The non-equilibrium attractor: Hydrodynamics and beyond

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Hirschegg 2019: From QCD matter to hadrons

International Workshop XLVII on Gross Properties of Nuclei and Nuclear Excitations
Hirschegg, Kleinwalsertal, Austria, January 13-19, 2019

January 18, 2019


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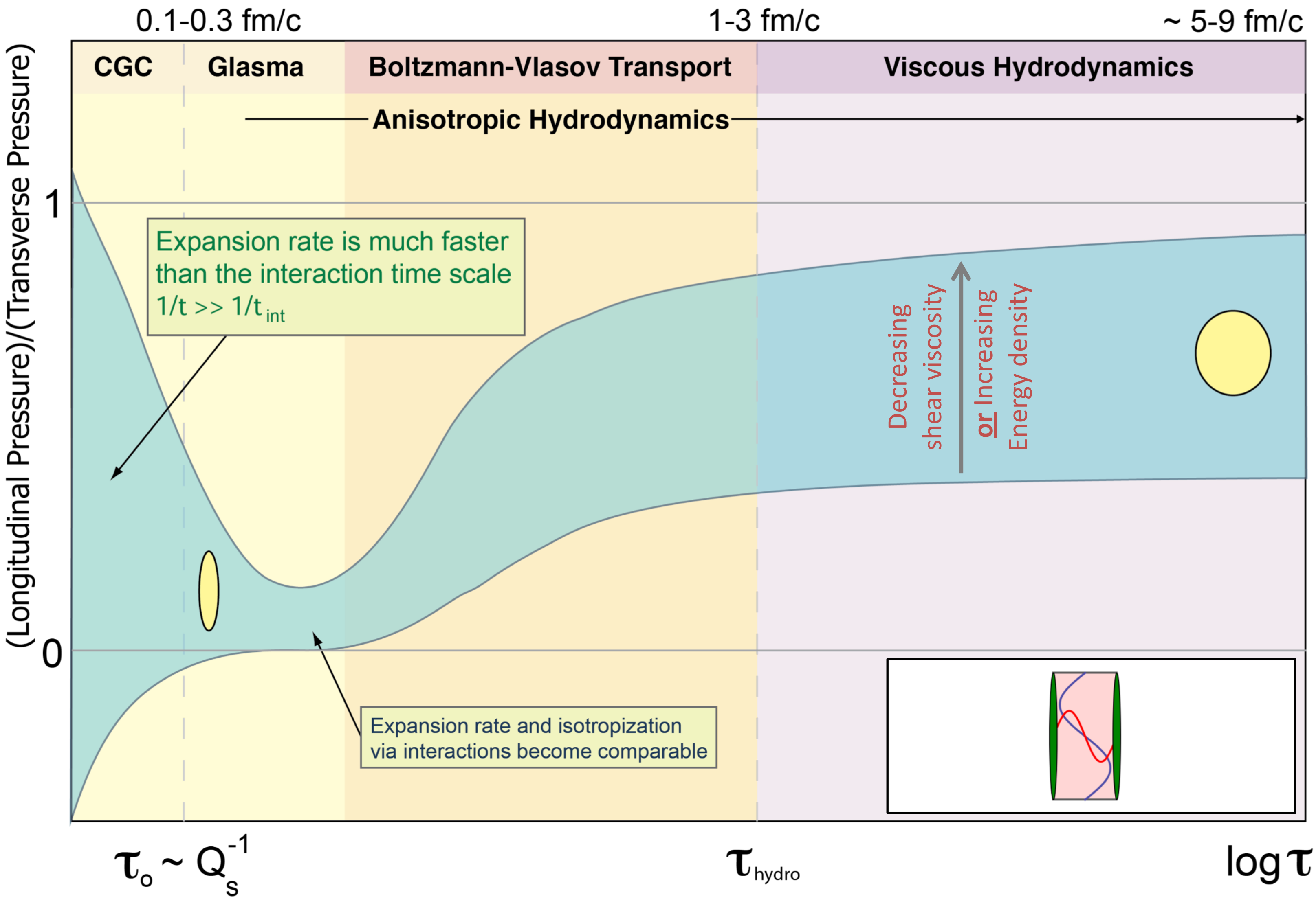
Introduction

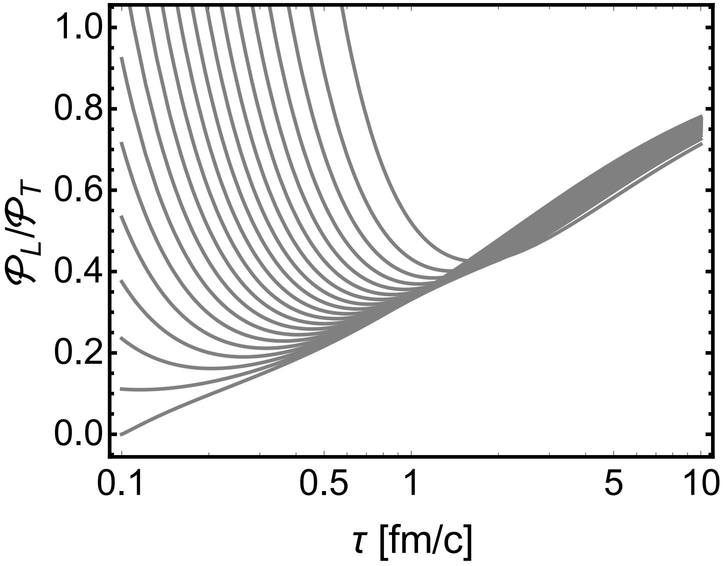
- Many disparate approaches find that the QGP is **highly momentum-space anisotropic in the local rest frame** (AdS-CFT, effective kinetic theory with/without plasma instabilities, 2d and 3d Glasma, hadronic transport, …).
- These EQ deviations persist for many fm/c and are very large near the edges.
- Despite these large momentum-space anisotropies, dissipative hydrodynamics has proven to be quite successful in describing experimental results → "**hydrodynamization**" instead of thermalization
- Why does it work and how can we use our understanding of hydrodynamization to improve existing hydrodynamics theories?
- I will discuss the unreasonable effectiveness of dissipative hydro using the concept of the **non-equilibrium dynamical attractor**.
  [see e.g. Heller and Spalinski, Phys. Rev. Lett. 115 (7), 072501 (2015); 1503.07514]
- I extend earlier treatments by studying higher-order moments of the one-particle distribution function.
- I demonstrate that, in RTA, **higher-order moments converge more quickly to their respective attractors than lower-order ones**.
  [Strickland, JHEP2018, 128; 1809.01200]

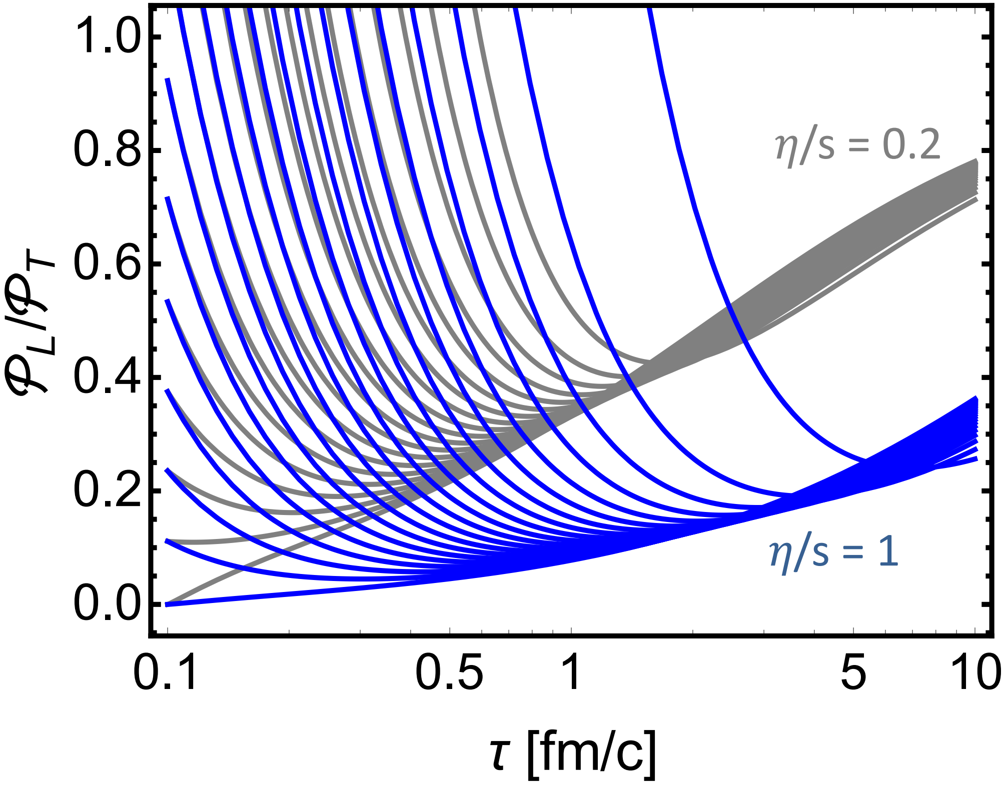
QGP momentum anisotropy cartoon



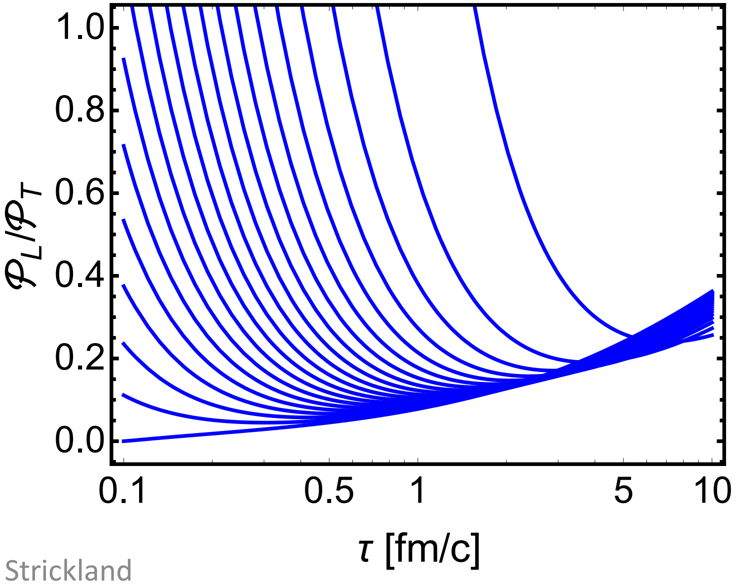
The
non-equilibrium
attractor

Assuming η/s = 0.2 and T_0 = 500 MeV



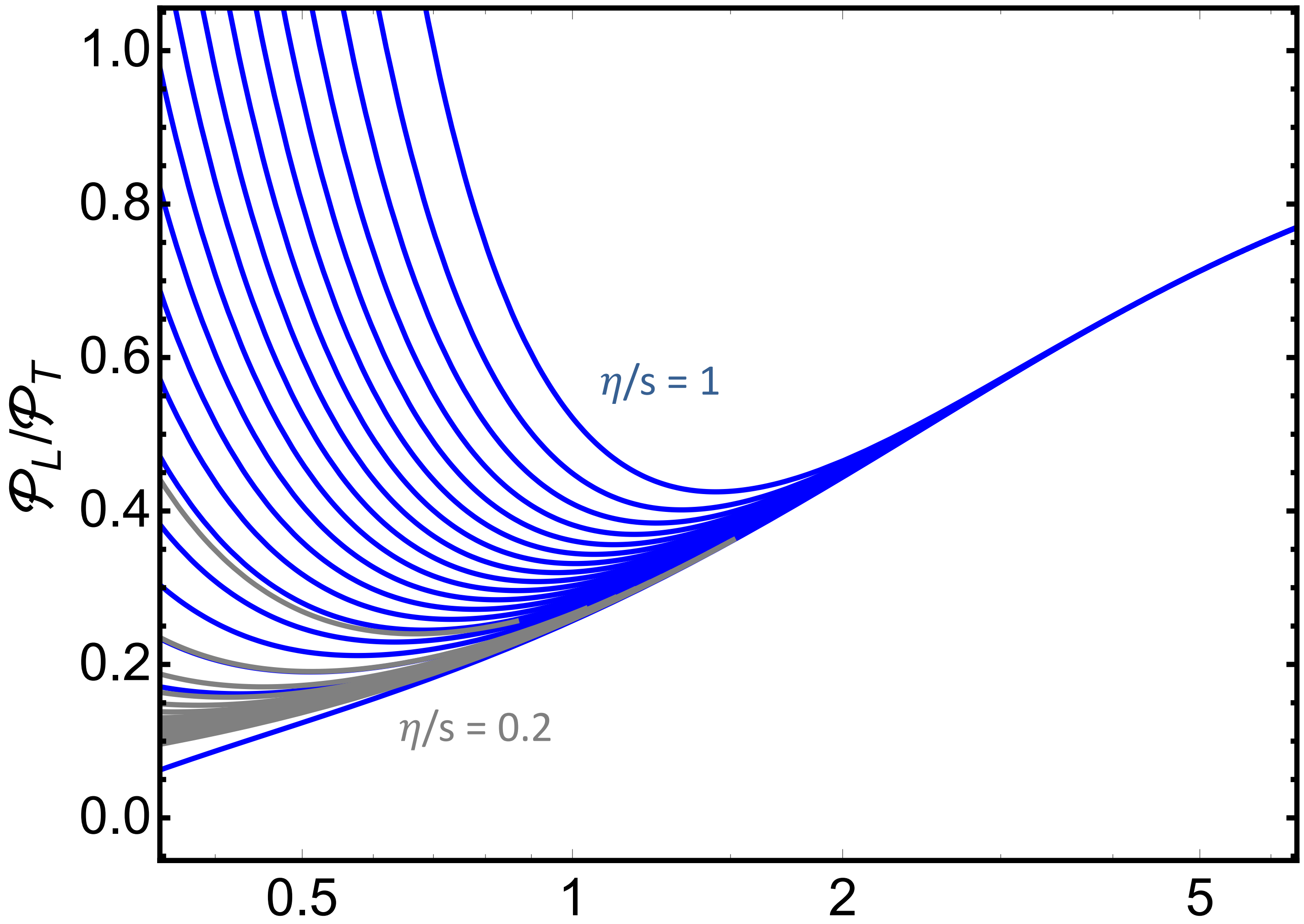


Assuming η/s = 1 and T_0 = 500 MeV



- Solve equations for different initial conditions and different values of the shear viscosity (gray vs blue)
- Hints of universal behavior at late times visible (similar levels of momentum anisotropy)

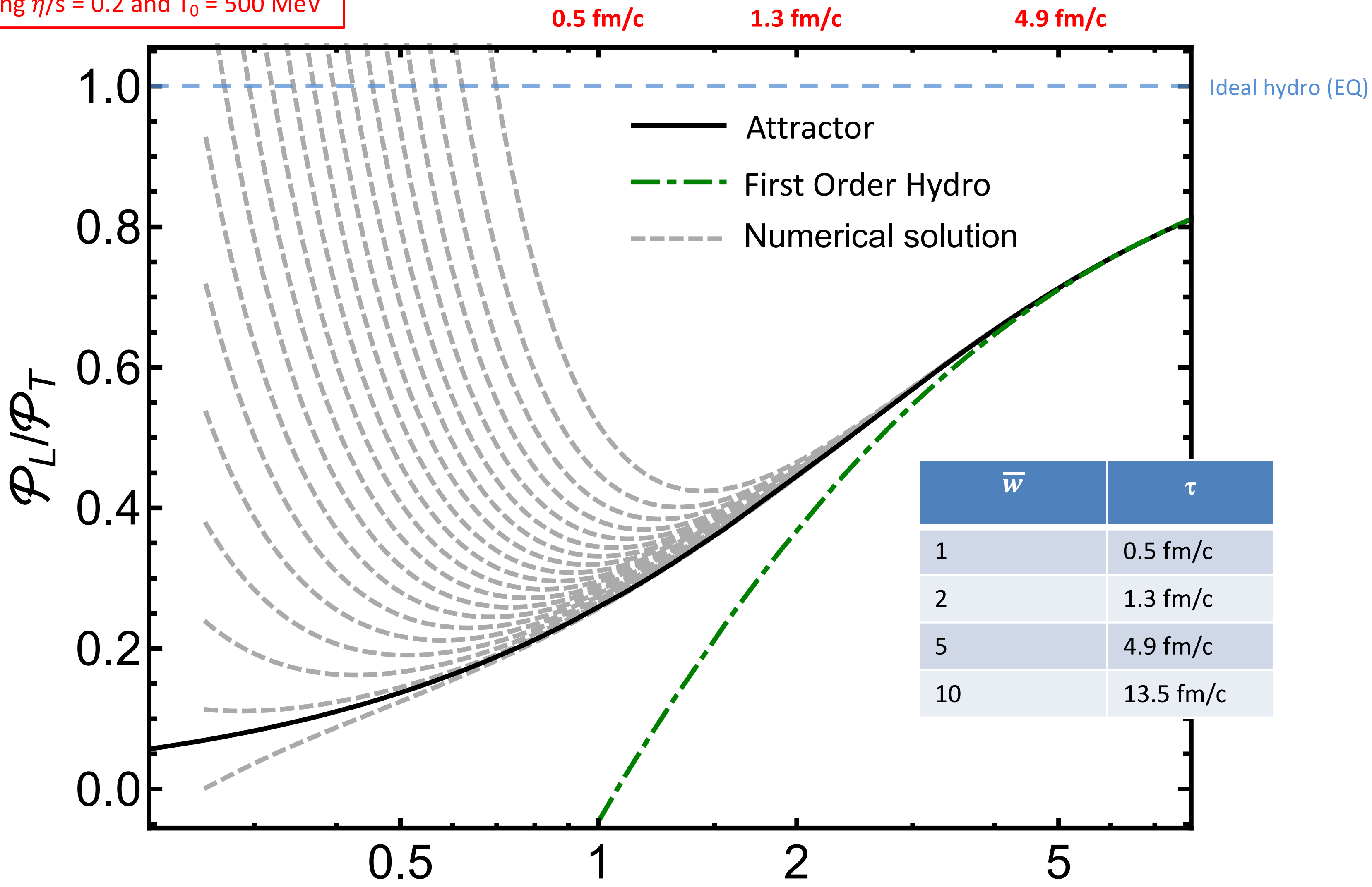
Evidence for an attractor



$$\overline{w} = \frac{\tau}{\tau_{\rm eq}(\tau)} = \frac{\tau T(\tau)}{5\bar{\eta}(\tau)}$$

