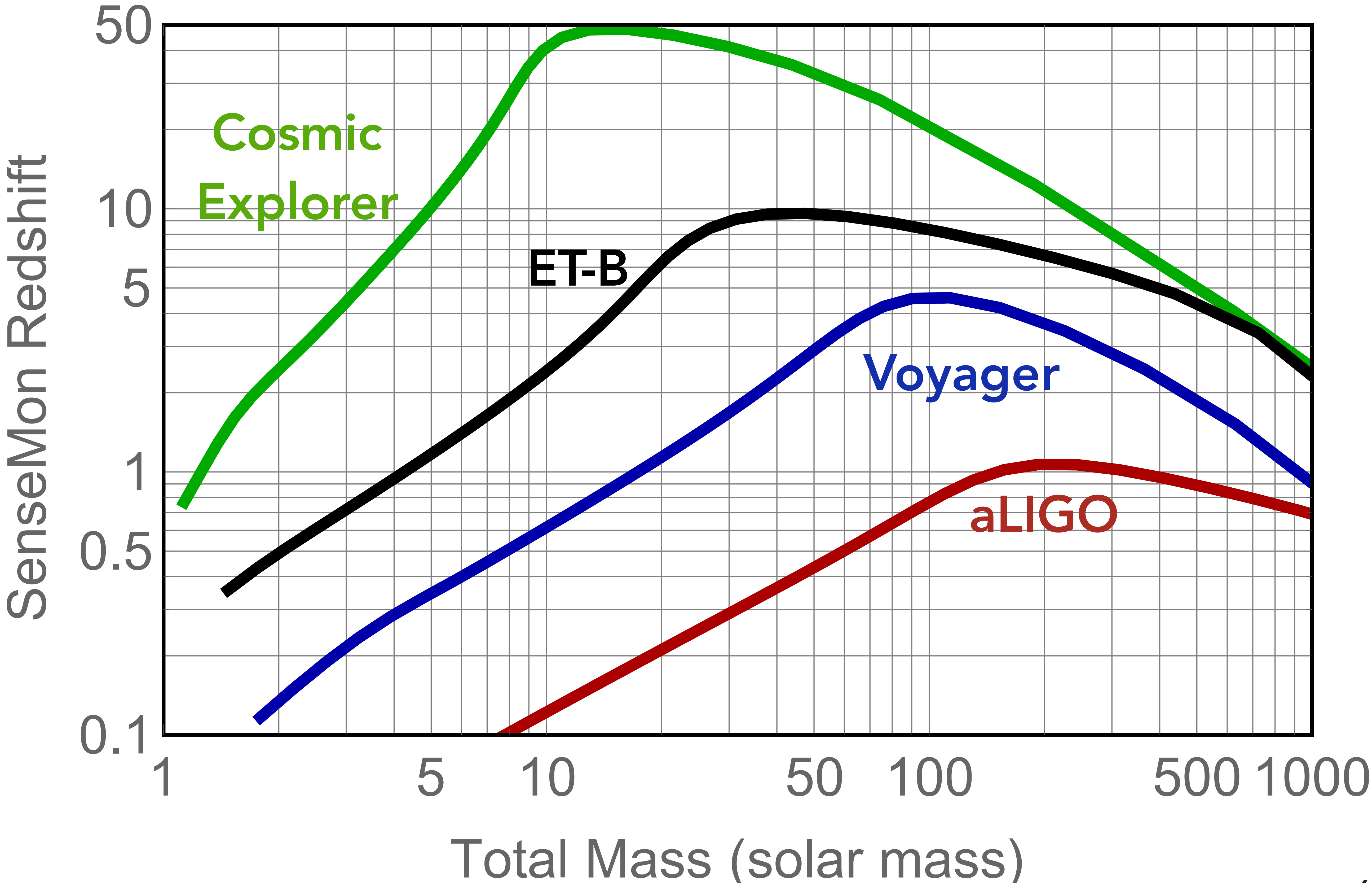
The chemical content of the Universe from NS-NS and NS-BS

