Relativistic fluid dynamics with spin and evolution of spin polarization in a Bjorken flow background

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Rotation and Polarization

Barnett Effect

S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)

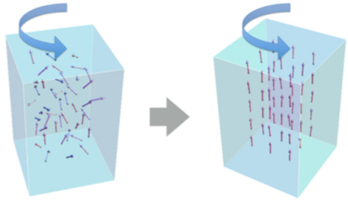

Figure: Mechanical rotation of an unmagnetized metallic object induces magnetization, an effective magnetic field emerges.

$$B_{\Omega} = \Omega/\rho$$

Einstein-de Haas Effect

A. Einstein and W. de Haas, Deutsche Physikalische Gesellschaft, Verhandlungen 17, 152 (1915)

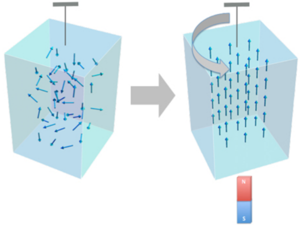

Figure: Application of magnetic field on an unmagnetized metallic object induces magnetization, body start rotating (mechanical angular momentum emerges)

Case of Heavy ion collision experiment

Global angular momentum $J \approx 10^4 \hbar$ (RHIC Au-Au 200 GeV, b=2.5 fm) [F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C77, 024906 (2008)].

Global rotation of the matter created in the non-central collisions can induce spin polarization, similar to magnetomechanical Barnett effect and Einstein and de Haas effect.

Emerging particle are expected to be globally polarized with their spins on average pointing along the system angular momentum.

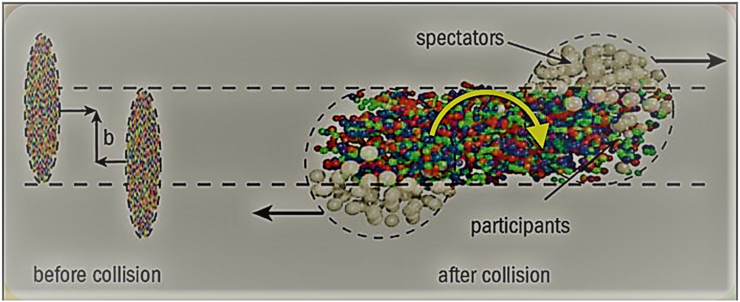

Figure: Geometry of a non-central heavy ion collision

Global Λ-polarization in RHIC experiment

The average polarization $\bar{P}_H$ (where $H = \Lambda$ or $\bar{\Lambda}$) from $20 - 50\%$ central Au+Au collisions [L. Adamczyk *et al.* (STAR), Nature 548 (2017) 62-65, arXiv:1701.06657 [nucl-ex]].

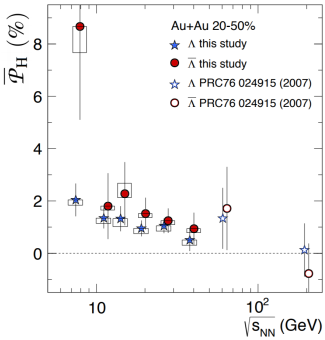

Figure: The average polarization versus collision energy

Present phenomenological prescription used to describe the data make use of hydrodynamic framework which deals with the spin polarization of particles at freeze-out. The main idea is to identify the thermal vorticity with the spin polarization tensor $\omega_{\mu\nu}$ and then to obtain the results for spin polarization.

[F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, Phys. Rev. C 95, 054902 (2017)] **.**

Note that the fact that thermal vorticity and spin polarization are same holds in global equilibrium if energy momentum tensor $T^{\mu\nu}$ is asymmetric.

[D. Zubarev, Nonequilibrium Statistical Thermodynamics (Springer, 1974); F. Becattini, Phys. Rev. Lett. 108, 244502 (2012)]

For symmetric $T^{\mu\nu}$ thermal vorticity and spin polarization tensor are independent.

[W. Florkowski, AK, and R. Ryblewski, Phys. Rev. C 98, 044906 (2018)]

$\Longrightarrow$ Talk by Wojtek

A natural framework for dealing simultaneously with polarization and vorticity would be relativistic hydrodynamics with spin.