The attractor concept

Assuming η/s = 0.2 and T_0 = 500 MeV



$\overline{w}$	τ
1	0.5 fm/c
2	1.3 fm/c
5	4.9 fm/c
10	13.5 fm/c

$$\overline{w} = \frac{\tau}{\tau_{\rm eq}} = \frac{\tau T}{5\bar{\eta}}$$

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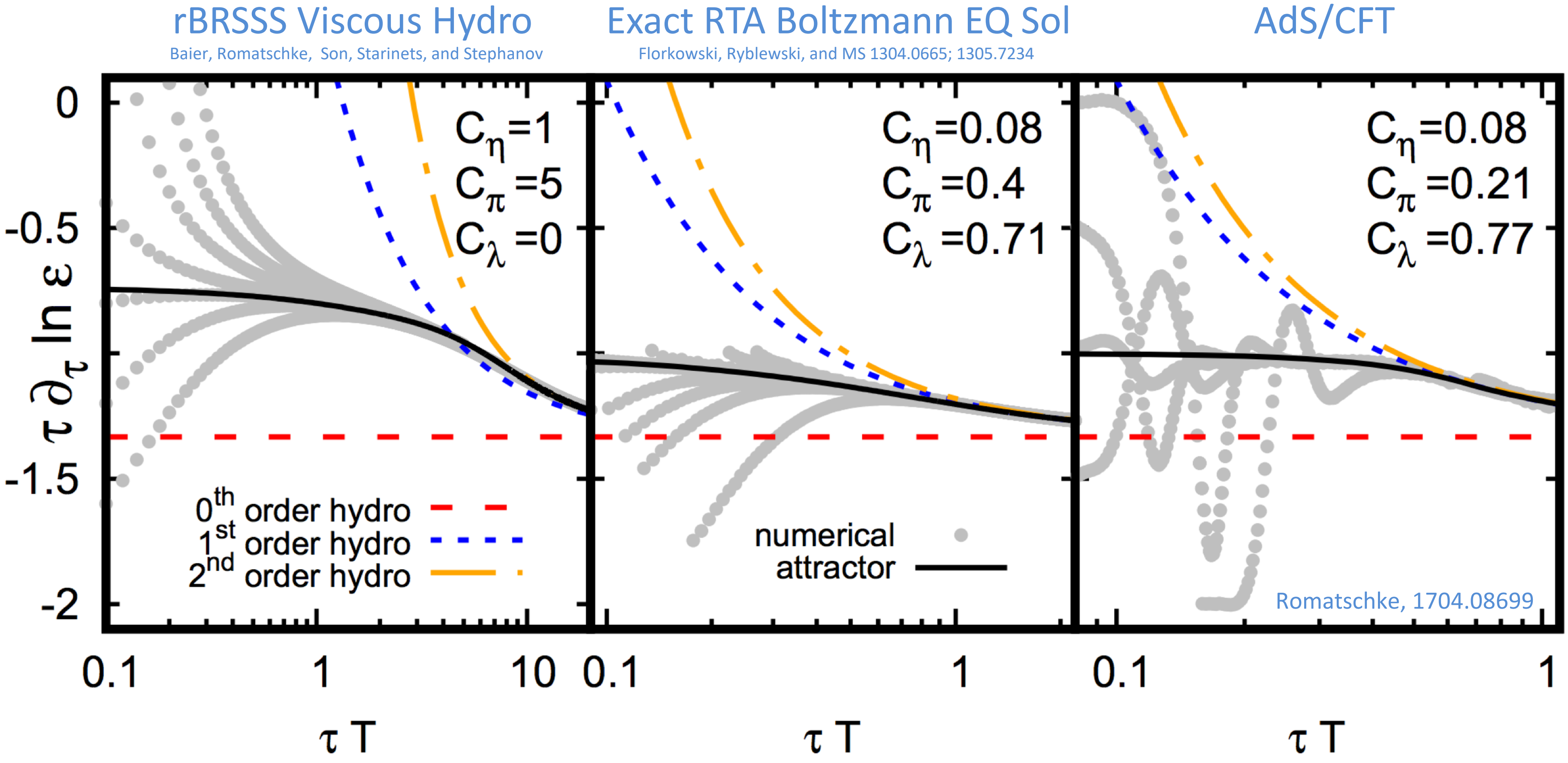
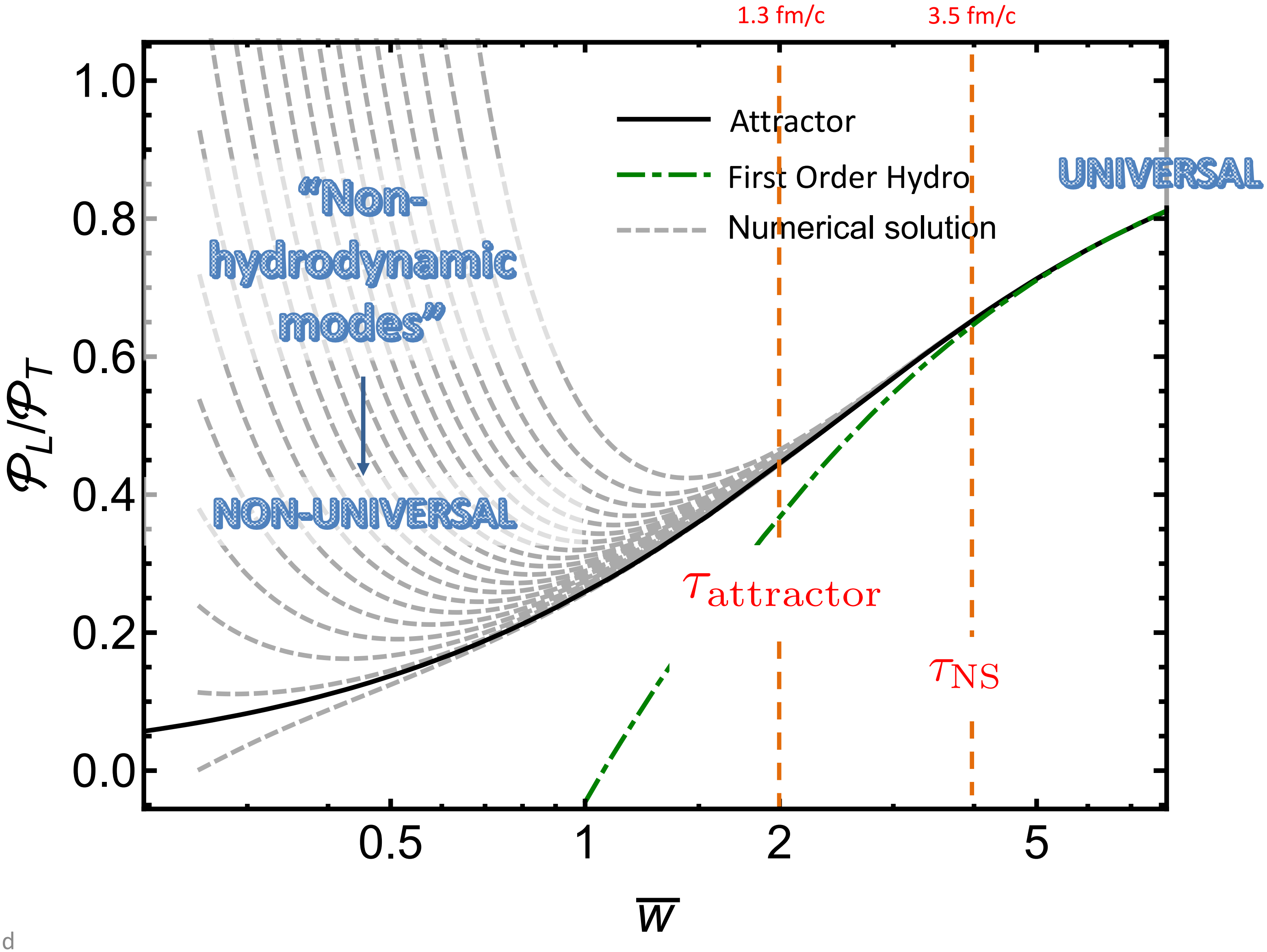
Attractor exists in many theories

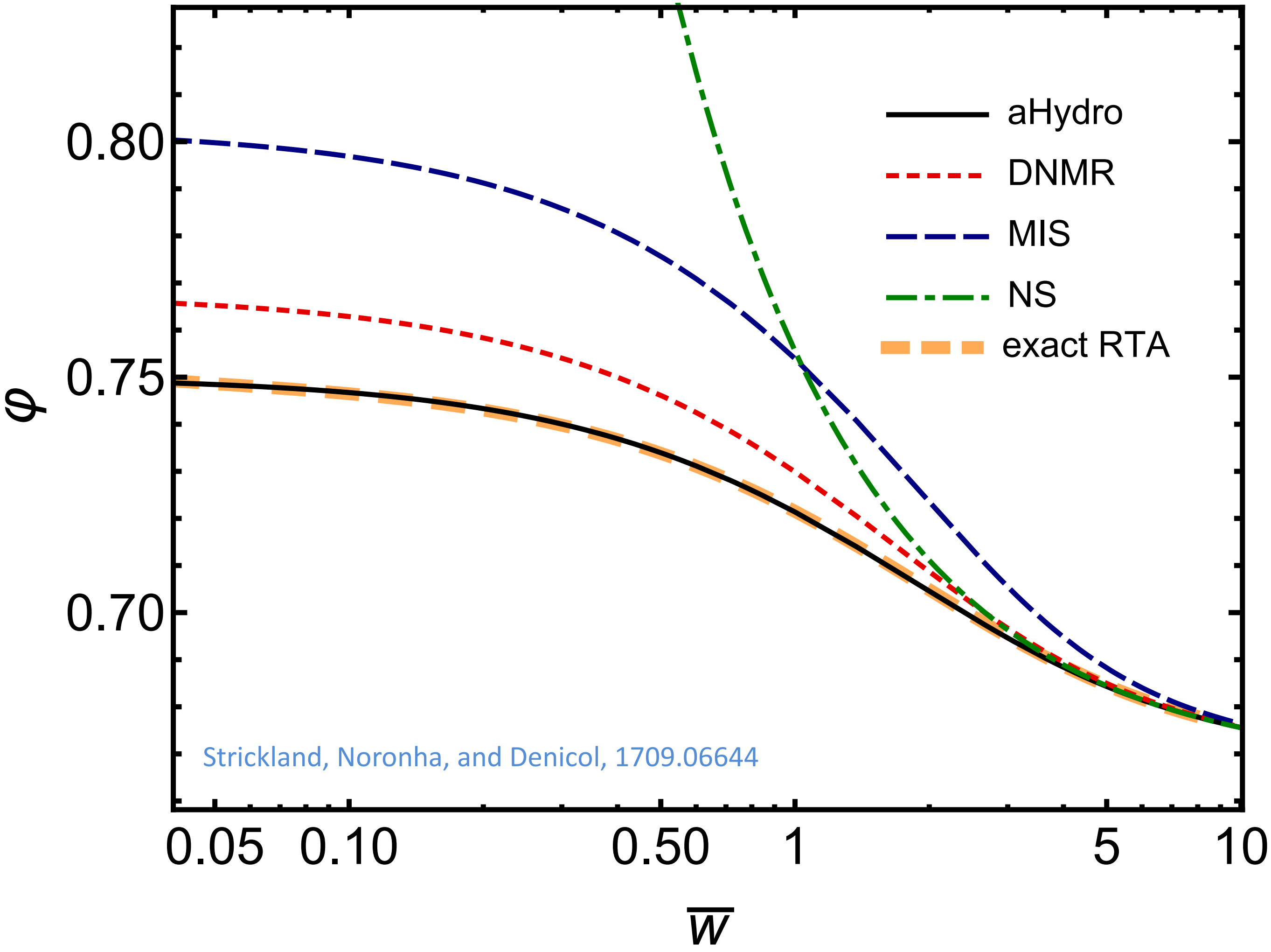


Romatschke, 1704.08699; see also Keegan et al, 1512.05347 for EKT QCD

The attractor concept



Hydro attractors vs the true attractor



Strickland, Noronha, and Denicol, 1709.06644

Beyond hydrodynamics?

M. Strickland, JHEP2018, 128; 1809.01200

- Can the concept of a non-equilibrium attractor be extended beyond the 14 degrees of freedom described using the energy-momentum tensor, number density, and diffusion current?

- In kinetic theory we describe things in terms of a one-particle distribution function **f(x,p)** and the energy-momentum tensor is obtained from low-order moments:

$$T^{\mu\nu} = \int dP \, p^\mu p^\nu f(x,p) \qquad \int dP \equiv \int \frac{d^3p}{(2\pi)^2 E}$$

- What about more general moments of f? Particularly ones that are sensitive to higher momenta?

Beyond hydrodynamics?

- For a conformal system it suffices to consider

$$\mathcal{M}^{nm}[f] \equiv \int dP \, (p \cdot u)^n \, (p \cdot z)^{2m} \, f(x,p)$$

- This encompasses the moments necessary to construct the energy momentum tensor, e.g. below, and more

$$\varepsilon = \mathcal{M}^{20} = \int dP \, (p \cdot u)^2 \, f(\tau, w, p_T) = T^{00}_{\rm LRF}$$

$$P_L = \mathcal{M}^{01} = \int dP \, (p \cdot z)^2 \, f(\tau, w, p_T) = T^{zz}_{\rm LRF}$$

Bjorken Expansion: Exact RTA Solution

- Simple model: Boost-invariant, transversally homogeneous Boltzmann equation in relaxation time approximation (RTA).
- Many results in this model, so we can compare with the literature.