Cosmic string bursts and backgrounds

STRAIN SENSITIVITIES OF FUTURE DETECTORS



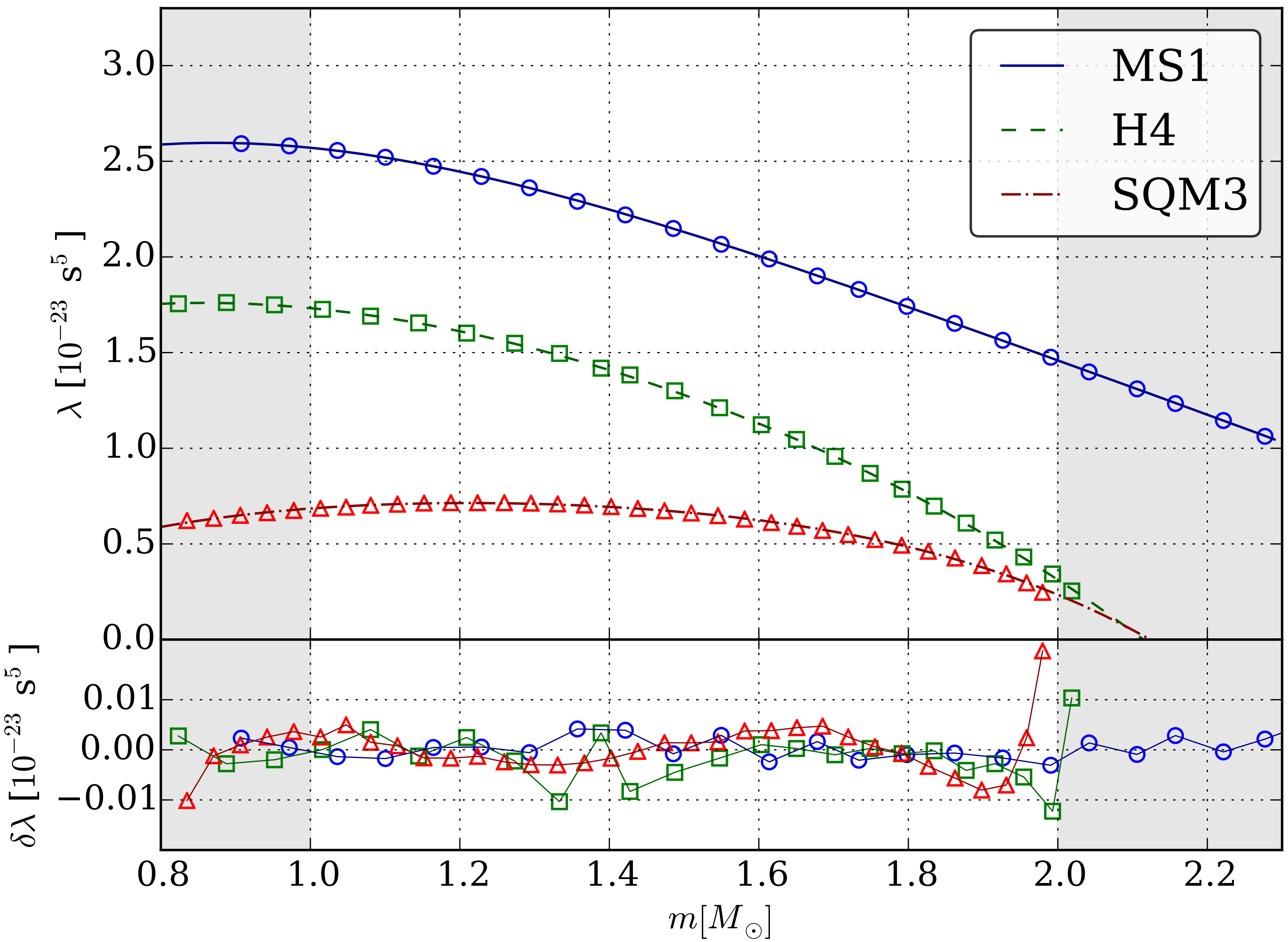
HOW FAR CAN WE SEE SOURCES?



SUMMARY

- binary neutron star signals are to GW observations as atomic spectra are to EM observations
    - signature of nuclear equation of state is imprinted in the inspiral and post-merger signal
    - GW amplitude gives us distance and spectra could give us redshift
- measuring the NS-EoS and radius via GW observations will take sometime
    - lack of accurate waveform models and systematic biases
    - unknown distribution of neutron star masses and spins
    - insufficient sensitivity frequencies beyond ~ 500 Hz
    - difficulties with calibration of phase and amplitude of the data
- third generation detectors, and probably new ideas, are needed to impact microphysics from GW observations

EOS USED IN AGATHOS+ PAPER



The tidal deformability parameter λ(m) as a function of neutron star mass for three different EOS: a soft one (SQM3), a moderate one (H4), and a stiff one (MS1). Adapted from [18]. Curves are fitted quartic polynomials, whose residuals are shown in the lower subplot. Only masses within the unshaded region [1, 2]M⊙ will be considered in our analyses.

QUADRUPOLE-MONOPOLE TERM

- Spin-induced deformation leads to quadrupole that depends as spin-square

$$\Psi_{\rm QM}(v) = -\frac{30}{128\eta}\sigma_{\rm QM} v^{-1},$$

$$\sigma_{\rm QM} = -\frac{5}{2}\sum_{A=1,2} q_A \left(\frac{m_A}{M}\right)^2 \left[3(\hat{\chi}_A\cdot\hat{L})^2 - 1\right]$$
$$\simeq \frac{5}{2}\sum_{A=1,2} a(m_A) \left(\frac{m_A}{M}\right)^2 \left[3(\hat{\chi}_A\cdot\hat{L})^2 - 1\right] \chi_A^2$$

$$q \simeq -a\chi^2,$$

$$\mathcal{C} = 0.371 - 3.91\times 10^{-2}\ln\frac{\lambda}{m^5} + 1.056\times 10^{-3}\left(\ln\frac{\lambda}{m^5}\right)^2$$