In this talk

We use relativistic hydrodynamics with spin to determine the space-time evolution of the spin polarization in a boost-invariant and transversely homogeneous background.

In our approach we use the forms of the energy-momentum and spin tensors proposed by de Groot, van Leeuwen, and van Weert.

Relativistic fluid dynamics with spin

Conservation of charge:

$$\partial_\alpha N^\alpha(x) = 0,$$

$$N^\alpha = nU^\alpha, \quad n = 4\,\sinh(\xi)\, n_{(0)}(T).$$

The quantity $n_{(0)}(T)$ defines the number density of spinless and neutral Boltzmann particles,

$$n_{(0)}(T) = \langle p \cdot U \rangle_0\,, \quad \text{where} \quad \langle \cdots \rangle_0 \equiv \int dP\, (\cdots)\, e^{-\beta \cdot p}.$$

$$= \frac{1}{2\pi^2}\, T^3\, \hat{m}^2 K_2\,(\hat{m})\,,$$

$\hat{m} \equiv m/T$ is the ratio of the particle mass and the temperature, and $K_2\,(\hat{m})$ denotes the modified Bessel function.

The variable ξ is the ratio of the chemical potential μ and the temperature, $\xi = \mu/T$.

The factor $4\,\sinh(\xi) = 2\left(e^{\xi} - e^{-\xi}\right)$ accounts for spin degeneracy and presence of both particles and antiparticles in the system. The variable ξ is the ratio of the chemical potential μ and the temperature, $\xi = \mu/T$.

[W. Florkowski, B. Friman, A. Jaiswal, and E. Speranza, Phys. Rev. C97, 041901 (2018); W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, and E. Speranza, Phys. Rev. D97, 116017 (2018); W. Florkowski, AK, and R. Ryblewski, Phys. Rev. C 98, 044906 (2018)]

Relativistic fluid dynamics with spin

Conservation of energy and linear momentum:

$$\partial_\alpha T^{\alpha\beta}_{\rm GLW}(x) = 0,$$

$$T^{\alpha\beta}_{\rm GLW} = (\varepsilon + P) U^\alpha U^\beta - P g^{\alpha\beta}$$

Energy density and pressure is given by,

$$\varepsilon = 4\cosh(\xi)\,\varepsilon_{(0)}(T)$$
$$P = 4\cosh(\xi)\,P_{(0)}(T).$$

In analogy to the density $n_{(0)}(T)$, the auxiliary quantities $\varepsilon_{(0)}(T)$ and $P_{(0)}(T)$ are defined as $\varepsilon_{(0)}(T) = \langle (p\cdot U)^2\rangle_0$ and $P_{(0)}(T) = -(1/3)\langle p\cdot p - (p\cdot U)^2\rangle_0$.
For an ideal relativistic gas of classical massive particles one finds

$$\varepsilon_{(0)}(T) = \frac{g}{2\pi^2} T^4\, \hat{m}^2 \Big[3K_2(\hat{m}) + \hat{m}K_1(\hat{m})\Big],$$
$$P_{(0)}(T) = T\, n_{(0)}(T).$$

At this point we have five equations for five unknown functions: ξ, T, and three independent components of U^μ.

[W. Florkowski, AK, and R. Ryblewski, Phys. Rev. C 98, 044906 (2018)]

Relativistic fluid dynamics with spin

Conservation of total angular momentum

$$\partial_\mu J^{\mu,\alpha\beta}(x) = 0, \qquad J^{\mu,\alpha\beta}(x) = -J^{\mu,\beta\alpha}(x) \qquad \text{6 equations}$$

Total angular momentum is the sum of orbital and spin parts:

$$J^{\mu,\alpha\beta}(x) = L^{\mu,\alpha\beta}(x) + S^{\mu,\alpha\beta}(x),$$

$$L^{\mu,\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x),$$

Conservation of energy momentum and total angular momentum implies

$$\partial_\mu T^{\mu\nu}(x) = 0, \quad \partial_\lambda J^{\lambda,\mu\nu}(x) = 0, \Rightarrow \partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x).$$

Thus spin tensor $\hat{S}^{\mu,\alpha\beta}(x)$ is conserved in GLW formulation.
$\Longrightarrow$ Wojtek's talk for different forms of spin tensors and the connection between them.