Boltzmann EQ $p^\mu \partial_\mu f(x,p) = C[f(x,p)]$

RTA $C[f] = \frac{p_\mu u^\mu}{\tau_{\rm eq}} \left[f_{\rm eq}\Big(p_\mu u^\mu, T(x)\Big) - f(x,p) \right]$

Massless Particles
W. Florkowski, R. Ryblewski, and MS, 1304.0665 and 1305.7234

Massive Particles
W. Florkowski, E. Maksymiuk, R. Ryblewski, and MS, 1402.7348

Solution for the energy density (massless particle case)

$$T^4(\tau) = D(\tau,\tau_0) T_0^4 \, \frac{\mathcal{H}\left(\frac{\alpha_0 \tau_0}{\tau}\right)}{\mathcal{H}(\alpha_0)} + \int_{\tau_0}^{\tau} \frac{d\tau'}{2\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, T^4(\tau') \mathcal{H}\left(\frac{\tau'}{\tau}\right)$$

Time-dependent relaxation time $\tau_{\rm eq}(\tau) = \frac{5\bar\eta}{T(\tau)}$	Damping Function $D(\tau_2,\tau_1) = \exp\left[-\int_{\tau_1}^{\tau_2} d\tau \, \tau_{\rm eq}^{-1}(\tau)\right]$

0+1d RTA Exact Solution

$$T^4(\tau) = D(\tau,\tau_0) T_0^4 \, \frac{\mathcal{H}\big(\frac{\alpha_0\tau_0}{\tau}\big)}{\mathcal{H}(\alpha_0)} + \int_{\tau_0}^{\tau} \frac{d\tau'}{2\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, T^4(\tau') \mathcal{H}\left(\frac{\tau'}{\tau}\right)$$

Once this integral equation is solved (by numerical iteration), we can construct the full one-particle distribution function f(τ,p) OR we can compute general moments:

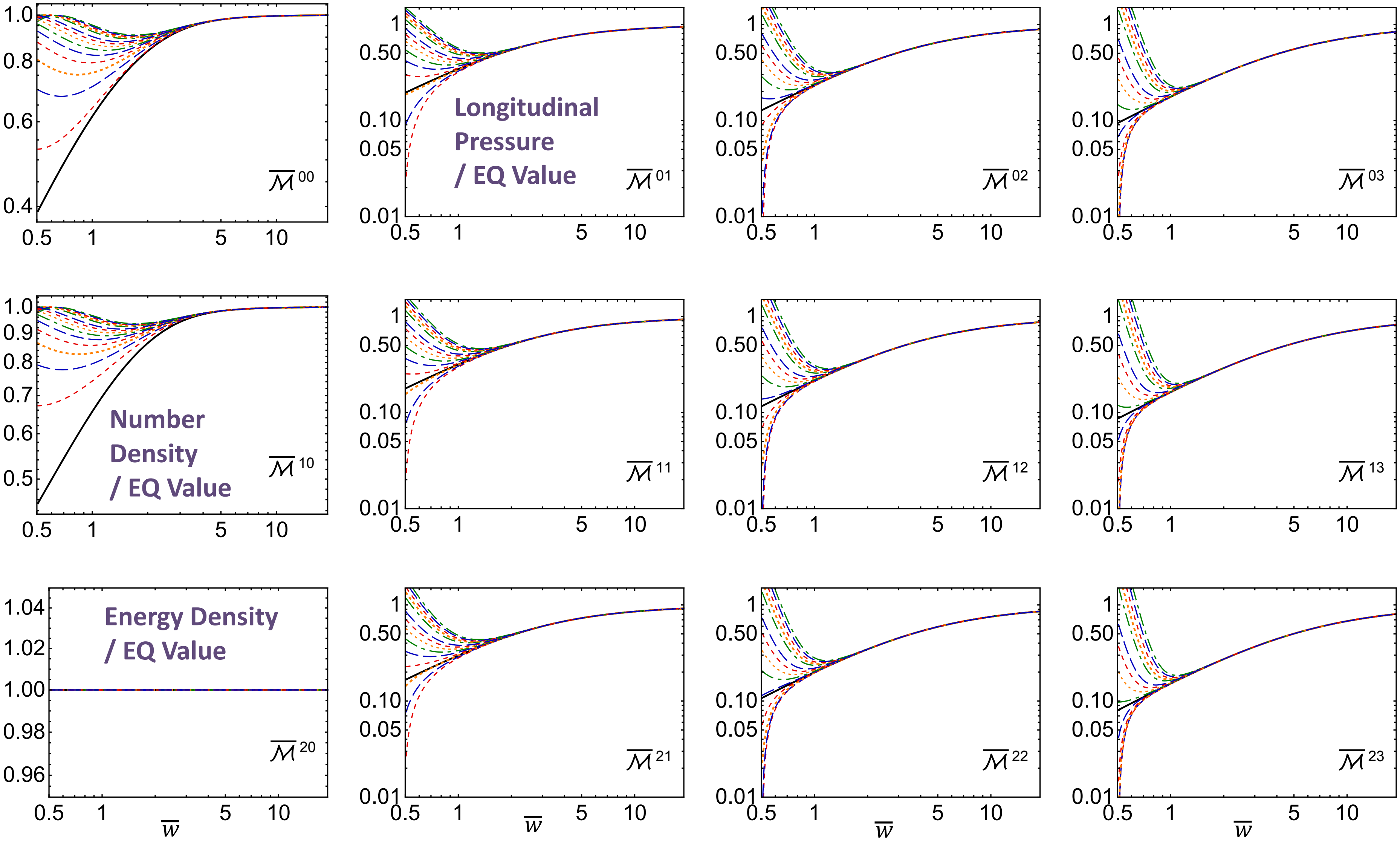
$$f(\tau,w,p_T) = D(\tau,\tau_0) f_0(w,p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, f_{\rm eq}(\tau',w,p_T)$$

$$\mathcal{M}^{nm}(\tau) = \frac{\Gamma(n+2m+2)}{(2\pi)^2}\Bigg[D(\tau,\tau_0) 2^{(n+2m+2)/4} T_0^{n+2m+2} \frac{\mathcal{H}^{nm}\big(\frac{\alpha_0\tau_0}{\tau}\big)}{[\mathcal{H}^{20}(\alpha_0)]^{(n+2m+2)/4}} + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\rm eq}(\tau')} \, D(\tau,\tau') \, T^{n+2m+2}(\tau') \mathcal{H}^{nm}\left(\frac{\tau'}{\tau}\right)\Bigg],$$

$$\mathcal{H}^{nm}(y) = \tfrac{2y^{2m+1}}{2m+1} {}_2F_1\big(\tfrac{1}{2}+m, \tfrac{1-n}{2}; \tfrac{3}{2}+m; 1-y^2\big)\,.$$

Behavior of higher order moments

M. Strickland, JHEP2018, 128; 1809.01200



Longitudinal Pressure / EQ Value

Number Density / EQ Value

Energy Density / EQ Value

Black Line = Attractor Solution

Dashed colored lines = scan of initial conditions

Time scales

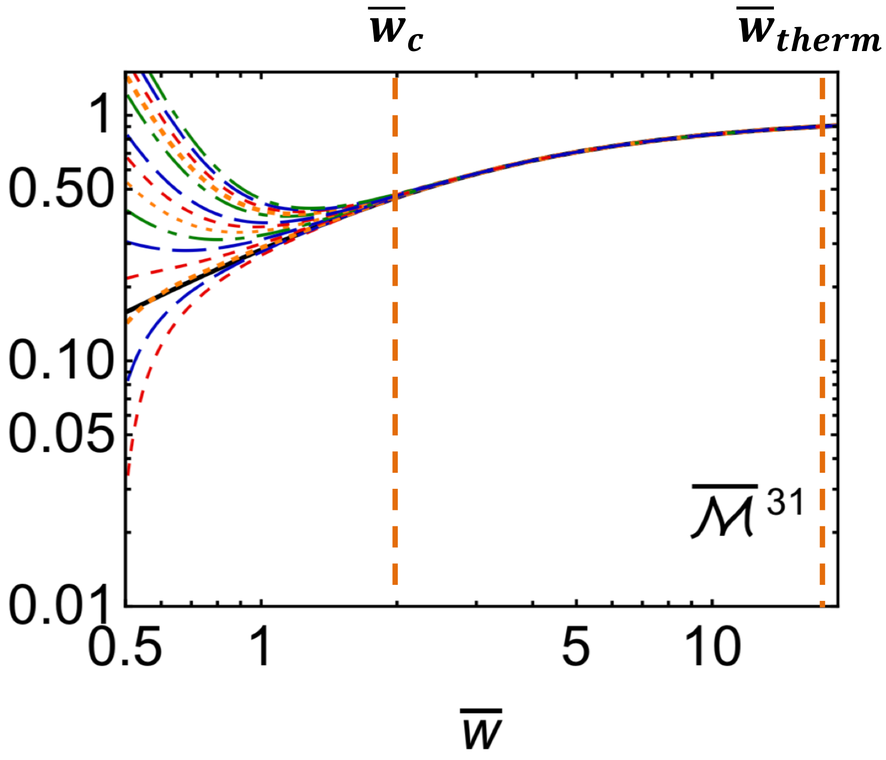
From each of these results, we can obtain

1. An estimate of the thermalization time by requiring that the scaled moment reaches 0.9.

   → **Thermalization time $\overline{w}_{therm}$**

2. An estimate of the time it takes each moment to approach its respective attractor solution to within a given tolerance.

   → **Convergence (or pseudo-thermalization) time $\overline{w}_c$**



$\overline{w}$	τ
1	0.5 fm/c
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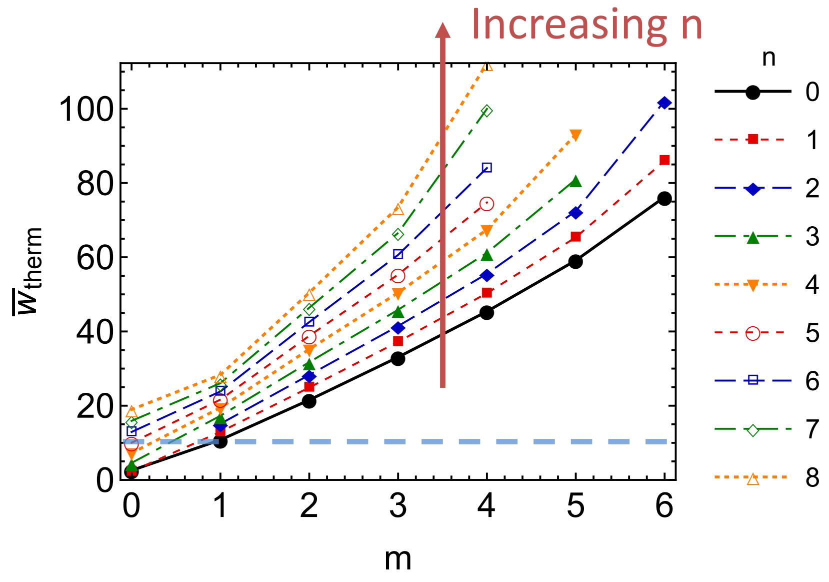
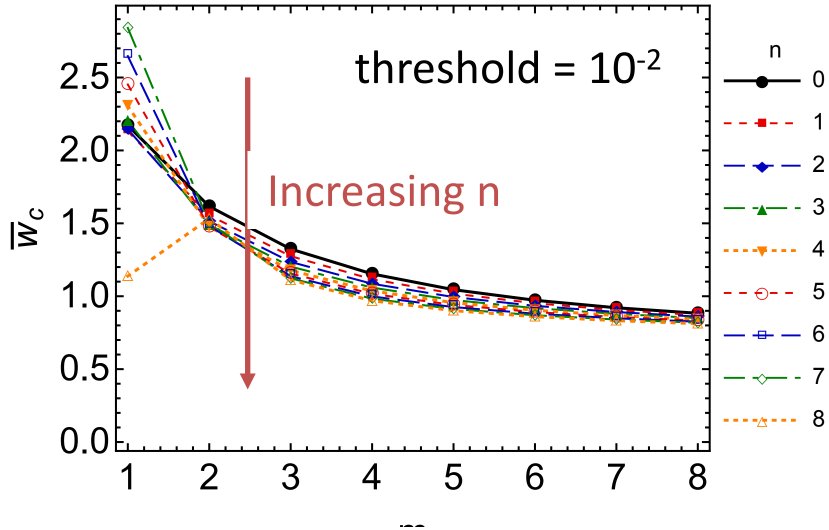
10	13.5 fm/c

Assuming $\eta/s = 0.2$ and T = 500 MeV

Time scales



- As m or n increase the thermalization time increases
- For m>2 (n>1) convergence time decrease as n (m) increases
- This demonstrates that at large n,m there is a parametric separation of scales



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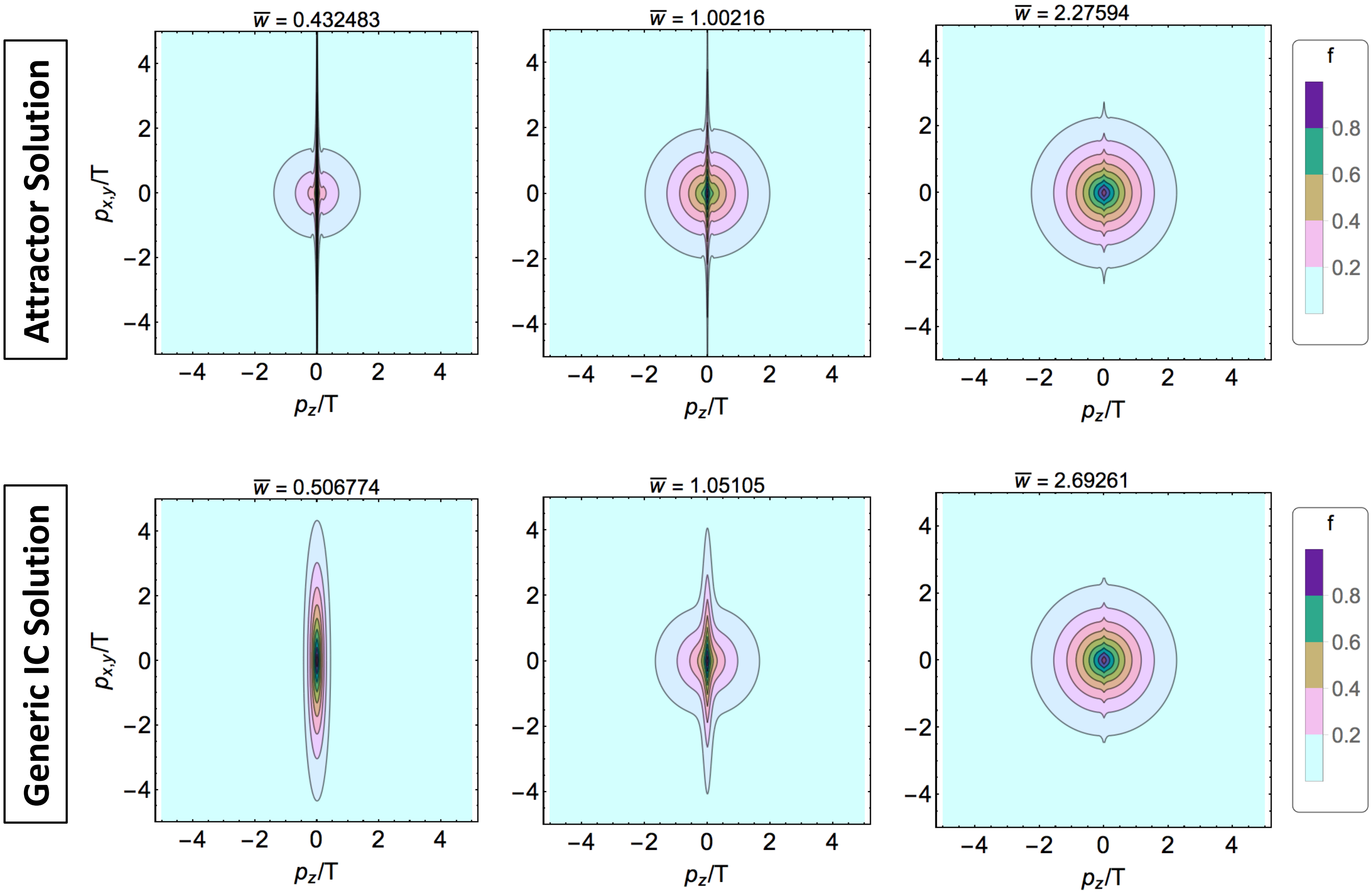
The attractor
for the distribution
function itself

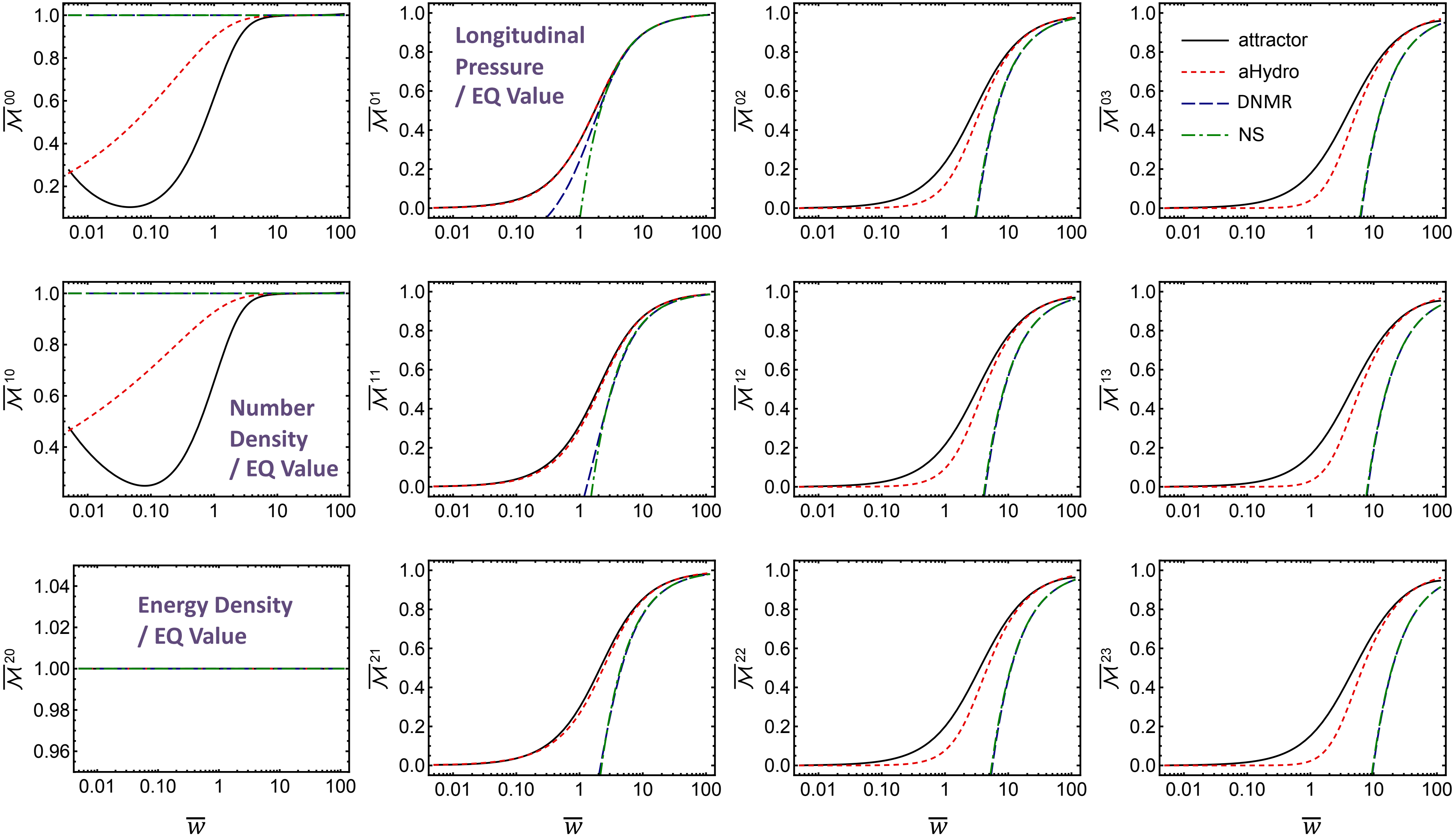
Attractor distribution function

M. Strickland, JHEP2018, 128; 1809.01200



Comparison of exact attractor for moments with different hydrodynamics approximations

