Relativistic hydrodynamics for polarized media and the classical treatment of spin

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W. Florkowski (Jagiellonian U. & IFJ PAN)

reference:
arXiv: 1811.04409

From QCD matter to hadrons
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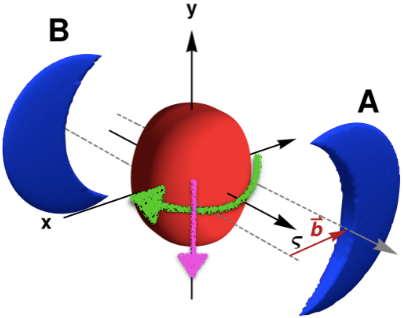
Motivation

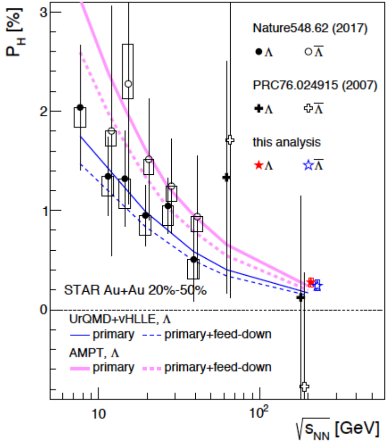
- Non-central HIC at low/intermediate energies create systems with large global angular momenta $\sim 10^4\hbar$
  F. Becattini, F. Piccinini, J. Rizzo, Phys.Rev. C77 (2008) 024906, arXiv:0711.1253

- Global rotation of the system may produce a spin polarization of the matter (similarly to Einstein-de Haas and Barnett effects)
  S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)
  A. Einstein and W. de Haas, Deutsche Physikalische Gesellschaft, Verhandlungen 17, 152 (1915)



Motivation

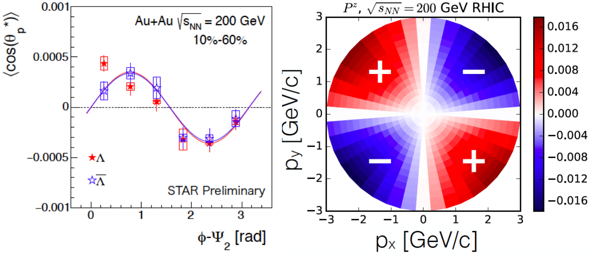
- The first positive measurement of average polarization of Λ hyperons within BES @STAR
  STAR Collaboration,
  L. Adamczyk et al. Nature 548 (2017) 62-65,
  arXiv:1701.06657
- Initiated vast theoretical studies analyzing the spin polarization and vorticity formation in heavy-ion collisions
- Present phenomenological prescription used to describe the data employs the fact that spin polarization tensor is equal to thermal vorticity (which holds in global equilibrium)
  UrQMD+vHLLE: I. Karpenko, F. Becattini, Eur. Phys. J. C 77, 213 (2017), arXiv:1610.04717
  AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, Phys. Rev. C 96, 054908 (2017), arXiv:1704.01507



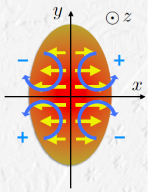
STAR Collaboration, Phys.Rev. C98 (2018) 014910, arXiv:1805.04400

Motivation

- Present works are limited to the calculation of polarization at freeze-out and say little about the changes of spin polarization during HIC
- So far the HIC evolution is best described with the help of relativistic hydrodynamics, it is thus of great interest to include the polarization effects in this approach
- The quadrupole structure of longitudinal polarization is not described within current approach - oposite sign observed
- Some explanation of the sign of polarization based on BW model possible
  S. Voloshin, EPJ Web Conf. 17 (2018) 10700, arXiv:1710.08934



(LEFT) STAR Collaboration, preliminary (RIGHT) F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302, arXiv:1707.07984



S. Voloshin, SQM2017

Motivation

- The relativistic hydrodynamics with spin was formulated very recently
  W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C97 (4) (2018) 041901. arXiv:1705.00587
  W. Florkowski, B. Friman, A. Jaiswal, R. R., E. Speranza, Phys. Rev. D97 (11) (2018) 116017. arXiv:1712.07676

- The latter was based on a particular choice of the forms of energy-momentum and spin tensors. Recent works clarified the use of de Groot - van Leeuwen - van Weert (GLW) forms of these tensors
  F. Becattini, W. Florkowski, E. Speranza, Phys. Lett. B. 789, (2019) 419-425 arXiv:1807.10994,
  W. Florkowski, A. Kumar, R. R., Phys. Rev. C98 (2018) 044906. arXiv:1806.02616