Relativistic fluid dynamics with spin

Conservation of Spin:

$$\partial_\alpha S^{\alpha,\beta\gamma}_{\rm GLW}(x) \quad = \quad 0,$$

The GLW spin tensor is given by

$$S^{\alpha,\beta\gamma}_{\rm GLW} = \cosh(\xi) n_{(0)}(T) U^\alpha \omega^{\beta\gamma} + \frac{2\cosh(\xi)}{m^2} S^{\alpha\beta\gamma}_{\Delta \rm GLW}.$$

Here, $\omega^{\beta\gamma}$ is the spin polarization tensor.
The auxiliary tensor $S^{\alpha,\beta\gamma}_{\Delta \rm GLW}$ is defined as

$$S^{\alpha,\beta\gamma}_{\Delta \rm GLW} \quad = \quad \mathcal{A}\, U^\alpha U^\delta U^{[\beta} \omega^{\gamma]}{}_\delta + \mathcal{B} \left(U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]}{}_\delta + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]}{}_\delta + U^\delta \Delta^{\alpha[\beta} \omega^{\gamma]}{}_\delta \right),$$

$$\mathcal{B} = -T \left(\varepsilon_{(0)}(T) + P_{(0)}(T) \right), \quad \mathcal{A} = T \left[3\varepsilon_{(0)}(T) + \left(3 + \frac{m^2}{T^2} \right) P_{(0)}(T) \right] = -3\mathcal{B} + \frac{m^2}{T} P_{(0)}(T).$$

[W. Florkowski, AK, and R. Ryblewski, Phys. Rev. C 98, 044906 (2018)]

Relativistic fluid dynamics with spin

Spin Polarization tensor:

The spin polarization tensor $\omega_{\mu\nu}$ is antisymmetric and can be decomposed in terms of four-vectors κ^μ and ω^μ,

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta,$$

$$\kappa \cdot U = 0, \quad \omega \cdot U = 0.$$

Using above conditions we can write

$$\kappa_\mu = \omega_{\mu\alpha} U^\alpha, \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu\alpha\beta\gamma} \omega^{\alpha\beta} U^\gamma.$$

Boost-invariant flow

Basis for transversely homogeneous longitudinal expansion

$$
\begin{aligned}
U^\alpha &= \frac{1}{\tau}(t,0,0,z) = (\cosh\eta, 0, 0, \sinh\eta)\,, \\
X^\alpha &= (0,1,0,0)\,, \\
Y^\alpha &= (0,0,1,0)\,, \\
Z^\alpha &= \frac{1}{\tau}(z,0,0,t) = (\sinh\eta, 0, 0, \cosh\eta)\,.
\end{aligned}
$$

Here $\tau = \sqrt{t^2 - z^2}$ is the longitudinal proper time, while $\eta = \frac{1}{2}\ln((t+z)/(t-z))$ is the space-time rapidity.

$$U \cdot U = 1$$

$$
\begin{aligned}
X \cdot X &= Y \cdot Y = Z \cdot Z = -1, \\
X \cdot U &= Y \cdot U = Z \cdot U = 0, \\
X \cdot Y &= Y \cdot Z = Z \cdot X = 0.
\end{aligned}
$$

Boost-invariant flow and spin polarization tensor

One can introduce the following representation of the vectors κ^μ and ω^μ

$$\begin{aligned}\kappa^\alpha &= C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \\ \omega^\alpha &= C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.\end{aligned}$$

Here, the scalar coefficients $C_{\kappa X}$, $C_{\kappa Y}$, $C_{\kappa Z}$, $C_{\omega X}$, $C_{\omega Y}$, and $C_{\omega Z}$ are functions of the proper time τ only.
It is important to note that due to the orthogonality connditions $\kappa \cdot U = 0, \ \omega \cdot U = 0$, there are no terms proportional U^μ.
The following boost-invariant expression for the spin polarization tensor $\omega_{\mu\nu}$ can be obtained by using above decomposition of vectors κ^μ and ω^μ,

$$\begin{aligned}\omega_{\mu\nu} &= C_{\kappa Z}(Z_\mu U_\nu - Z_\nu U_\mu) + C_{\kappa X}(X_\mu U_\nu - X_\nu U_\mu) + C_{\kappa Y}(Y_\mu U_\nu - Y_\nu U_\mu) \\ &\quad + \epsilon_{\mu\nu\alpha\beta} U_\alpha (C_{\omega Z} Z^\beta + C_{\omega X} X^\beta + C_{\omega Y} Y^\beta).\end{aligned}$$