Hydrodynamic comparisons



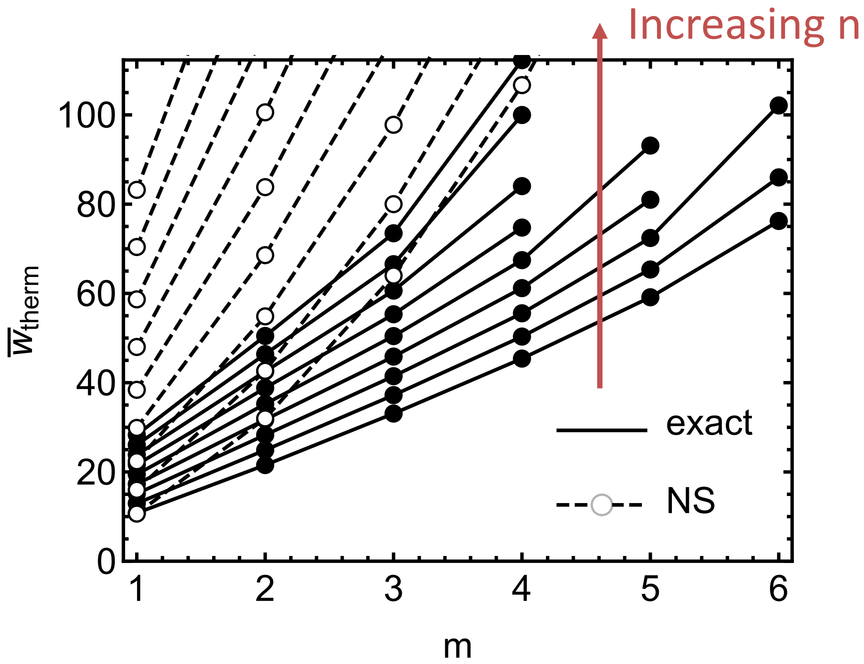
Longitudinal Pressure / EQ Value

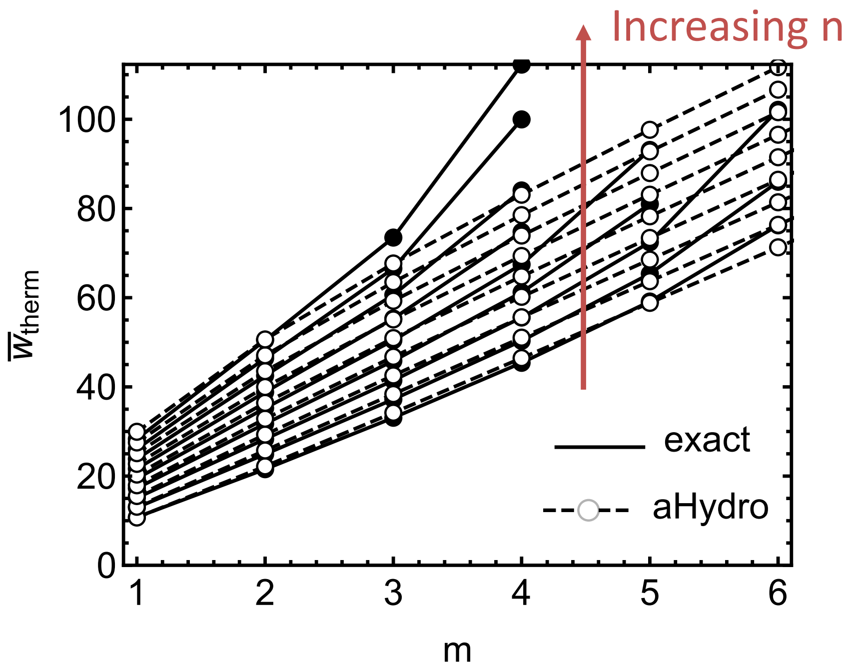
Number Density / EQ Value

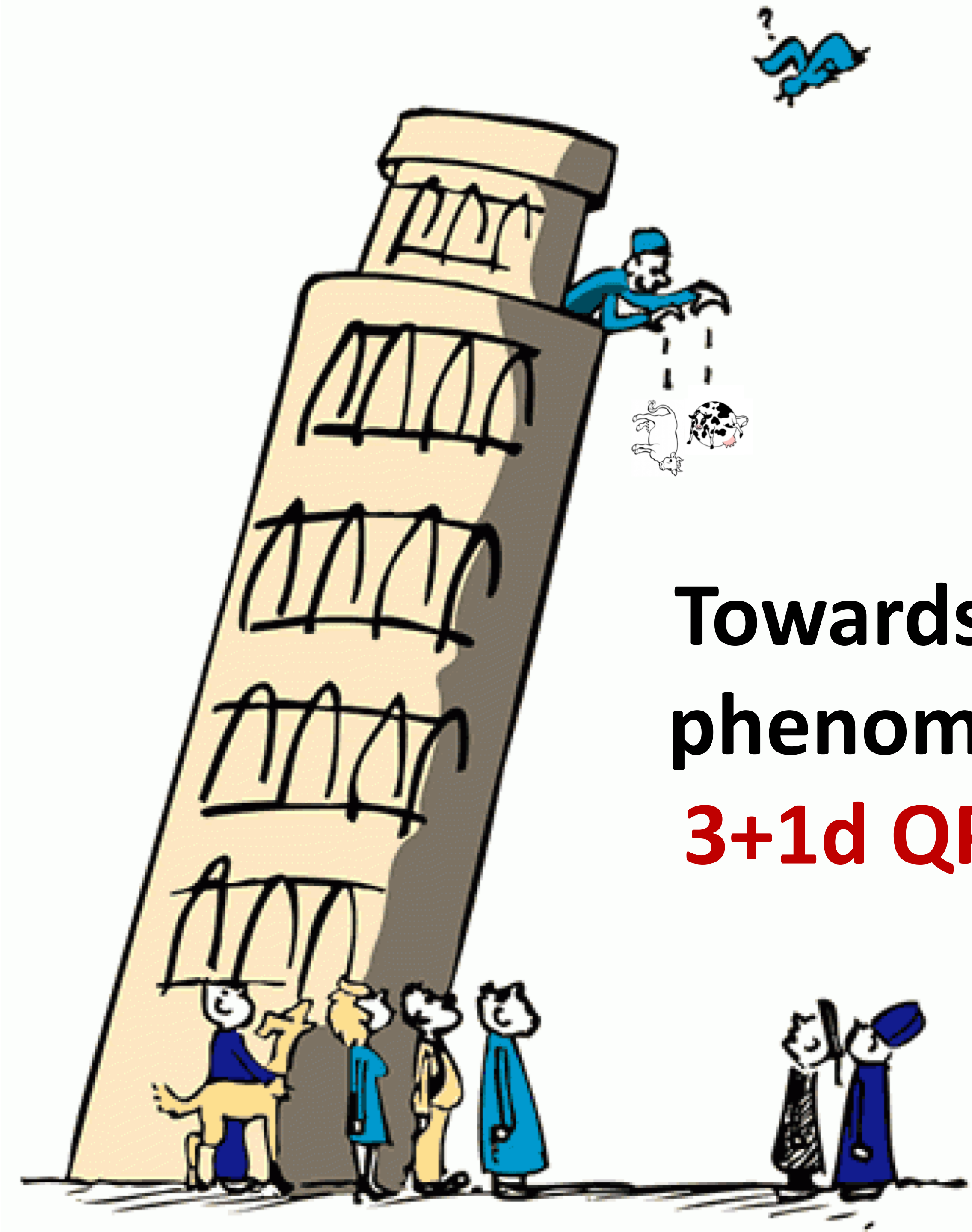
Energy Density / EQ Value

Hydrodynamic comparisons

- aHydro does best job in reproducing the exact moments.
- Failure of standard viscous hydro to describe high-order moments means that the system does not “hydrodynamize” in the traditional sense.
- aHydro works reasonably well, even for high-order moments.
- Fast convergence of exact solution to attractor does suggest fast ”pseudothermalization”







Towards realistic phenomenology: 3+1d QP aHydro

Generalized aHydro formalism

In aHydro, one starts from kinetic theory and assumes that the distribution function is of the form

$$f(x,p) = f_{\text{eq}}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\substack{\text{LRF four}\\ \text{velocity}}} + \underbrace{\xi^{\mu\nu}}_{\substack{\text{Traceless}\\ \text{symmetric}\\ \text{anisotropy}\\ \text{tensor}}} - \underbrace{\Delta^{\mu\nu}}_{\substack{\uparrow\\ \text{Transverse}\\ \text{projector}}} \underbrace{\Phi}_{\text{“Bulk”}}$$

$$\begin{aligned} u^\mu u_\mu &= 1 \\ \xi^\mu{}_\mu &= 0 \\ \Delta^\mu{}_\mu &= 3 \\ u_\mu \xi^{\mu\nu} &= u_\mu \Delta^{\mu\nu} = 0 \end{aligned}$$

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Implementing the equation of state

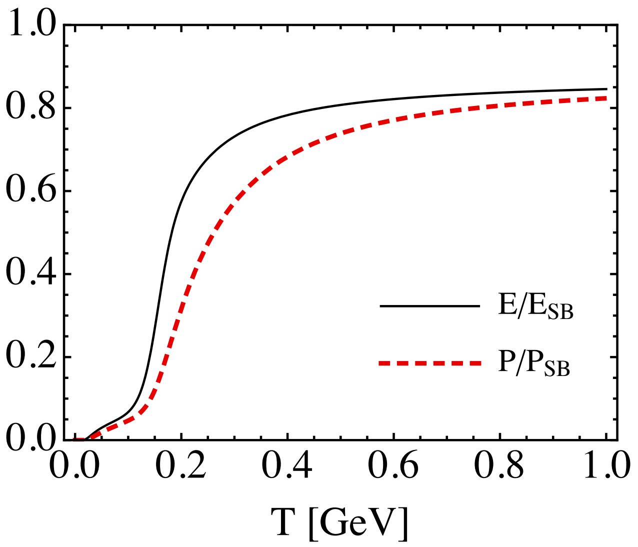
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

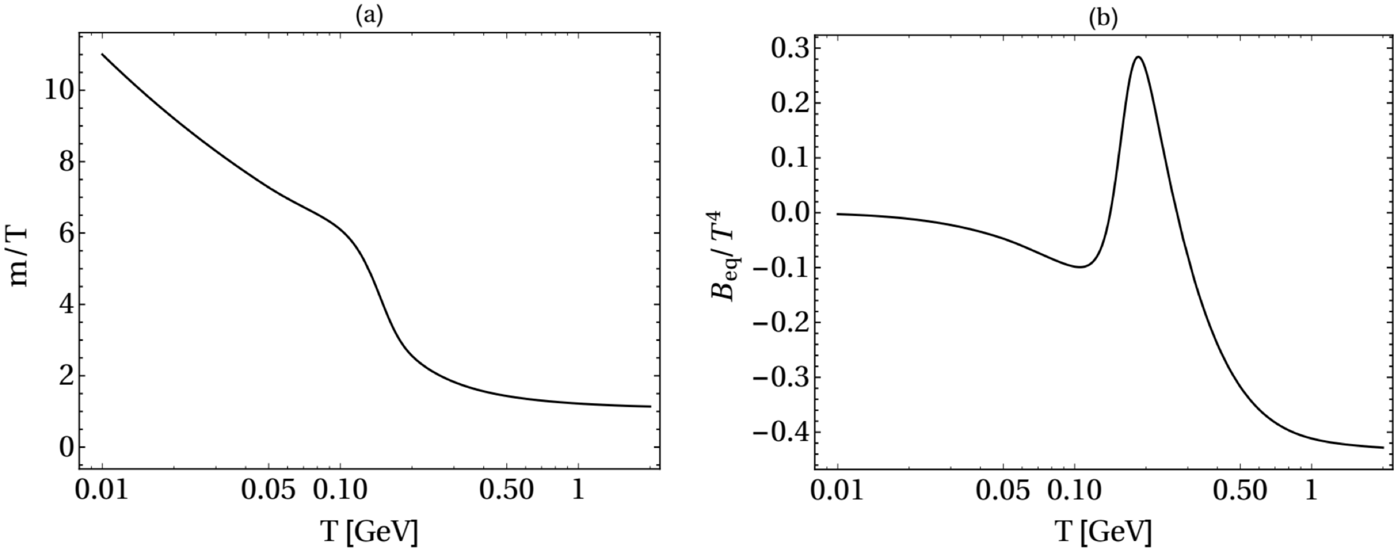
Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$





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$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entropy density ratio.

$$\frac{\eta}{\tau_{\text{eq}}} = \frac{1}{T} I_{3,2}(\hat{m}_{\text{eq}})$$

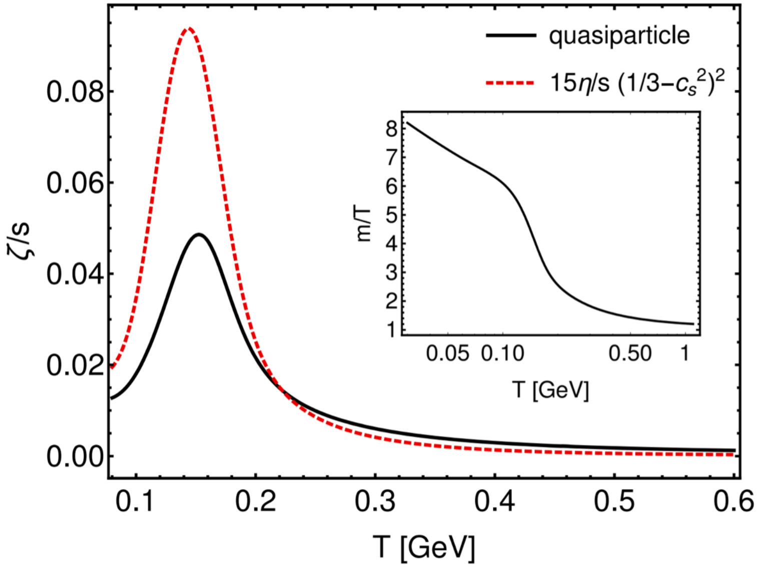
Bulk viscosity

$$\frac{\zeta}{\tau_{\text{eq}}} = \frac{5}{3T} I_{3,2} - c_s^2(\mathcal{E} + \mathcal{P}) + T\hat{m}^3 \frac{dm}{dT} I_{1,1}$$

$$I_{3,2}(x) = \frac{N_{\text{dof}} T^5\, x^5}{30\pi^2}\left[\frac{1}{16}\Big(K_5(x) - 7K_3(x) + 22K_1(x)\Big) - K_{i,1}(x)\right],$$

$$K_{i,1}(x) = \frac{\pi}{2}\left[1 - xK_0(x)\mathcal{S}_{-1}(x) - xK_1(x)\mathcal{S}_0(x)\right],$$

$$I_{1,1} = \frac{g\, m^3}{6\pi^2}\left[\frac{1}{4}(K_3 - 5K_1) + K_{i,1}\right]$$



Anisotropic Cooper-Frye Freezeout

M. Alqahtani, M. Nopoush, and MS, 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

- Use same form for "anisotropic freeze-out" at LO
- Form includes both shear and bulk corrections to the distribution function
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\boxed{\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{isotropic}} + \underbrace{\xi^{\mu\nu}}_{\text{anisotropy tensor}} - \underbrace{\Phi\Delta^{\mu\nu}}_{\text{bulk correction}}}$$