  $\rightarrow$ **talk by W. Florkowski**

- Realistic applications of the GLW-based hydrodynamics with spin to the HIC data in the small polarization limit is the work in progress
  W. Florkowski, A. Kumar, R. R. and R. Singh, forthcoming

  $\rightarrow$ **talk by A. Kumar**

- However in this approach for large momenta of particles one encounters problems with normalization of the mean polarization three-vector

$$\pi_*^0 = 0 \quad \boldsymbol{\pi}_* = -\frac{1}{4}\boldsymbol{P} \qquad\qquad \boldsymbol{P} = \frac{1}{m}\left[E_p\,\boldsymbol{b} - \boldsymbol{p}\times\boldsymbol{e} - \frac{\boldsymbol{p}\cdot\boldsymbol{b}}{E_p + m}\boldsymbol{p}\right] = \boldsymbol{b}_*$$

Strategy

We (in parallel) study the polarization effects in the framework which treats spin classically in order to improve the quantum description based on the spin density matrices so that it is not restricted to small values of the spin chemical potential and small particle momenta

Internal angular momentum tensor

In the classical treatment of spin one introduces internal angular momentum tensor
M. Mathisson, Acta Phys. Polon. 6 (1937) 163–2900

$$s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta} p_\gamma s_\delta$$

It satisfies the Frenkel (or Weyssenhoff) condition

$$p_\alpha s^{\alpha\beta} = 0$$

Since $s \cdot p = 0$ one can write

$$s^\alpha = \frac{1}{2m}\epsilon^{\alpha\beta\gamma\delta} p_\beta s_{\gamma\delta}$$

In PRF, where $p^\mu = (m, 0, 0, 0)$, one has $s^\alpha = (0, \boldsymbol{s}_*)$.
For spin-$^1/_2$ particles the length of $\boldsymbol{s}_*$ is given by the value of the Casimir operator,

$$|\boldsymbol{s}_*|^2 = \mathfrak{s}^2 = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4}$$

Angular momentum conservation

To construct the equilibrium function we have to identify the collisional invariants of the Boltzmann equation. In addition to four-momentum and conserved charges one can include the total angular momentum

$$j_{\alpha\beta} = l_{\alpha\beta} + s_{\alpha\beta} = x_\alpha p_\beta - x_\beta p_\alpha + s_{\alpha\beta}$$

The locality of the standard Boltzmann equation suggests that the orbital part can be eliminated, and the spin part can be considered separately.
For elastic binary collisions of particles 1 and 2 going to $1'$ and $2'$, this suggests that
C. G. van Weert, Henkes- Holland N.V. – Haarlem, 1970.

$$s_1^{\alpha\beta} + s_2^{\alpha\beta} = s_{1'}^{\alpha\beta} + s_{2'}^{\alpha\beta}$$

These equations admit two types of simple solutions: either the sum of two spin three-vectors or their difference (before and after the collision) vanishes.
They may be interpreted as collisions in the spin singlet and triplet states.

If the collision integral allows for processes such as discussed above, the spin angular momentum conservation law should be included among the other conservation laws.

Non-local effects ?

Locality of the collision kernel suggests that we cannot describe the Barnett effect - the fact realized in 1966.

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Kinetic Theory for a Dilute Gas of Particles with Spin

S. Hess and L. Waldmann

Institut für Theoretische Physik der Universität Erlangen-Nürnberg, Erlangen

(Z. Naturforschg. **21 a**, 1529—1546 [1966]; received 6 April 1966)

conserved. There is another effect which we cannot describe with a local collision operator (even in thermal equilibrium): the orientation of the spin by a local or uniform rotation of the system (Barnett effect).

Further developments require the use of non-local collisional kernels.

A. Jaiswal, R. S. Bhalerao, S. Pal, Phys. Lett. B720 (2013) 347–351

Spin-dependent distribution function and invariant measure

We introduce a spin-dependent equilibrium distribution functions for particles and antiparticles

$$f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x)s^{\alpha\beta}\right)$$

with $\beta^\mu = u^\mu/T$ and $\xi^\mu = \mu/T$.