In the plane $z = 0$ we find

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & C_{\kappa X} & C_{\kappa Y} & C_{\kappa Z} \\ -C_{\kappa X} & 0 & -C_{\omega Z} & C_{\omega Y} \\ -C_{\kappa Y} & C_{\omega Z} & 0 & -C_{\omega X} \\ -C_{\kappa Z} & -C_{\omega Y} & C_{\omega X} & 0 \end{bmatrix}$$

Boost-invariant form of fluid dynamics with spin

Charge conservation:

$$\dot{n} + \frac{n}{\tau} = 0.$$

Energy-momentum conservation:

$$\dot{\varepsilon} + \frac{(\varepsilon + P)}{\tau} = 0.$$

Spin conservation:

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

$$\begin{aligned}
\mathcal{L}(\tau) &= \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3, \\
\mathcal{P}(\tau) &= \mathcal{A}_1, \\
\mathcal{Q}_1(\tau) &= -\left(\dot{\mathcal{A}}_1 - \frac{1}{2}\dot{\mathcal{A}}_2 - \dot{\mathcal{A}}_3 + \frac{\mathcal{A}_1}{\tau} - \frac{1}{2}\frac{\mathcal{A}_2}{\tau} - \frac{1}{2}\frac{\mathcal{A}_3}{\tau}\right), \\
\mathcal{Q}_2(\tau) &= -\left(\dot{\mathcal{A}}_1 - \frac{1}{2}\dot{\mathcal{A}}_2 - \dot{\mathcal{A}}_3 + \frac{\mathcal{A}_1}{\tau} - \frac{1}{2}\frac{\mathcal{A}_2}{\tau} - \frac{\mathcal{A}_3}{\tau}\right), \\
\mathcal{R}_1(\tau) &= -\left(\dot{\mathcal{A}}_1 + \frac{\mathcal{A}_1}{\tau} - \frac{1}{2}\frac{\mathcal{A}_3}{\tau}\right), \\
\mathcal{R}_2(\tau) &= -\left(\dot{\mathcal{A}}_1 + \frac{\mathcal{A}_1}{\tau}\right).
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_1 &= \mathcal{C}\left(n_{(0)} - \frac{2\mathcal{B}}{m^2}\right), \\
\mathcal{A}_2 &= \frac{2\mathcal{C}}{m^2}(\mathcal{A} - 3\mathcal{B}), \\
\mathcal{A}_3 &= \frac{2\mathcal{C}\mathcal{B}}{m^2},
\end{aligned}$$

$$\mathcal{C} = \cosh(\xi).$$

Initial baryon chemical potential $\mu_0 = 800$ MeV, initial temperature $T_0 = 155$ MeV. The particle mass, $m = 1116$ MeV. The initial proper time is $\tau_0 = 1$ fm and final time $\tau_f = 10$ fm.
$C_{\kappa X0} = C_{\kappa Z0} = C_{\omega X0} = C_{\omega Z0} = 0.1$.

Numerical Solutions

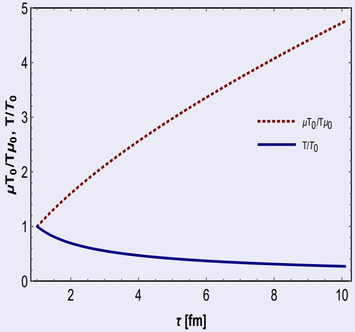

Figure: Proper-time dependence of T divided by its initial value T_0 and the ratio of baryon chemical potential μ and temperature T rescaled by the initial ratio μ_0/T_0.

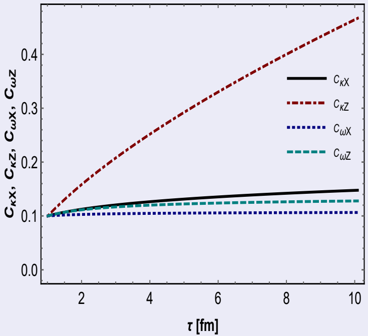

Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$.