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0, \xi_x, \xi_y, \xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

NOTE: Usual 2nd-order viscous hydro form

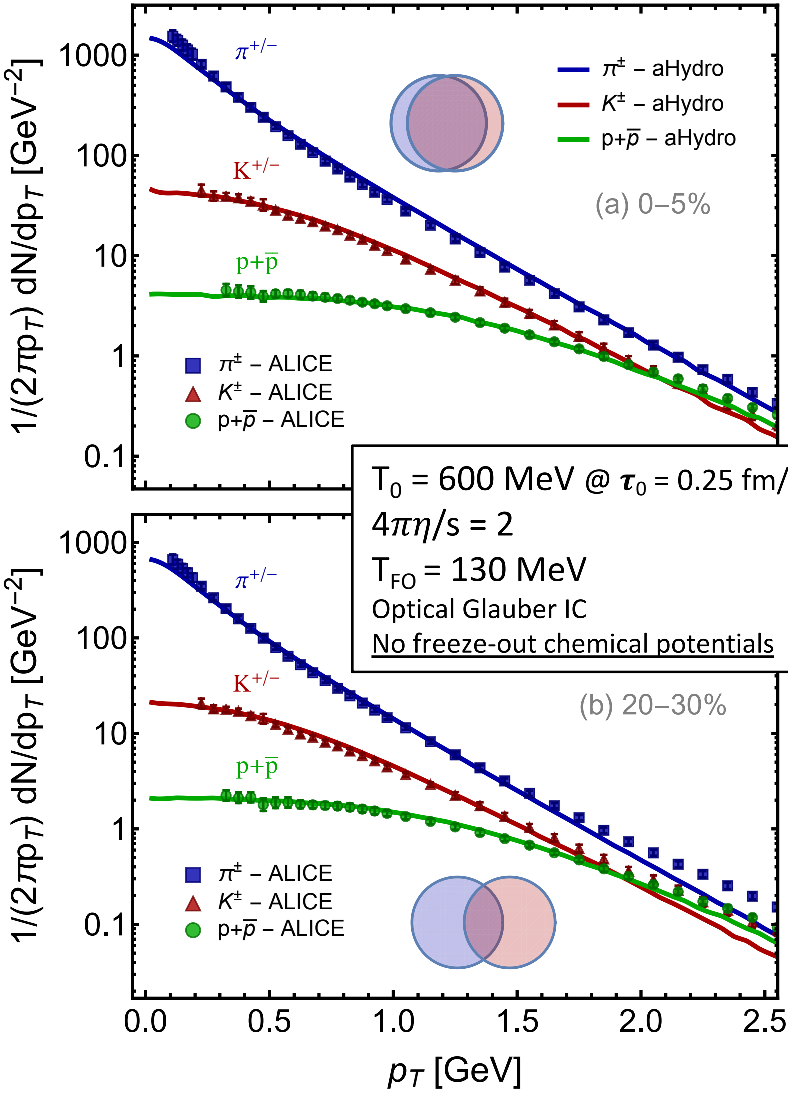
$$f(p,x) = f_{\rm eq}\left[1 + (1 - a f_{\rm eq})\frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon + P)T^2}\right]$$

$$f_{\rm eq} = 1/[\exp(p \cdot u/T) + a] \qquad a = \text{-}1,\ +1,\ \text{or}\ 0$$

- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- **Problem becomes much worse when including the bulk viscous correction**.

Identified particle spectra (LHC)

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



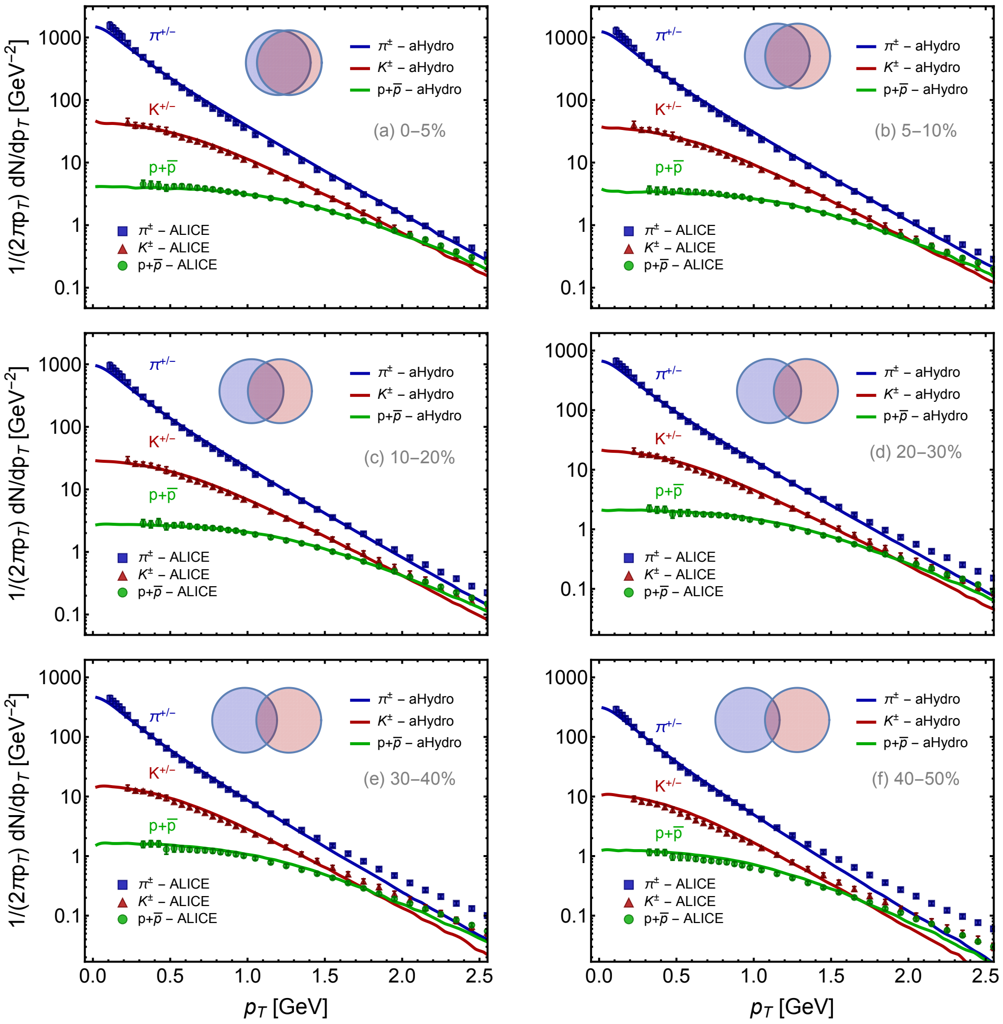
T_0 = 600 MeV @ τ_0 = 0.25 fm/c

$4\pi\eta/s = 2$

T_{FO} = 130 MeV

Optical Glauber IC

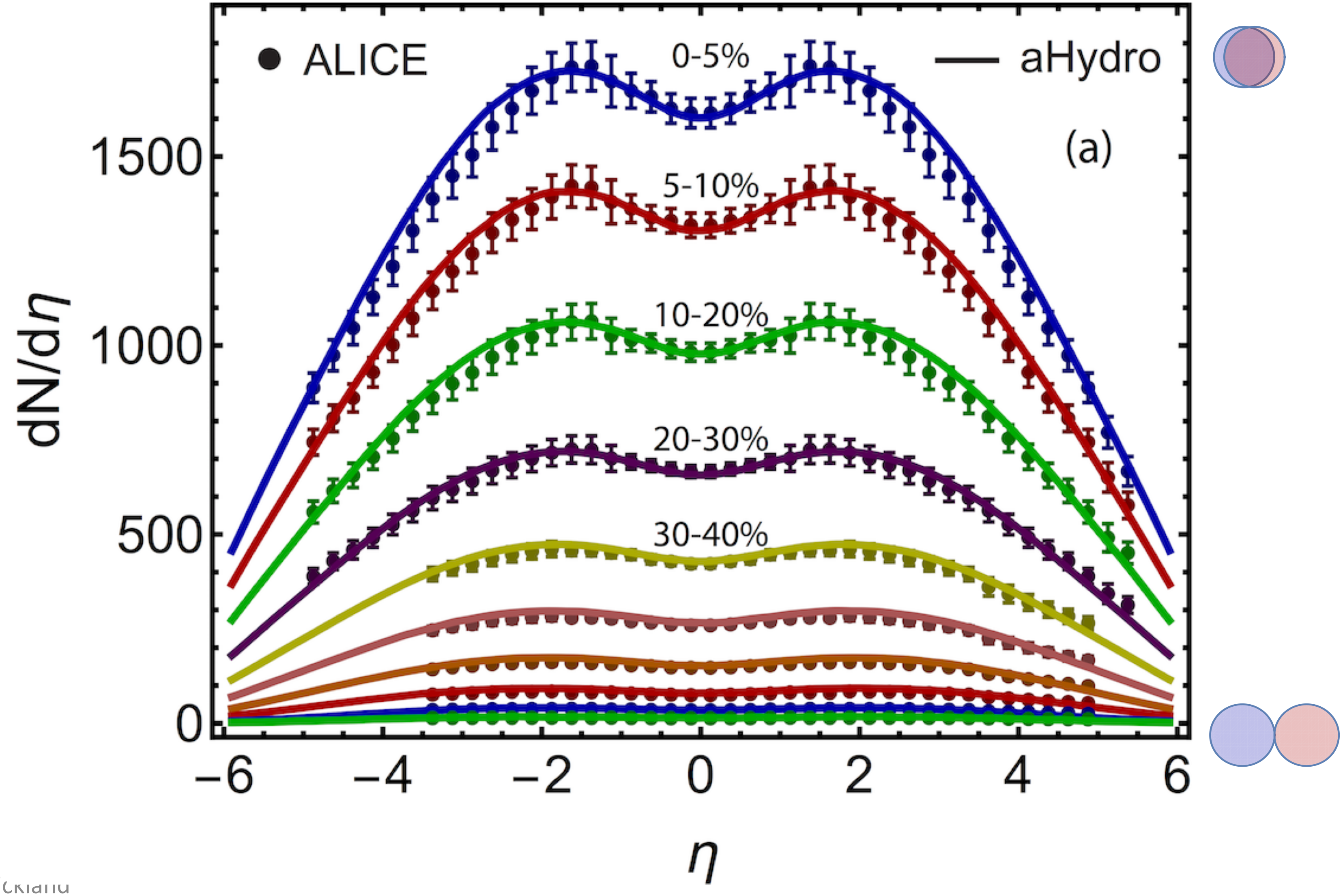
No freeze-out chemical potentials



Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

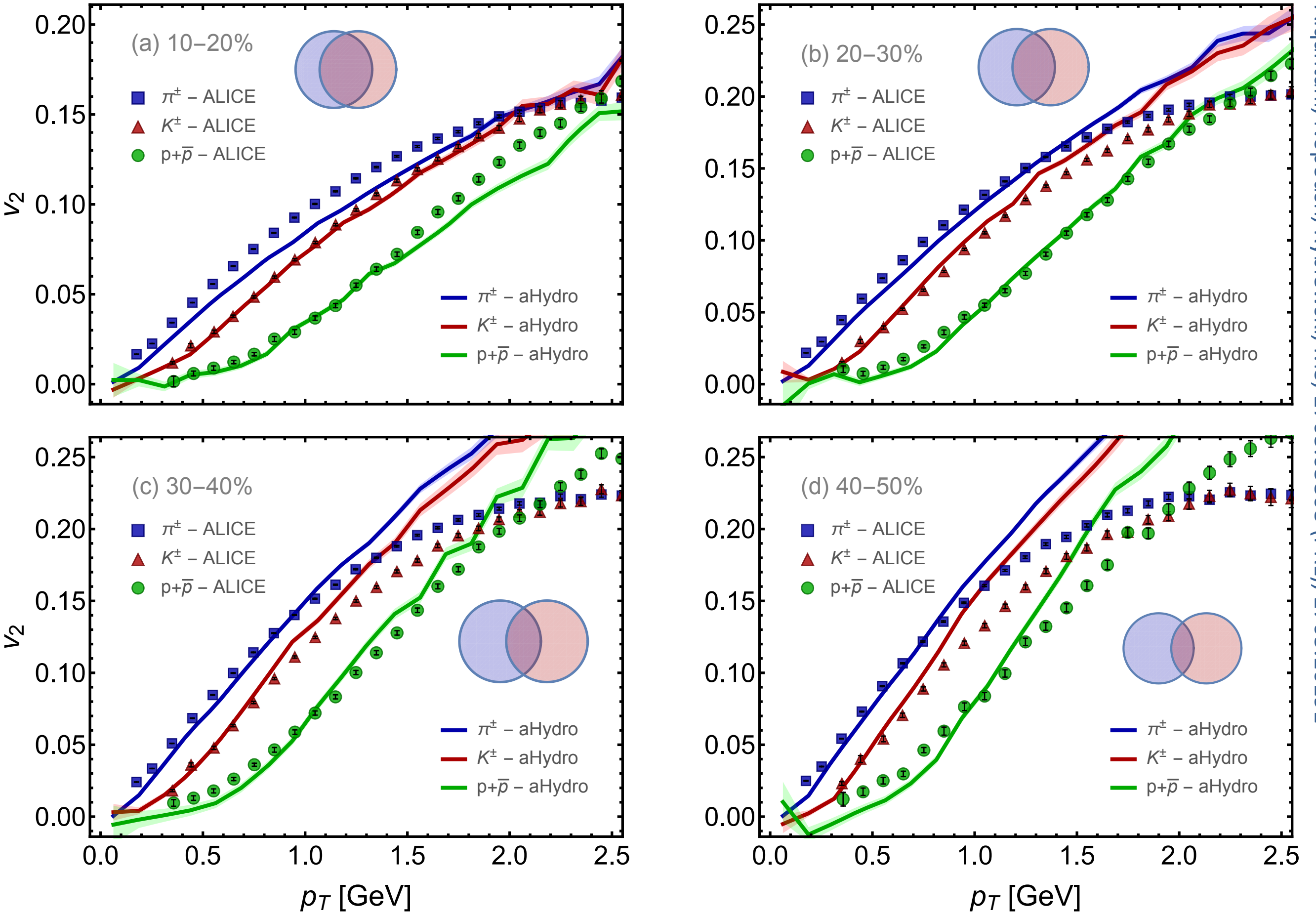
Charged particle multiplicity (LHC)

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



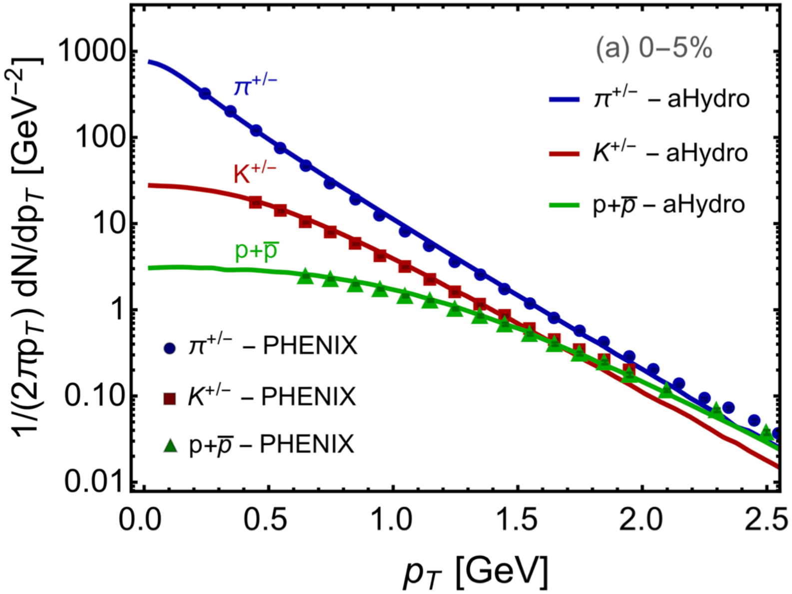
Elliptic flow (LHC)

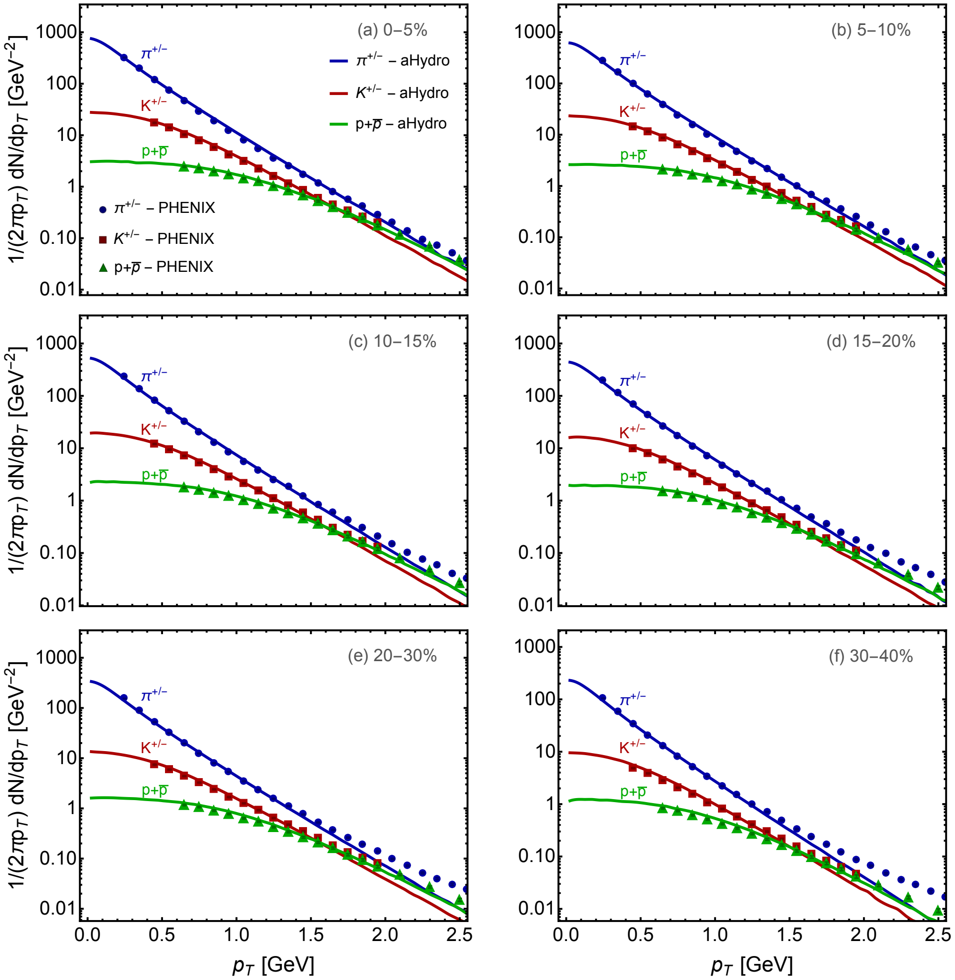
- Quite good description of identified particle elliptic flow as well
- Central collisions → need to include fluctuating init. conditions!



Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191

ID'd particle spectra (RHIC)

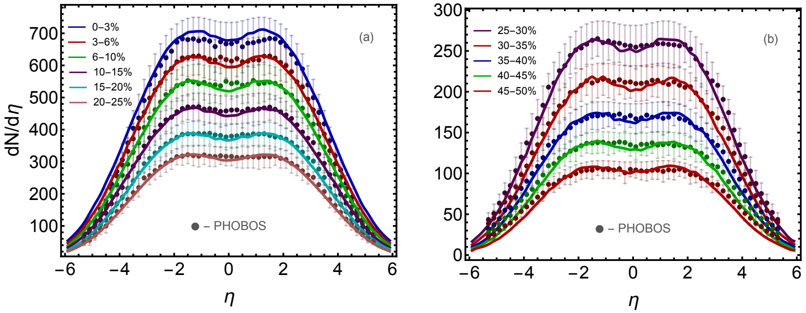


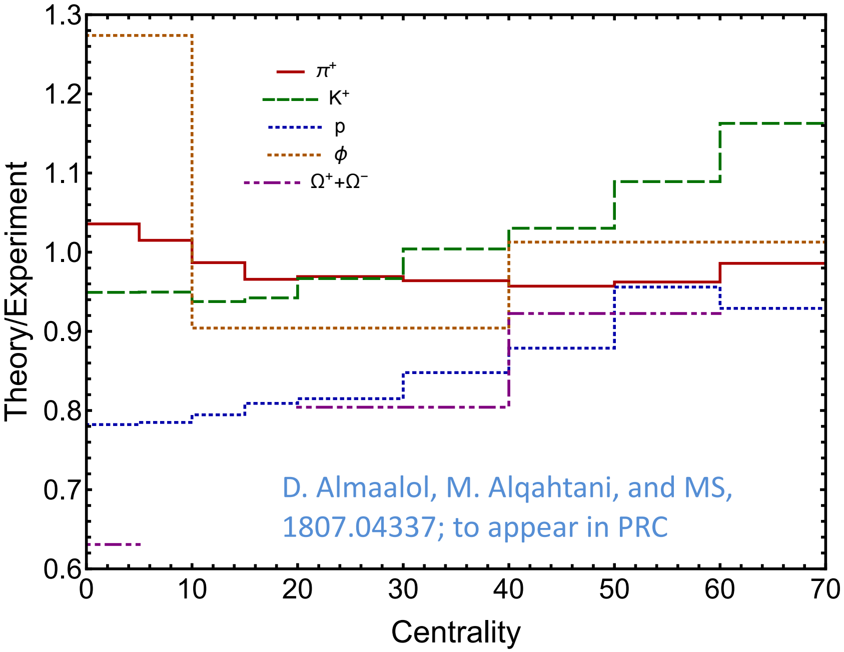


T_0 = 455 MeV @ τ_0 = 0.25 fm/c
$4\pi\eta/s$ = 2.25
T_{FO} = 130 MeV
Optical Glauber IC
No freeze-out chemical potentials

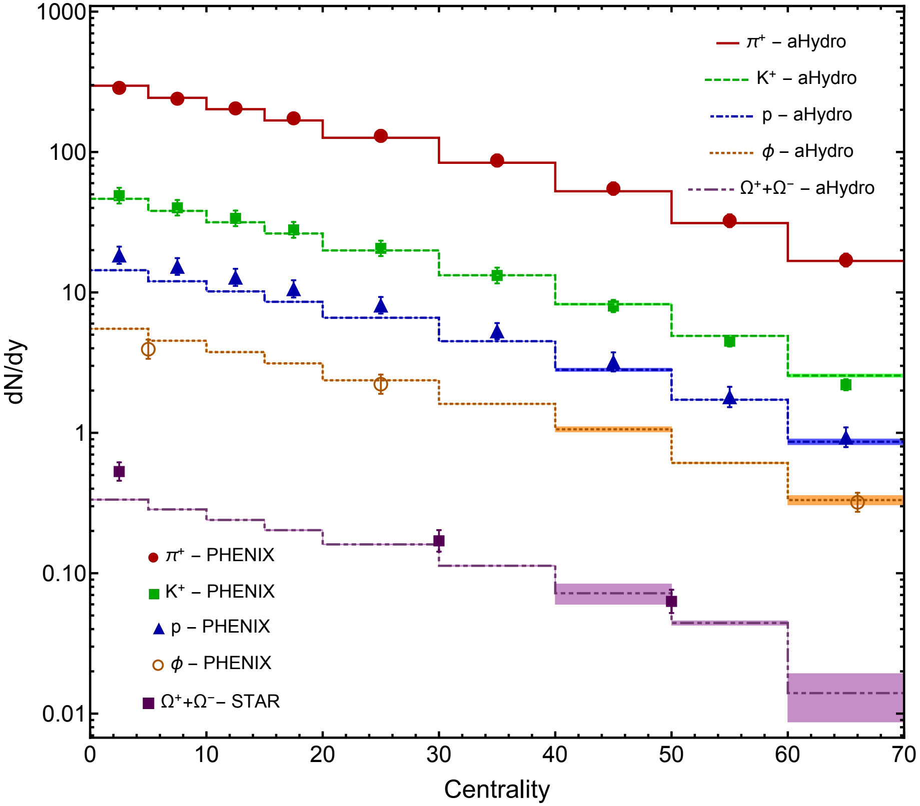
D. Almaalol, M. Alqahtani, and MS, 1807.04337; to appear in PRC

Particle multiplicity (RHIC)





D. Almaalol, M. Alqahtani, and MS, 1807.04337; to appear in PRC



T_0 = 455 MeV @ τ_0 = 0.25 fm/c

$4\pi\eta/s$ = 2.25

T_{FO} = 130 MeV

Optical Glauber IC

No freeze-out chemical potentials

Also works well for v2(p_T,η) for non-central collisions

Conclusions

- Momentum-space anisotropic non-equilibrium attractor is generic
- Attractor for low-order moments well-approximated by hydro but not in equilibrium → **hydrodynamization instead of thermalization**
- Higher-order moments are poorly described by standard hydrodynamics; however, there is a very fast convergence to the non-equilibrium attractor for higher moments → **pseudo-thermalization instead of hydrodynamization**
- One can extract an **attractor for full one-particle distribution function** and it approaches equilibrium in a bottom-up manner
- **aHydro RTA attractor agrees best with the exact attractor for all moments.**
- In the second half I showed comparisons at RHIC and LHC energies which demonstrate that our first aHydro comparisons with data are quite successful given the simple setup.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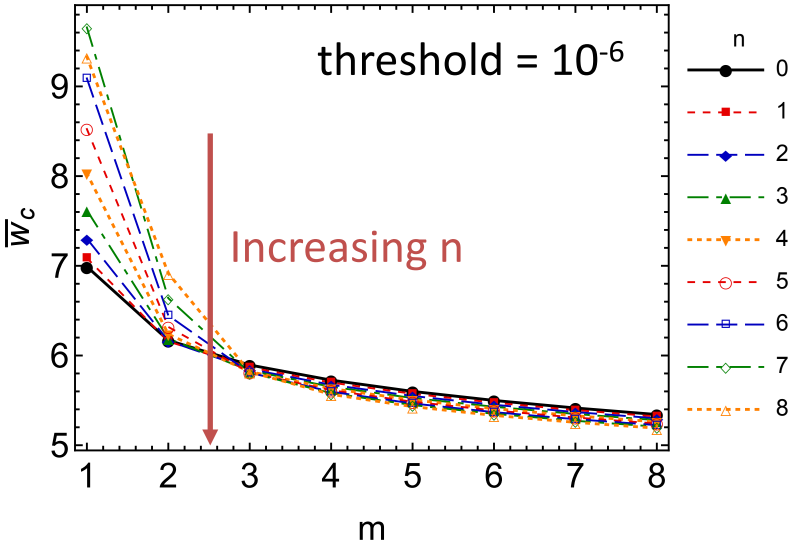
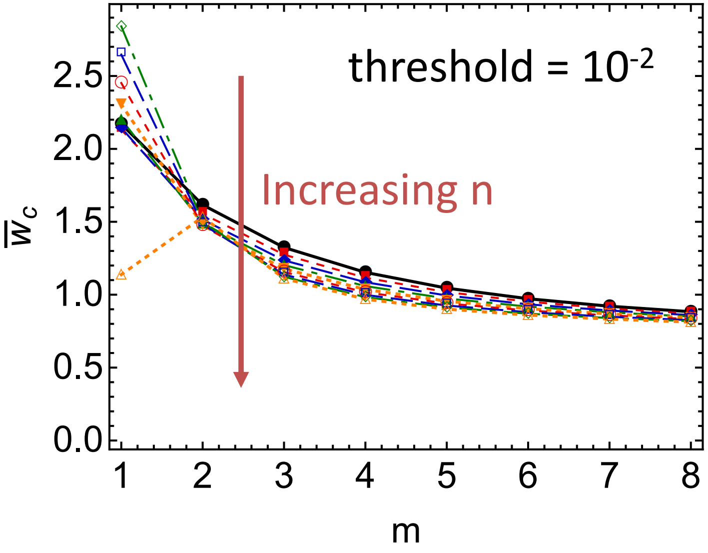
Outlook

- Analytic understanding of higher moment attractors seems possible based on the work of Heller, Kurkela, and Spalinski, 1609.04803 [See also Heller and Svensson, 1802.08225]
- Work is underway to extend this analysis to more **realistic scattering kernels**
- First work along these lines using a scalar kernel has appeared last January [D. Almaalol, and M. Strickland 1801.10173] and extended to enforce number conservation in August [D. Almaalol, M. Alqahtani, and M. Strickland 1808.07038]
- Next step is to look at QCD [in progress with D. Almaalol and A. Kurkela]

Backup Slides

Time scales (convergence threshold)



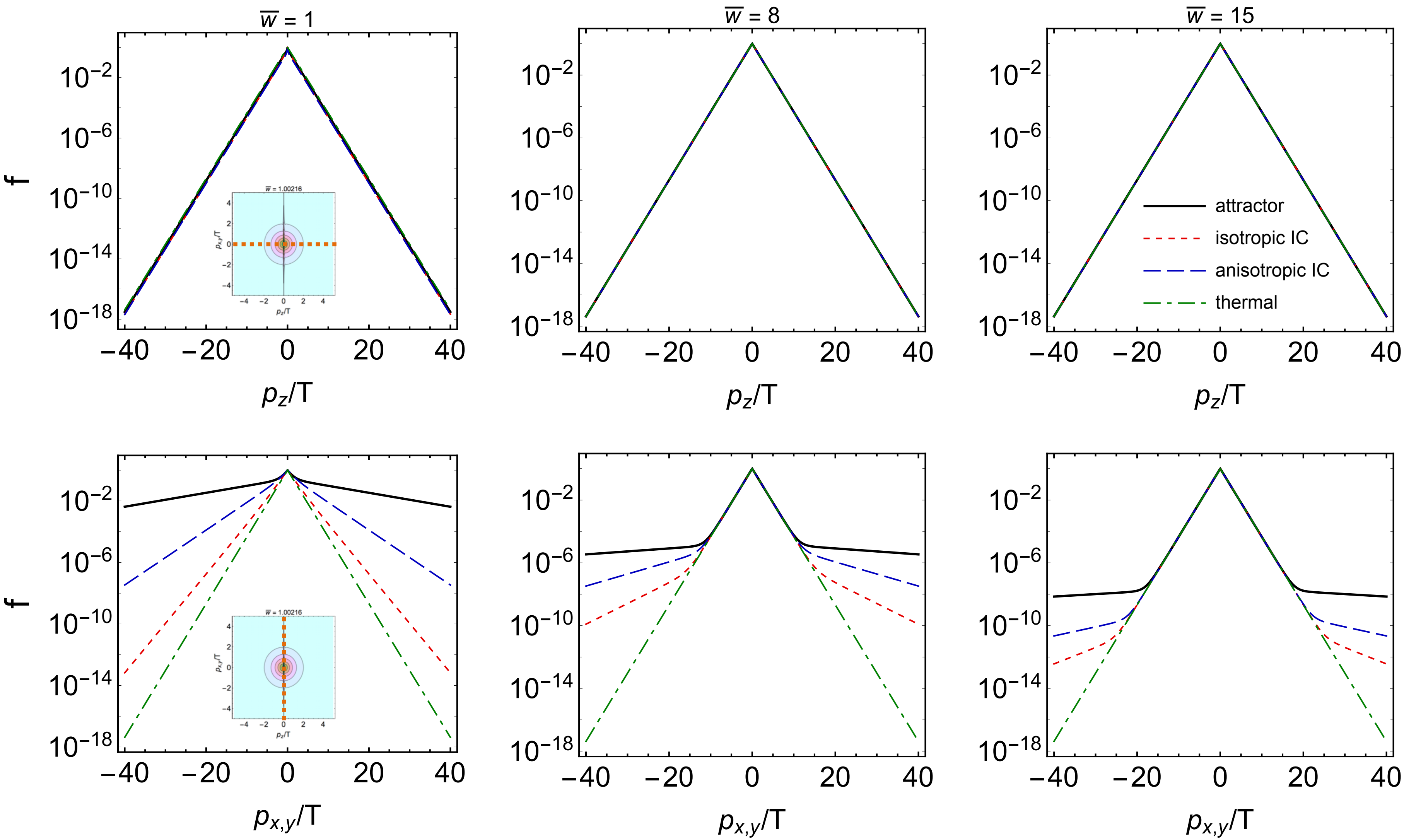


If we only require that the solutions converge to the attractor to within 1% then the convergence times are dramatically shorter