Different orientations of spin can be integrated out with the help of a covariant measure

$$\int dS \ldots = \frac{m}{\pi\,\mathfrak{s}} \int d^4s\, \delta(s\cdot s + \mathfrak{s}^2)\, \delta(p\cdot s) \ldots$$

The prefactor $m/(\pi\,\mathfrak{s})$ is chosen to obtain the normalization

$$\int dS = \frac{m}{\pi\,\mathfrak{s}} \int d^4s\, \delta(s\cdot s + \mathfrak{s}^2)\, \delta(p\cdot s) = 2$$

Charge current

The charge current is obtained from the generalization of the standard definition

$$N^\mu_{\rm eq} = \int dP \int dS \; p^\mu \left[f^+_{\rm eq}(x,p,s) - f^-_{\rm eq}(x,p,s) \right]$$

which after using the forms of the equilibrium functions leads to

$$N^\mu_{\rm eq} = 2\sinh(\xi) \int dP \, p^\mu e^{-p\cdot\beta} \int dS \, \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right)$$

For small values of ω one gets

$$\begin{aligned} N^\mu_{\rm eq} &= 2\sinh(\xi) \int dP \, p^\mu \, e^{-p\cdot\beta} \int dS \left[1 + \frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right] \\ &= 4\sinh(\xi) \int dP \, p^\mu \, e^{-p\cdot\beta} \end{aligned}$$

Agrees up to the first order in ω with the charge current obtained by taking zeroth moment of the kinetic equation for scalar coefficient function of the Wigner function decomposition obtained at NLO from the semi-classical expansion of the kinetic equation in powers of $\hbar$

Energy-momentum tensor (I)

The energy-momentum tensor is given by

$$T^{\mu\nu}_{\rm eq} = \int dP \int dS \; p^\mu p^\nu \left[f^+_{\rm eq}(x,p,s) + f^-_{\rm eq}(x,p,s) \right]$$

which leads to

$$T^{\mu\nu}_{\rm eq} \quad = \quad 2\cosh(\xi) \int dP \, p^\mu p^\nu \, e^{-p\cdot\beta} \int dS \; \exp\left(\frac{1}{2}\omega_{\alpha\beta} s^{\alpha\beta}\right)$$

For small values of ω one gets

$$T^{\mu\nu}_{\rm eq} \quad = \quad 4\cosh(\xi) \int dP \, p^\mu p^\nu \, e^{-p\cdot\beta}$$

Agrees up to the first order in ω with the energy-momentum tensor obtained by taking first moment of the kinetic equation for scalar coefficient function of the Wigner function decomposition obtained at NLO from the semi-classical expansion of the kinetic equation in powers of $\hbar$ and the GLW energy-momentum tensor

Energy-momentum tensor (II)

$$T^{\mu\nu}_{\rm eq} = 2\cosh(\xi) \int dP\, p^\mu p^\nu\, e^{-p\cdot\beta} \int dS\, \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right)$$

Using definition of the dual polarization tensor $\tilde{\omega}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\,\omega^{\alpha\beta}$ one may write

$$\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta} = \frac{p_\gamma}{m}\,\tilde{\omega}^{\gamma\delta}s_\delta \qquad \Rightarrow^{PRF} \qquad \frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta} = \boldsymbol{b}_* \cdot \boldsymbol{s}_* = \boldsymbol{P}\cdot\boldsymbol{s}_*$$

For arbitrary value of ω we may write

$$\int dS\, \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right) = \frac{m}{\pi\,\mathfrak{s}} \int ds_0 \int |\boldsymbol{s}_*|^2 d|\boldsymbol{s}_*| \int d\Omega\, \delta(|\boldsymbol{s}_*|^2 - \mathfrak{s}^2)\, \delta(m s_0) e^{\boldsymbol{P}\cdot\boldsymbol{s}_*}$$

Here $d\Omega = \sin(\theta)d\theta d\phi$ denotes the integration over the solid angle. One gets a result

$$\int dS\, \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right) = \int_{-1}^{+1} e^{\mathfrak{s}\,P\,x}dx = \frac{2\sinh(\mathfrak{s}P)}{\mathfrak{s}\,P} \qquad \text{with} \qquad P = |\boldsymbol{P}| = |\boldsymbol{b}_*|$$

Since P depends on momentum, the energy-momentum tensor for large values of ω induces momentum anisotropy - to be studied with the methods of anisotropic hydrodynamics

W. Florkowski, R. Ryblewski, Phys. Rev. C83 (2011) 034907. arXiv:1007.0130
M. Martinez, M. Strickland, Nucl. Phys. A848 (2010) 183–197. arXiv:1007.0889

Spin tensor (I)