Relaxation of towards thermal vorticity

If the spin polarization relaxes towards thermal vorticity, it mean equations of each C-coefficient must be modified similar to the following

$$\dot{C}_{\kappa X} = \frac{\mathcal{Q}_1(\tau)}{\mathcal{L}(\tau)} C_{\kappa X} + \left(\frac{\bar{C}_{\kappa X} - C_{\kappa X}}{\tau_\omega} \right)$$

Thermal vorticity is defined as

$$\varpi_{\mu\nu}(x) = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$$

Since, $\beta_\mu = \frac{U^\mu}{T}$, we will have $\bar{C}_{\kappa X} = \bar{C}_{\kappa Y} = \bar{C}_{\kappa Z} = \bar{C}_{\omega X} = \bar{C}_{\omega Y} = \bar{C}_{\omega Y} = 0$

$$\dot{C}_{\kappa X} = \frac{\mathcal{Q}_1(\tau)}{\mathcal{L}(\tau)} C_{\kappa X} - \frac{C_{\kappa X}}{\tau_\omega}$$

[F. Becattini, W. Florkowski, and E. Speranza, Phys.Lett. B789, 419 (2019)]
[Giorgio Torrieri, arXiv:1810.12468[hep-th]]

Numerical Solutions

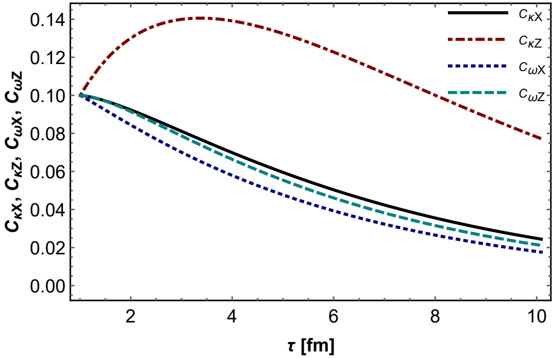

Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$ at $\tau_\omega = 5$ fm.

Spin polarization of particles at freeze-out

The average spin polarization is given by

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

where for GLW form of spin tensor, total Pauli-Lubański (PL) vector for particles with momentum p is given by,

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta \cdot p} \, \tilde{\omega}_{\mu\beta} p^\beta .$$

while,

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4\cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta \cdot p} .$$

We can carryout the integration very easily by assuming that freeze-out takes place at a constant value of the proper time τ, in this case

$$\Delta\Sigma_\lambda = U_\lambda dx dy \tau d\eta.$$

We can parametrize of the particle four-momentum p^λ in terms of the transverse mass m_T and rapidity y_p,

$$p^\lambda = (m_T \cosh(y_p), p_x, p_y, m_T \sinh(y_p))$$

$$\Delta\Sigma_\lambda p^\lambda = m_T \cosh(y_p - \eta) \, dx dy \tau d\eta.$$

Boost to local rest frame (LRF) of the particle

In the local rest frame of the particle, polarization vector $\langle \pi^{\star}_{\mu} \rangle$ can be obtained by using the canonical boost [E. Leader, "Spin in Particle Physics," Cambridge University Press (2001)]

$$\Lambda^{\mu}_{\nu}(-\mathbf{v_p}) = \begin{bmatrix} \frac{E_p}{m} & -\frac{p_x}{m} & -\frac{p_y}{m} & -\frac{p_z}{m} \\ -\frac{p_x}{m} & 1+\alpha_p p_x^2 & \alpha_p p_x p_y & \alpha_p p_x p_z \\ -\frac{p_y}{m} & \alpha_p p_y p_x & 1+\alpha_p p_y^2 & \alpha_p p_y p_z \\ -\frac{p_z}{m} & \alpha_p p_z p_x & \alpha_p p_z p_y & 1+\alpha_p p_z^2 \end{bmatrix}$$

where, $\mathbf{v_p} = \boldsymbol{p}/E_p$ and $\alpha_p = 1/(m(E_p + m))$.