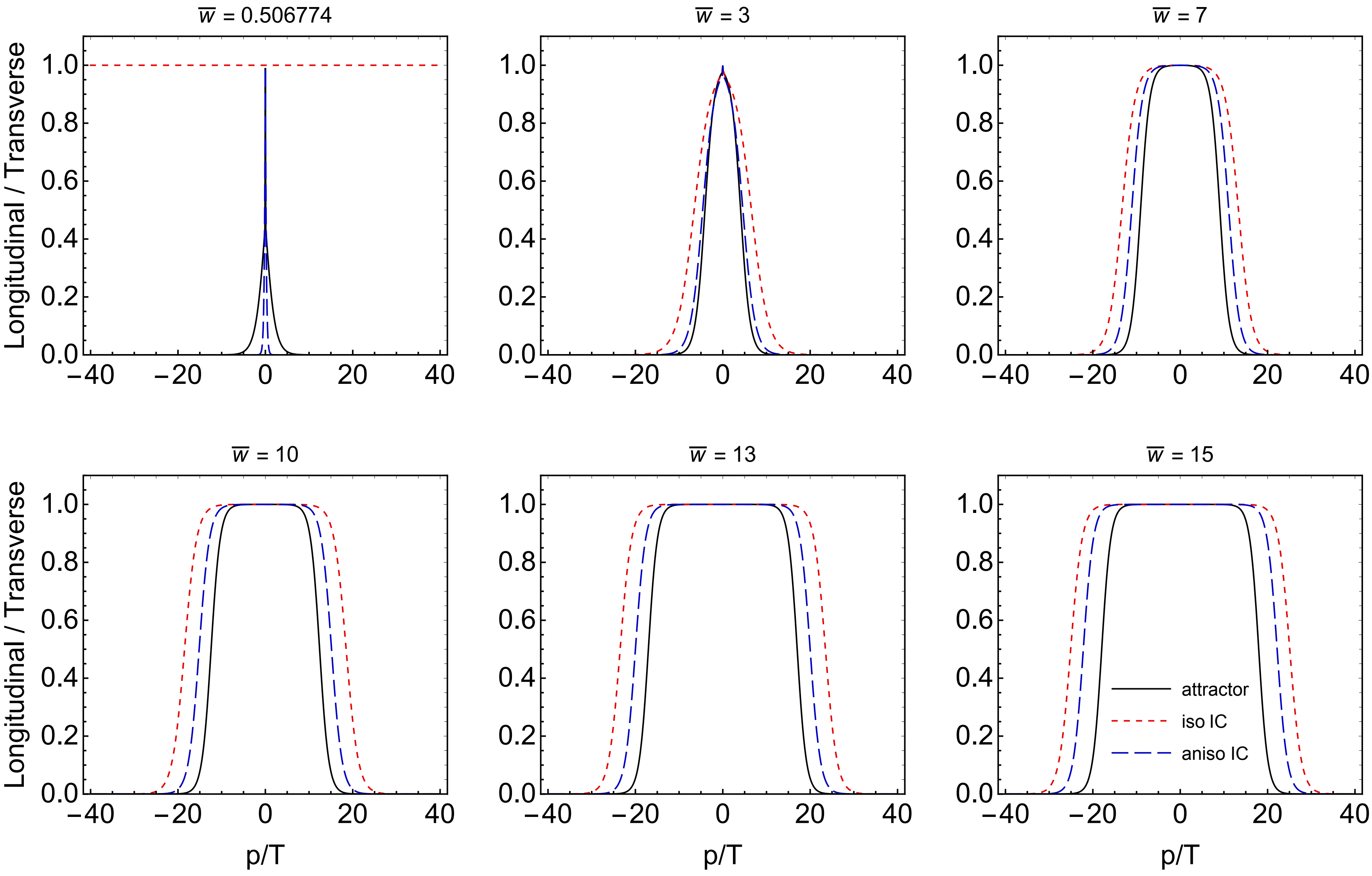
$\overline{w}$	τ
1	0.5 fm/c
2	1.3 fm/c
5	4.9 fm/c
10	13.5 fm/c

Assuming η/s = 0.2 and T = 500 MeV

Attractor distribution function



Isotropization front



Univerisality and non-universality

- Universality → All microscopic theories are equally good starting points; pick the one that's easiest to deal with
- Non-universality → We have to pick which microscopic theory we think best describes the system's dynamics
- I would argue that some sort of pQCD/kinetic theory inspired microscopic theory makes the most sense at early times (particles + CGC) and in the dilute regions (hadronic transport) which are precisely where "non-universal" physics pops up.
- And, since kinetic-theory based models also share the "universal properties" of hydrodynamics at later times, they will also work well in this region.

How does one obtain the attractor?

- Let’s look at hydrodynamics-like theories for simplicity (e.g. MIS, DNMR, aHydro, etc.)
- Start with the 0+1 d energy conservation equation

$$\tau\dot{\epsilon} = -\frac{4}{3}\epsilon + \Pi \qquad \Pi = \Pi^{\varsigma}{}_{\varsigma}$$

- Change variables to

$$\boxed{w = \tau T} \qquad \boxed{\varphi(w) \equiv \tau\frac{\dot{w}}{w} = 1 + \frac{\tau}{4}\partial_\tau \log\epsilon}$$

$$\boxed{w\varphi\frac{\partial\varphi}{\partial w} = -\frac{8}{3} + \frac{20}{3}\varphi - 4\varphi^2 + \frac{\tau}{4}\frac{\dot{\Pi}}{\epsilon}}$$

How does one obtain the attractor?

- Need the evolution equation for the viscous correction.
- To linear order in the shear correction (e.g. MIS, DNMR) one has

$$\dot{\Pi} = \frac{4\eta}{3\tau\tau_\pi} - \beta_{\pi\pi}\frac{\Pi}{\tau} - \frac{\Pi}{\tau_\pi}$$

For DNMR in RTA $\beta_{\pi\pi} = \frac{38}{21}$

- Plugging this into the energy-momentum conservation equation gives

$$\boxed{\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0}$$

$$\overline{w} \equiv \frac{w}{c_\pi} = \frac{\tau T}{5\bar{\eta}} \qquad c_{\eta/\pi} \equiv \frac{c_\eta}{c_\pi} = \frac{1}{5}$$

How does one solve for the attractor?

$$\boxed{\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0}$$

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- First try to approximate using "slow-roll" approx ($\varphi' = 0$)
- From this, we can read off the boundary condition as $w \to 0$

$$\lim_{\overline{w}\to 0} \varphi(\overline{w}) = \frac{1}{24}\left(-3\beta_{\pi\pi} + \sqrt{64c_{\eta/\pi} + (3\beta_{\pi\pi} - 4)^2} + 20\right)$$

- Then numerically solve the ODE at the top of the slide

Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808 (PRL); 1705.10191

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2} \partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2} \partial_\mu m^2 \int dP f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entropy density ratio.

$$\frac{\eta}{\tau_{\rm eq}} = \frac{1}{T} I_{3,2}(\hat{m}_{\rm eq})$$

Bulk viscosi

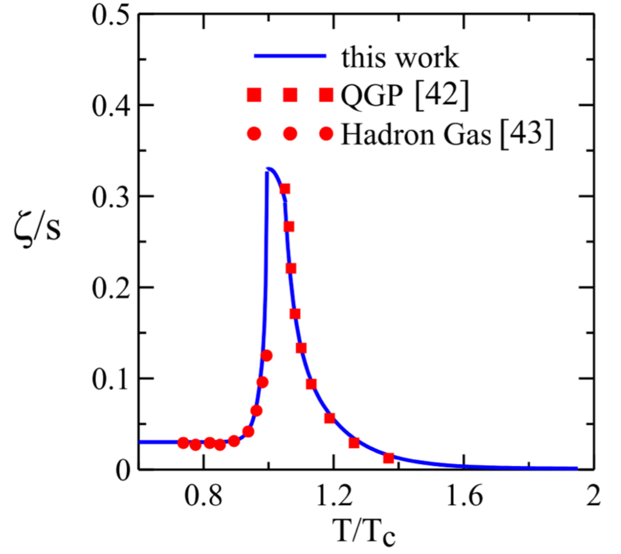
$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I_{3,}$$

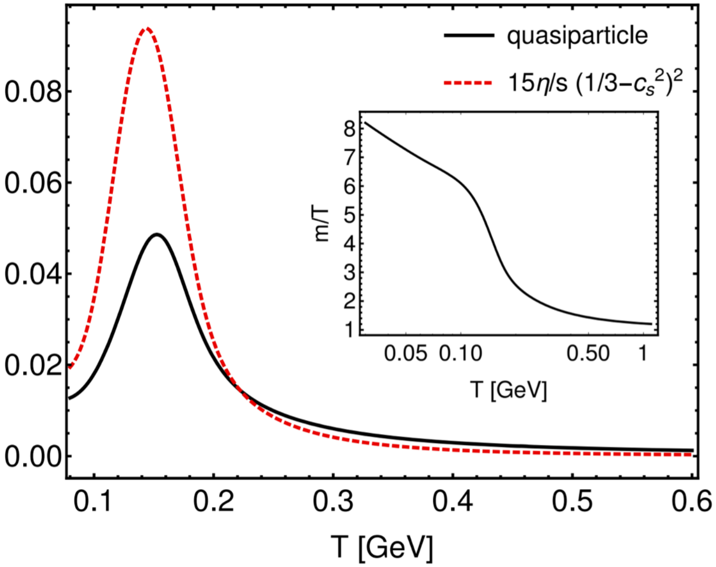
$$I_{3,2}(x) = \frac{N_{\rm dof}}{3}$$

$$K_{i,1}(x) = \frac{\pi}{2} \Big[1$$

$$I_{1,1} = \frac{g\, m}{6\pi^2}$$

Ryu et al, PRL 115 (2015) no.13, 132301





Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101

M. Al 91

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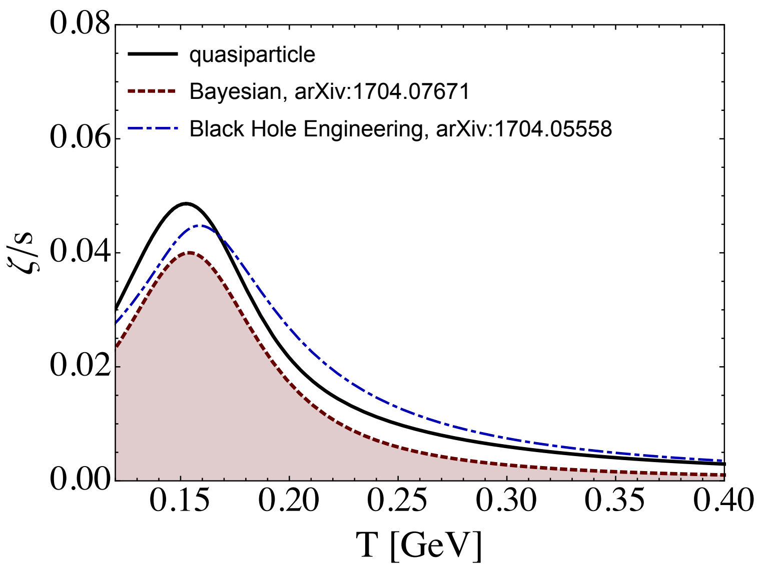

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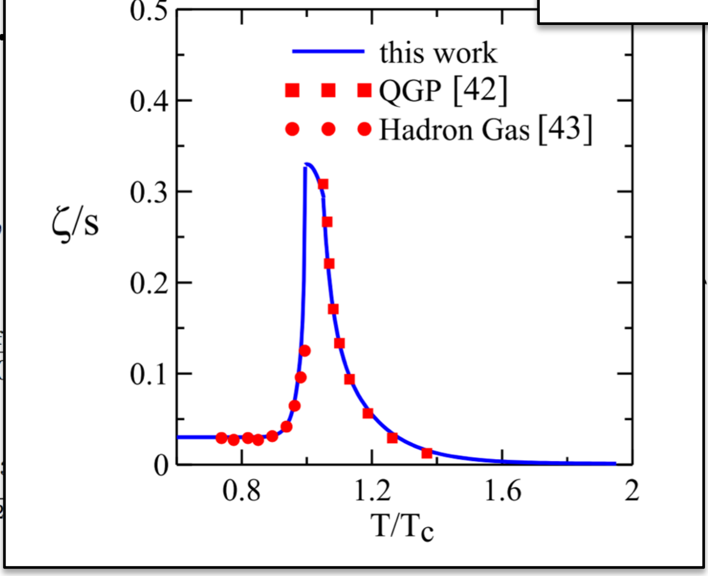
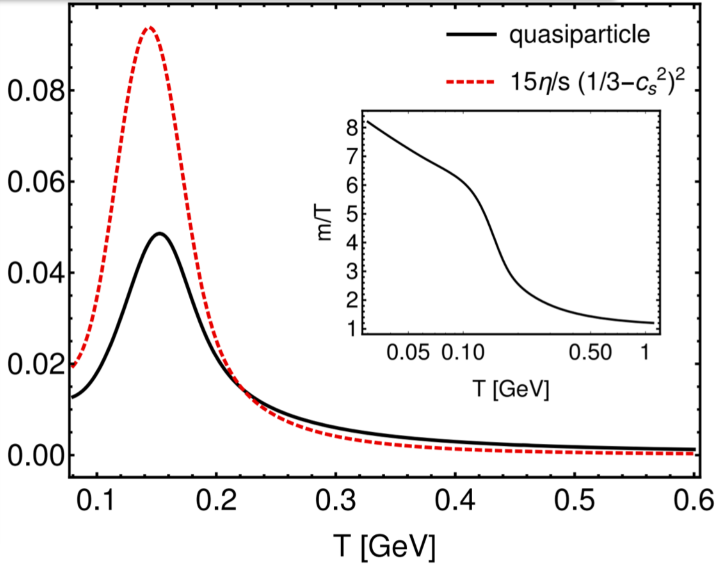
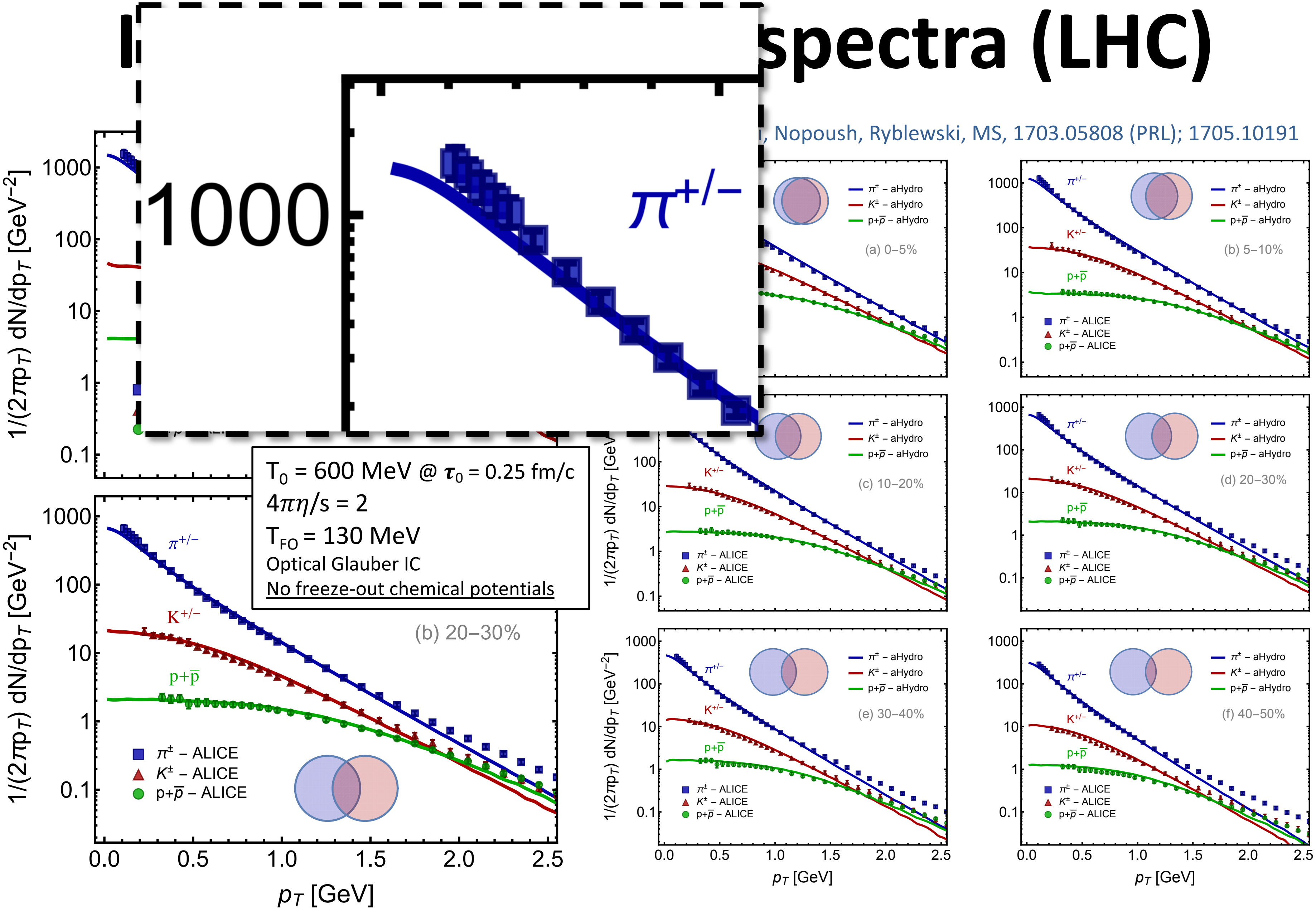
Ryu et al, PRL 115 (2015) no.





spectra (LHC)

Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



T_0 = 600 MeV @ τ_0 = 0.25 fm/c

$4\pi\eta/s = 2$

T_{FO} = 130 MeV

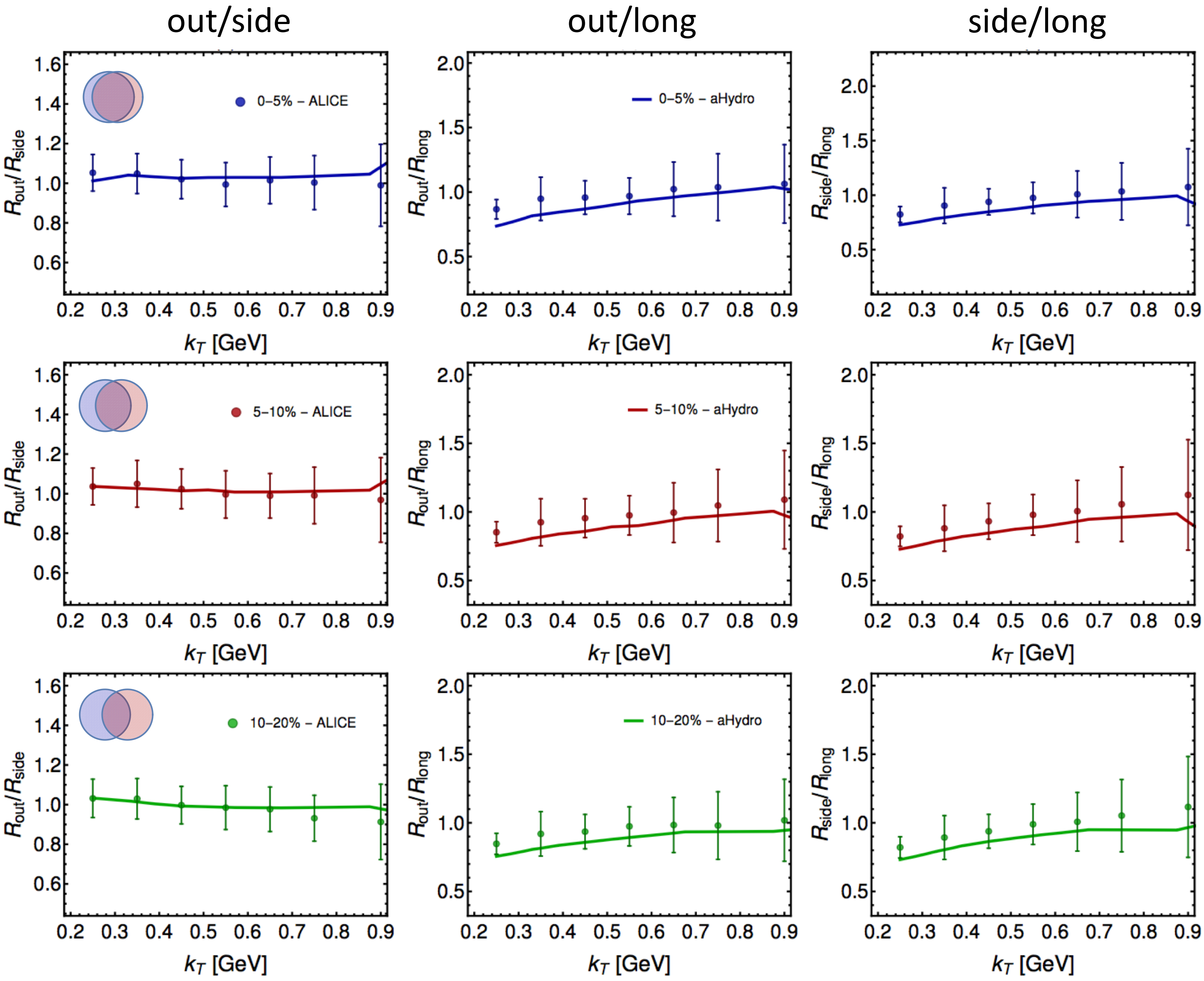
Optical Glauber IC

No freeze-out chemical potentials

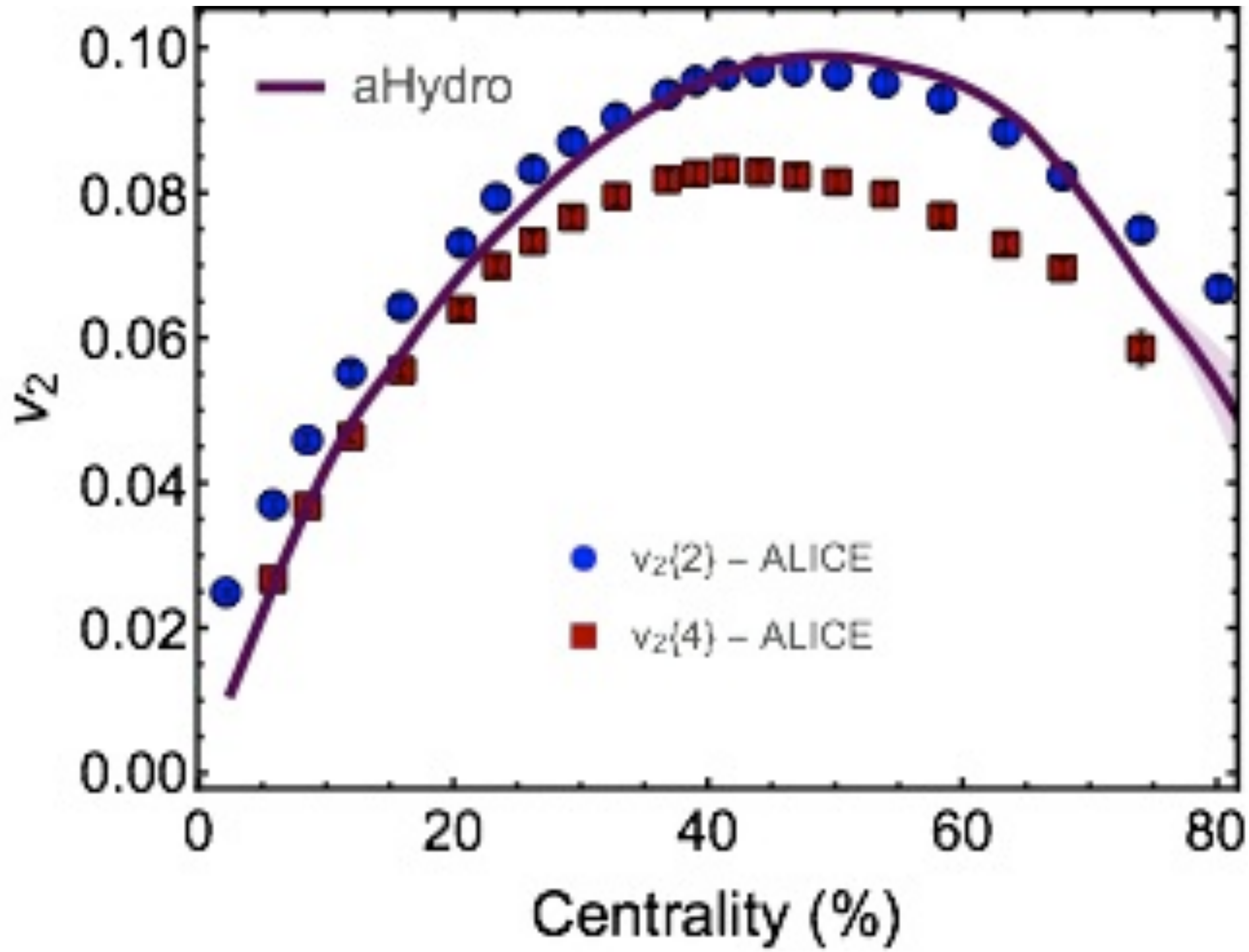
Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

Alqahtani, Nopoush, Ryblewski, MS, 1705.10191

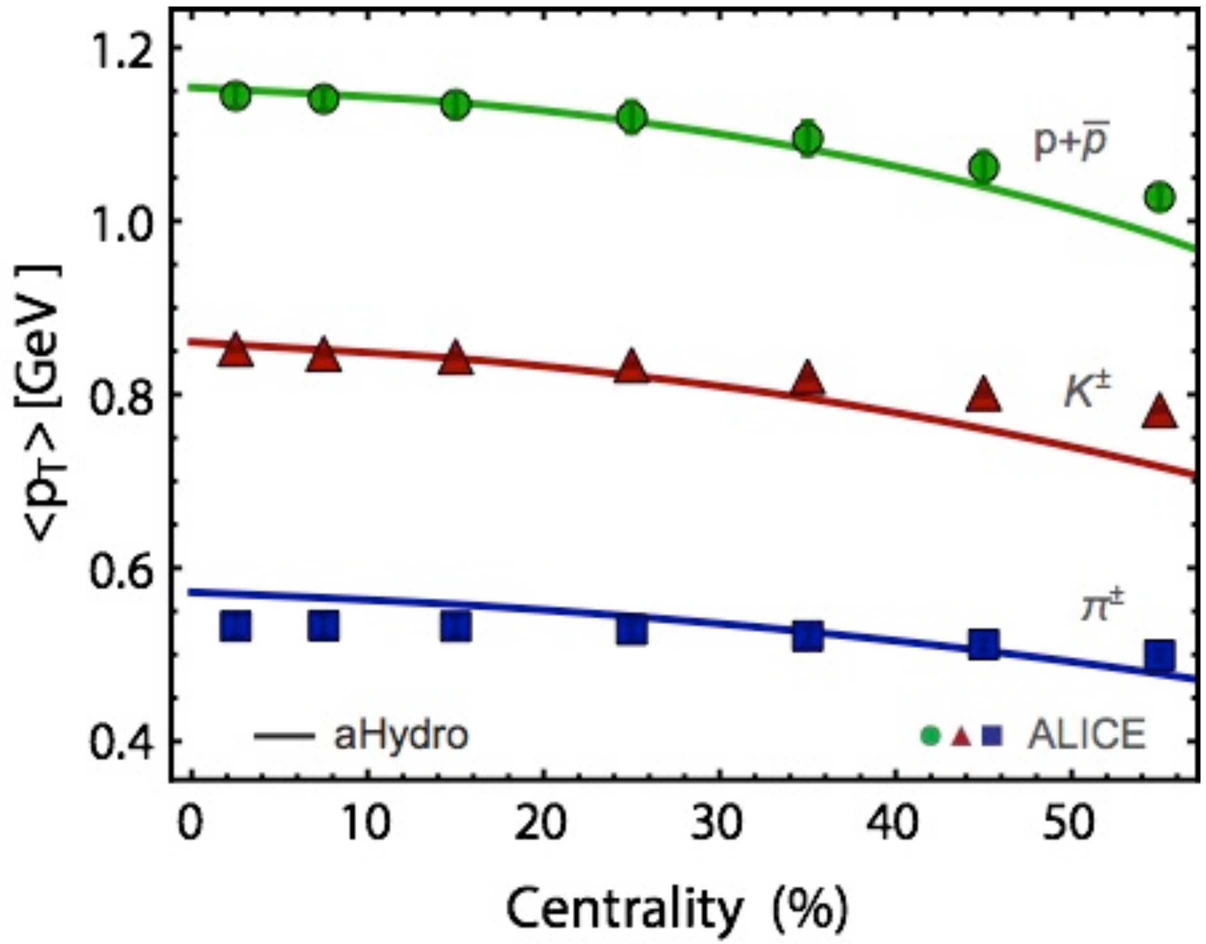
HBT Radii Ratios (LHC)



Integrated v_2 (LHC)

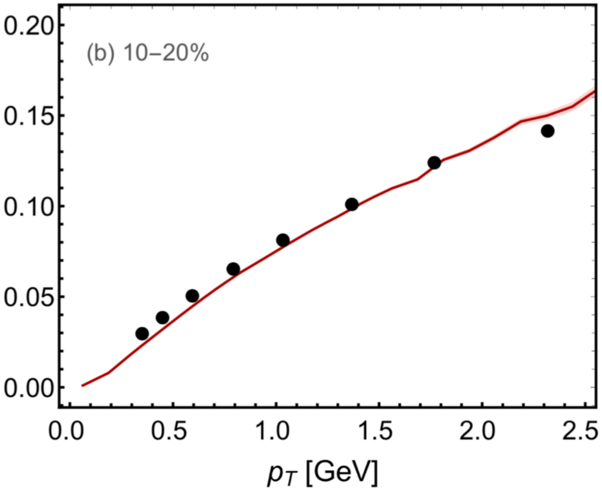


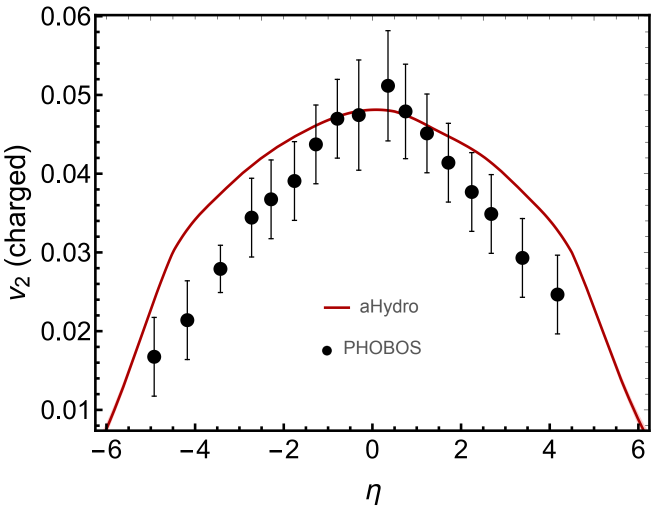
Average p_T (LHC)

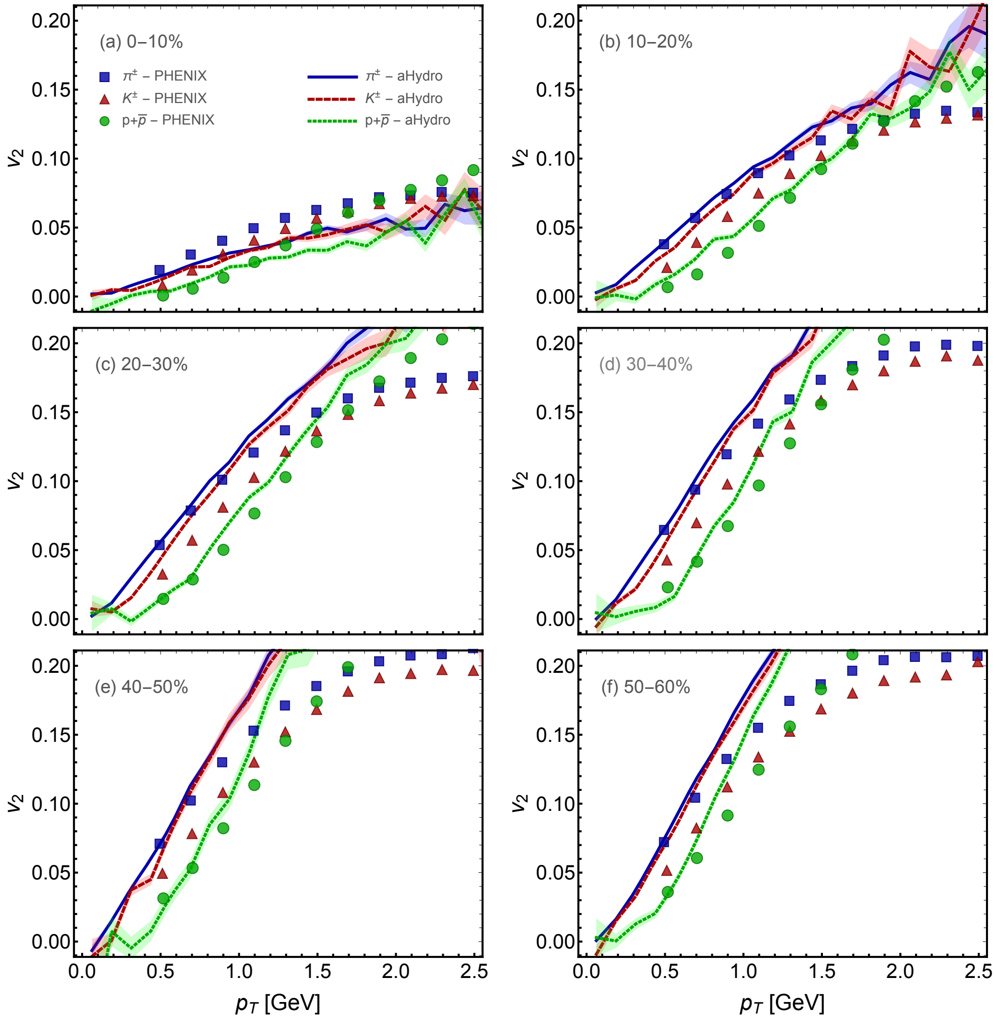


Elliptic Flow (RHIC)

T_0 = 455 MeV @ τ_0 = 0.25 fm/c
$4\pi\eta/s$ = 2.25
T_{FO} = 130 MeV
Optical Glauber IC







D. Almaalol, M. Alqahtani, and MS, 1807.04337; to appear in PRC