The spin tensor is defined as an expectation value of the internal angular momentum tensor,

$$S^{\lambda,\mu\nu}_{\rm eq} = \int dP \int dS \; p^\lambda \, s^{\mu\nu} \left[f^+_{\rm eq}(x,p,s) + f^-_{\rm eq}(x,p,s)\right]$$

$$S^{\lambda,\mu\nu}_{\rm eq} = 2\cosh(\xi) \int dP \, p^\lambda \, e^{-p\cdot\beta} \int dS \, s^{\mu\nu} \; \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right)$$

In the leading-order approximation in ω we find

$$\int dS \, s^{\mu\nu} \; \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right) = \frac{1}{2}\omega_{\alpha\beta} \int dS \, s^{\mu\nu}s^{\alpha\beta} = \frac{\omega_{\alpha\beta}}{2m^2}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}p_\rho p_\gamma \int dS \, s_\delta s_\sigma = \frac{2}{3m^2}\,\mathfrak{s}^2\left(m^2\omega^{\mu\nu} + 2p^\alpha p^{[\mu}\omega^{\nu]}_{\ \ \alpha}\right)$$

$$S^{\lambda,\mu\nu}_{\rm eq} = \frac{4}{3m^2}\,\mathfrak{s}^2 \cosh(\xi) \int dP \, p^\lambda \, e^{-p\cdot\beta}\left(m^2\omega^{\mu\nu} + 2p^\alpha p^{[\mu}\omega^{\nu]}_{\ \ \alpha}\right)$$

Agrees up to the first order in ω with the spin tensor obtained from the moment of the kinetic equation for axial vector coefficient function of the Wigner function obtained at NLO from the semi-classical expansion of the kinetic equation in powers of $\hbar$ and the GLW spin tensor

Spin tensor (II)

$$S_{\rm eq}^{\lambda,\mu\nu} = 2\cosh(\xi)\int dP\, p^\lambda\, e^{-p\cdot\beta}\int dS\, s^{\mu\nu}\ \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right)$$

In the case of arbitrary large ω or P one gets the result

$$\int dS\, s^{\mu\nu}\ \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right) = \frac{\chi(P\,\mathfrak{s})}{m^2}\left(m^2\omega^{\mu\nu} + 2p^\alpha p^{[\mu}\omega^{\nu]}_{\ \alpha}\right)$$

where

$$\chi(P\,\mathfrak{s}) = \frac{2\left[P\,\mathfrak{s}\cosh(P\,\mathfrak{s}) - \sinh(P\,\mathfrak{s})\right]}{P^3\,\mathfrak{s}}.$$

and

$$S_{\rm eq}^{\lambda,\mu\nu} \quad = \quad \frac{2}{m^2}\cosh(\xi)\int dP\, p^\lambda\, e^{-p\cdot\beta}\chi(P\,\mathfrak{s})\left(m^2\omega^{\mu\nu} + 2p^\alpha p^{[\mu}\omega^{\nu]}_{\ \alpha}\right)$$

Pauli-Lubański vector (I)

We define the particle number current for both particles and antiparticles as

$$\mathcal{N}^{\mu}_{\rm eq} = \int dP \int dS \; p^{\mu} \left[f^{+}_{\rm eq}(x,p,s) + f^{-}_{\rm eq}(x,p,s) \right].$$

Using this expression we obtain the momentum density of the total number of particles

$$E_p \frac{d\Delta\mathcal{N}}{d^3p} = \frac{\cosh(\xi)}{4\pi^3} \Delta\Sigma \cdot p \, e^{-p\cdot\beta} \int dS \, \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right)$$

The spin density is

$$E_p \frac{d\Delta\mathcal{S}^{\mu\nu}}{d^3p} = \frac{\cosh(\xi)}{4\pi^3} \Delta\Sigma \cdot p \, e^{-p\cdot\beta} \int dS \, s^{\mu\nu} \, \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right)$$

The phase-space density of the Pauli-Lubański (PL) vector is obtained as the ratio

$$\pi_{\mu}(x,p) = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} \frac{\int dS \, s^{\nu\alpha} \, \exp\left(\frac{1}{2}\omega_{\rho\sigma}s^{\rho\sigma}\right)}{\int dS \, \exp\left(\frac{1}{2}\omega_{\rho\sigma}s^{\rho\sigma}\right)} \frac{p^{\beta}}{m}$$

Pauli-Lubański vector (II)

For arbitrary values of the polarization one gets

$$\pi_\mu = -\mathfrak{s}\,\frac{\tilde{\omega}_{\mu\beta}}{P}\,\frac{p^\beta}{m}\,L(P\,\mathfrak{s})$$

where $L(x)$ is the Langevin function defined by the formula

$$L(x) = \coth(x) - \frac{1}{x} \qquad \text{where} \qquad L \approx 1 \quad \text{for} \quad x \gg 1 \quad \text{and} \quad L \approx \frac{x}{3} \quad \text{for} \quad x \ll 1$$