$$\langle \pi^{\star}_{\mu} \rangle = -\frac{1}{8mK_1(\hat{m}_T)} \begin{pmatrix} 0 \\ \left(\left(\frac{\sinh(y_p)p_x}{m_T\cosh(y_p)+m}\right)\left((K_0(\hat{m}_T)+K_2(\hat{m}_T))(C_{\kappa X}p_y - C_{\kappa Y}p_x) + 2C_{\omega Z}m_T K_1(\hat{m}_T)\right) + \frac{(K_0(\hat{m}_T)+K_2(\hat{m}_T))\cosh(y_p)p_x(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T\cosh(y_p)+m} + 2C_{\kappa Z}p_y K_1(\hat{m}_T) - (K_0(\hat{m}_T)+K_2(\hat{m}_T))C_{\omega X}m_T\right) \\ \left(\left(\frac{\sinh(y_p)p_y}{m_T\cosh(y_p)+m}\right)\left((K_0(\hat{m}_T)+K_2(\hat{m}_T))(C_{\kappa X}p_y - C_{\kappa Y}p_x) + 2C_{\omega Z}m_T K_1(\hat{m}_T)\right) + \frac{(K_0(\hat{m}_T)+K_2(\hat{m}_T))\cosh(y_p)p_y(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T\cosh(y_p)+m} - 2C_{\kappa Z}p_x K_1(\hat{m}_T) - (K_0(\hat{m}_T)+K_2(\hat{m}_T))C_{\omega Y}m_T\right) \\ \left(-\left(\frac{m\cosh(y_p)+m_T}{m_T\cosh(y_p)+m}\right)\left((K_0(\hat{m}_T)+K_2(\hat{m}_T))(C_{\kappa X}p_y - C_{\kappa Y}p_x) + 2C_{\omega Z}m_T K_1(\hat{m}_T)\right) - \frac{m(K_0(\hat{m}_T)+K_2(\hat{m}_T))\sinh(y_p)(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T\cosh(y_p)+m}\right) \end{pmatrix}$$

Here, $\hat{m}_T = \frac{m_T}{T}$.

Approxmimate expression for spin polarization

We consider particles with $y_p = 0$.
Mass of the Lambda hyperon is much larger than temperature *i.e.*, $\hat{m}_T \gg 1$.
In this case, we may use the approximation $\left(K_0\left(\hat{m}_T\right) + K_2\left(\hat{m}_T\right)\right)/K_1\left(\hat{m}_T\right) \approx 2$.

$$\langle \pi^\star_\mu \rangle = -\frac{1}{4m}\begin{pmatrix} 0 \\ \frac{p_x(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T+m} + C_{\kappa Z}p_y - C_{\omega X}m_T \\ \frac{p_y(C_{\omega X}p_x + C_{\omega Y}p_y)}{m_T+m} - C_{\kappa Z}p_x - C_{\omega Y}m_T \\ -(C_{\kappa X}p_y - C_{\kappa Y}p_x) - C_{\omega Z}m_T \end{pmatrix}.$$

$\langle \boldsymbol{\pi}^\star \rangle = (\langle \pi^{\star 1}\rangle, \langle \pi^{\star 2}\rangle, \langle \pi^{\star 3}\rangle) \equiv (\langle \pi^\star_x\rangle, \langle \pi^\star_y\rangle, \langle \pi^\star_z\rangle)$,
If we write coefficient functions C as,

$$\begin{aligned} \boldsymbol{C}_\kappa &= (C_{\kappa X}, C_{\kappa Y}, C_{\kappa Z}), \\ \boldsymbol{C}_\omega &= (C_{\omega X}, C_{\omega Y}, C_{\omega Z}). \end{aligned}$$

We can write,

$$\boxed{\langle \boldsymbol{\pi}^* \rangle = -\frac{1}{4m}\left[E_p \boldsymbol{C}_\omega - \boldsymbol{p} \times \boldsymbol{C}_\kappa - \frac{\boldsymbol{p}\cdot\boldsymbol{C}_\omega}{E_p + m}\boldsymbol{p}\right]}$$

where, $\boldsymbol{p} = (p_x, p_y, 0)$

Momentum dependence of polarization

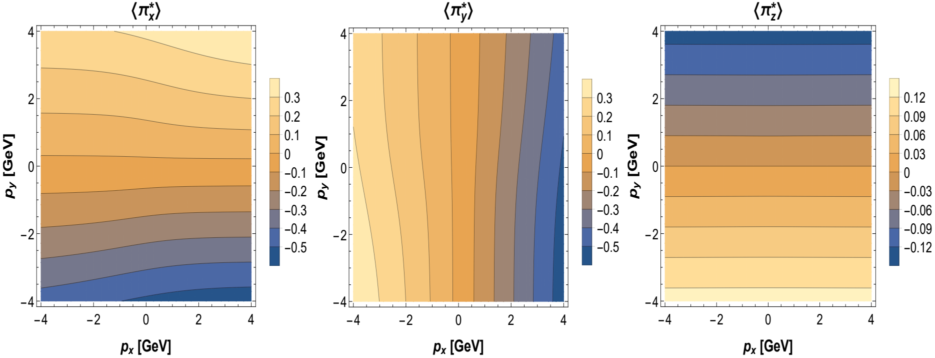

Figure: Components of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0.1, 0, 0.1)$, and $\boldsymbol{C}_{\omega,0} = (0.1, 0.1, 0)$ for $y_p = 0$.

Momentum dependence of polarization

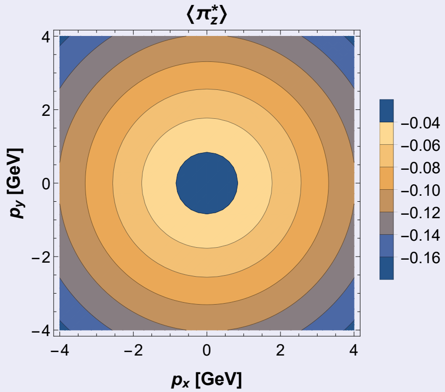

Figure: Longitudinal component of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0, 0, 0)$, and $\boldsymbol{C}_{\omega,0} = (0, 0, 0.1)$ for $y_p = 0$. The transverse components are in this case zero.

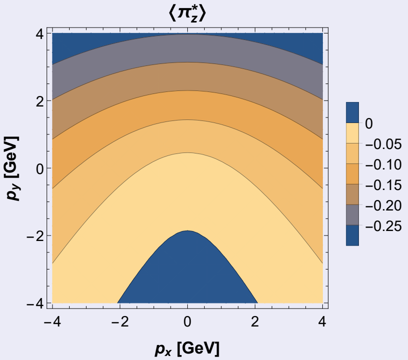

Figure: Longitudinal component of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0.1, 0, 0.1)$, and $\boldsymbol{C}_{\omega,0} = (0.1, 0.1, 0.1)$ for $y_p = 0$. The transverse components are in this case are same as shown in previous slide.

Freeze-out time dependence of spin polarization

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Figure: Propertime evolution of transverse components of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0.1, 0, 0.1)$, and $\boldsymbol{C}_{\omega,0} = (0.1, 0.1, 0)$ for $y_p = 0$.

Evolution of Spin polarization

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Figure: Propertime evolution of longitudinal component of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0.1, 0, 0.1)$, and $\boldsymbol{C}_{\omega,0} = (0.1, 0.1, 0)$ for $y_p = 0$.

Summary

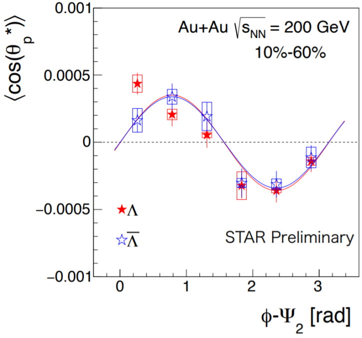
- We have discussed hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensor.
- Using the simple transversely homogeneous longitudinal expansion we show that hydrodynamic framework with spin can be used to determine the spin polarization observed in heavy ion collisions.
- Numerical results obtained by us can not be compared with the experimental results [This is because we have a simple 1+0 dimensional expansion].