In PRF

$$\pi_*^0 = 0, \quad \boldsymbol{\pi}_* = -\mathfrak{s}\,\frac{\boldsymbol{P}}{P}\,L(P\,\mathfrak{s})$$

For small and large P we obtain two important results:

$$\boldsymbol{\pi}_* = -\mathfrak{s}\,\frac{\boldsymbol{P}}{P}, \quad |\boldsymbol{\pi}_*| = \mathfrak{s} = \sqrt{\frac{3}{4}}, \quad \text{if} \quad P \gg 1$$

The normalization of the PL vector cannot exceed the value of $\mathfrak{s}$.

$$\boldsymbol{\pi}_* = -\mathfrak{s}^2\,\frac{\boldsymbol{P}}{3}, \quad |\boldsymbol{\pi}_*| = \mathfrak{s}^2\,\frac{P}{3} = \frac{P}{4}, \quad \text{if} \quad P \ll 1$$

For small values of P the classical treatment of spin reproduces the quantum mechanical result

Entropy conservation

Classical treatment of spin allows for explicit derivation of the entropy current conservation.
We adopt the Boltzmann definition

$$H^\mu = -\int dP \int dS\, p^\mu \left[f^+_{\rm eq}\left(\ln f^+_{\rm eq} - 1\right) + f^-_{\rm eq}\left(\ln f^-_{\rm eq} - 1\right)\right]$$

Using form of $f^\pm$ and the conservation laws for energy, linear and angular momentum, and charge, we obtain

$$H^\mu = \beta_\alpha T^{\mu\alpha}_{\rm eq} - \frac{1}{2}\omega_{\alpha\beta} S^{\mu,\alpha\beta}_{\rm eq} - \xi N^\mu_{\rm eq} + \mathcal{N}^\mu_{\rm eq} \quad \Rightarrow \quad \partial_\mu H^\mu = \left(\partial_\mu \beta_\alpha\right) T^{\mu\alpha}_{\rm eq} - \frac{1}{2}\left(\partial_\mu \omega_{\alpha\beta}\right) S^{\mu,\alpha\beta}_{\rm eq} - \left(\partial_\mu \xi\right) N^\mu_{\rm eq} + \partial_\mu \mathcal{N}^\mu_{\rm eq}$$

With the help of the relation $\mathcal{N}^\mu_{\rm eq} = \frac{\cosh(\xi)}{\sinh(\xi)} N^\mu_{\rm eq}$ and the conservation of charge one can easily show that

$$\partial_\mu H^\mu = 0$$

Contributions to H^μ, connected with the polarization tensor, start with quadratic terms in ω.

If we restrict ourselves to linear terms in ω, all thermodynamic quantities become independent of ω, while the conservation of the angular momentum determines the polarization evolution in a given hydrodynamic background.

Conclusions and Summary

We show that the classical approach to spin:

- allows for arbitrarily large values of spin chemical potential leading to the anisotropic hydrodynamics framework with spin
- agrees with the quantum GLW results for small values of the spin chemical potential
- indicates how to avoid problems with the normalization of the average polarization three-vector for large values of the spin potential
- helps to define microscopic conditions that validate the use of the proposed equilibrium functions
- can be used to define entropy current and prove its conservation within the perfect-fluid approach with spin

Thank you for your attention!

Backup slides

Barnett and Einstein-de Haas Effects

Barnett Effect

S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)

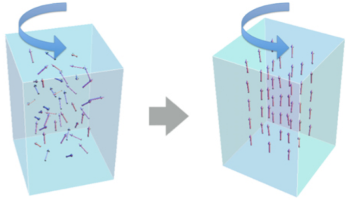

Figure: Mechanical rotation of an unmagnetized metallic object induces magnetization, an effective magnetic field emerges.

$$B_{\Omega} = \Omega/\gamma$$

Einstein-de Haas Effect

A. Einstein and W. de Haas, Deutsche Physikalische Gesellschaft, Verhandlungen 17, 152 (1915)

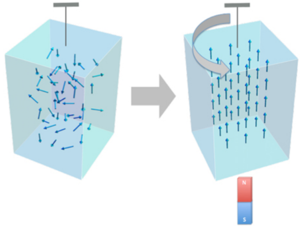

Figure: Application of magnetic field on an unmagnetized metallic object induces magnetization, body start rotating (mechanical angular momentum emerges)