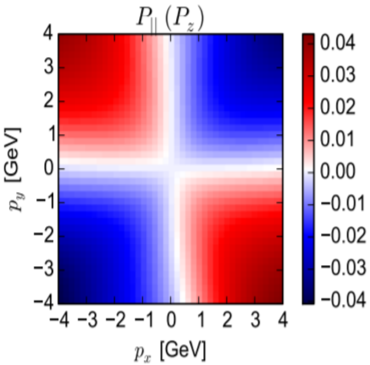
Outlook: Study of the polarization to a more realistic scenario *i.e.* for 3+1 dimensional expansion (Work in progress).

Note: With full 3+1 dimensional simulation of hydrodynamics with spin we hope to resolve the sign problem (sign of quadrupole structure of spin polarization).

THANK YOU

Polarization using the thermal vorticity model





Global and local thermodynamic equilibrium for particles with spin from quantum mechanics

The density operator [D. Zubarev, Nonequilibrium Statistical Thermodynamics (Springer, 1974); F. Becattini, Phys. Rev. Lett. 108, 244502 (2012)],

$$\hat{\rho}(t) = \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)b_\nu(x) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)\right].$$

Here $d^3\Sigma_\mu$ is an element of a space-like, three-dimensional hypersurface Σ_μ. We can take it as, $d^3\Sigma_\mu = (dV, 0, 0, 0)$.

The operators $\hat{T}^{\mu\nu}(x)$, $\hat{J}^{\mu,\alpha\beta}(x)$ and $\hat{N}^\mu(x)$ are the energy-momentum, angular momentum and charge operators respectively.

In global thermodynamic equilibrium the operator $\hat{\rho}(t)$ should be independent of time.

$$\partial_\mu\left(\hat{T}^{\mu\nu}(x)b_\nu(x) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)$$
$$= \hat{T}^{\mu\nu}(x)\left(\partial_\mu b_\nu(x)\right) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\left(\partial_\mu\omega_{\alpha\beta}(x)\right) - \hat{N}^\mu(x)\partial_\mu\xi(x) = 0.$$

From above equation we can conclude that $\omega_{\alpha\beta} = \omega^0_{\alpha\beta}$, $\xi = \xi^0$, But

For asymmetric energy momentum tensor, $b_\nu = b^0_\nu$.

For symmetric energy momentum tensor, $b_\nu = b^0_\nu + \delta\omega^0_{\nu\rho}x^\rho$.

Global equilibrium; particle with spin

Total angular momentum

$$\hat{J}^{\mu,\alpha\beta}(x) = \hat{L}^{\mu,\alpha\beta}(x) + \hat{S}^{\mu,\alpha\beta}(x).$$

Using above equation, we can write two cases discussed above can be expressed by a single form of the density operator

$$\hat{\rho}_{\rm EQ} \quad = \quad \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)\beta_\nu(x) - \frac{1}{2}\hat{S}^{\mu,\alpha\beta}(x)\omega^0_{\alpha\beta} - \hat{N}^\mu(x)\xi^0\right)\right].$$

For asymmetric energy-momentum tensor $\beta_\mu(x) = b^0_\mu + \omega^0_{\mu\gamma}x^\gamma$.
$\beta_\mu(x)$ is a Killing vector, $\omega_{\mu\gamma} = \omega^0_{\mu\gamma} = \varpi_{\mu\nu}$.
global equilibrium —
For symmetric energy-momentum tensor $\beta_\mu(x) = b^0_\mu + (\delta\omega^0_{\mu\gamma} + \omega^0_{\mu\gamma})x^\gamma$.
$\beta_\mu(x)$ is again a Killing vector, $\omega_{\mu\gamma} = \omega^0_{\mu\gamma} \neq \varpi_{\mu\nu}(= \delta\omega^0_{\mu\gamma} + \omega^0_{\mu\gamma})$.
extended global equilibrium —

Local thermodynamic equilibrium; particle with spin

We define the statistical operator for local equilibrium by the same form as

$$\hat{\rho}_{\rm eq} \quad = \quad \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)\beta_\nu(x) - \frac{1}{2}\hat{S}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)\right].$$

We allow for arbitrary form of $\beta_\mu(x)$ [not a killing vector] and $\xi = \xi(x)$ and two cases for $\omega_{\mu\nu}$.

$\omega_{\mu\nu} = \varpi_{\mu\nu}$.

local equilibrium —

$\omega_{\mu\nu} \neq \varpi_{\mu\nu}$.

extended local equilibrium —