Fluctuations of conserved charges and the QCD phase diagram - a lattice QCD perspective

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CRC-TR 211

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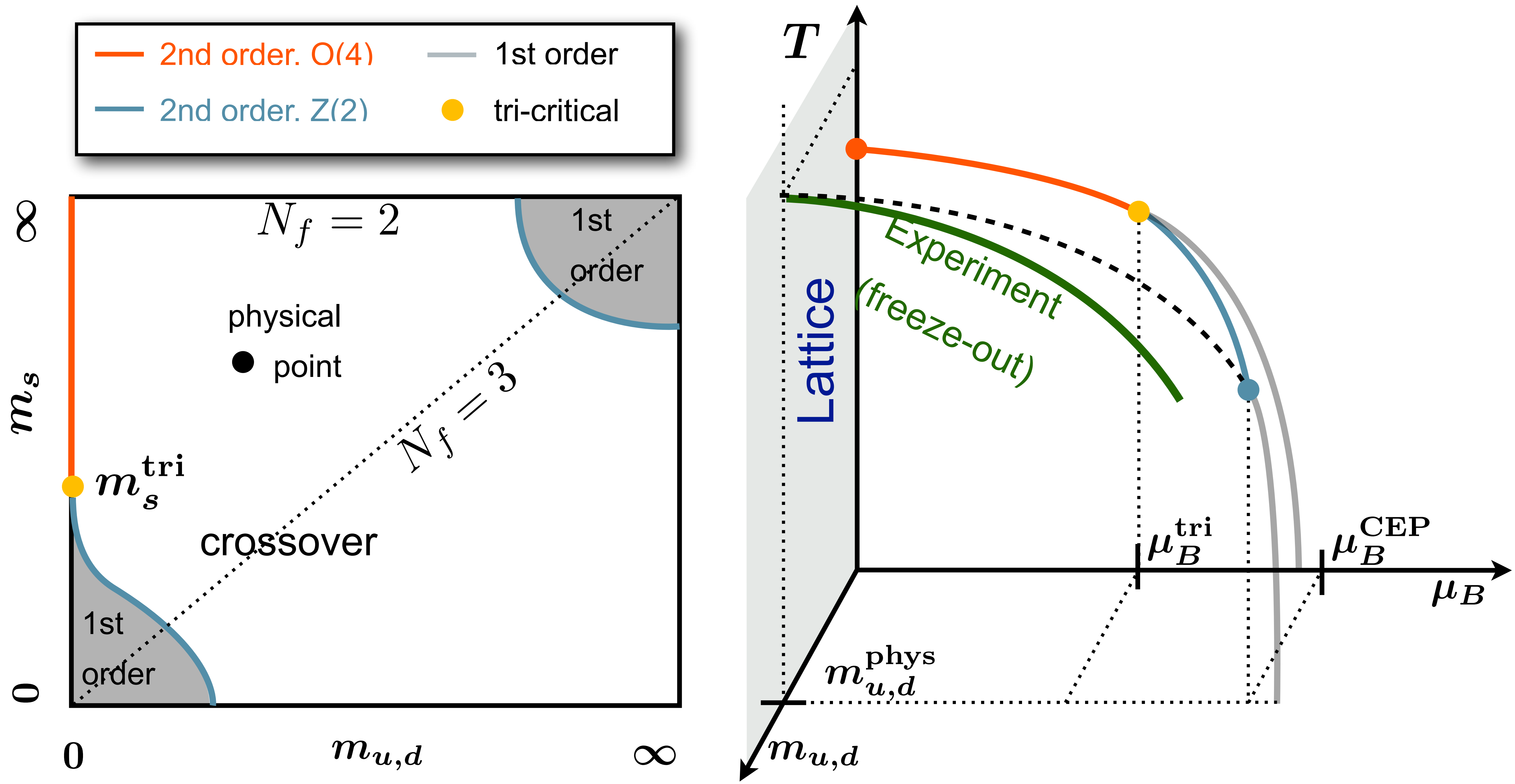


- Expectations based on Pisarski, Wilczek, PRD 29 (1984)

Overview

- The lattice setup
- Taylor Expansion Approach
- The equation of state
- The chiral crossover line
- Fluctuations of conserved charges at ALICE
- Fluctuations of conserved charges at RHIC
- The radius of convergence and the critical point
- Summary

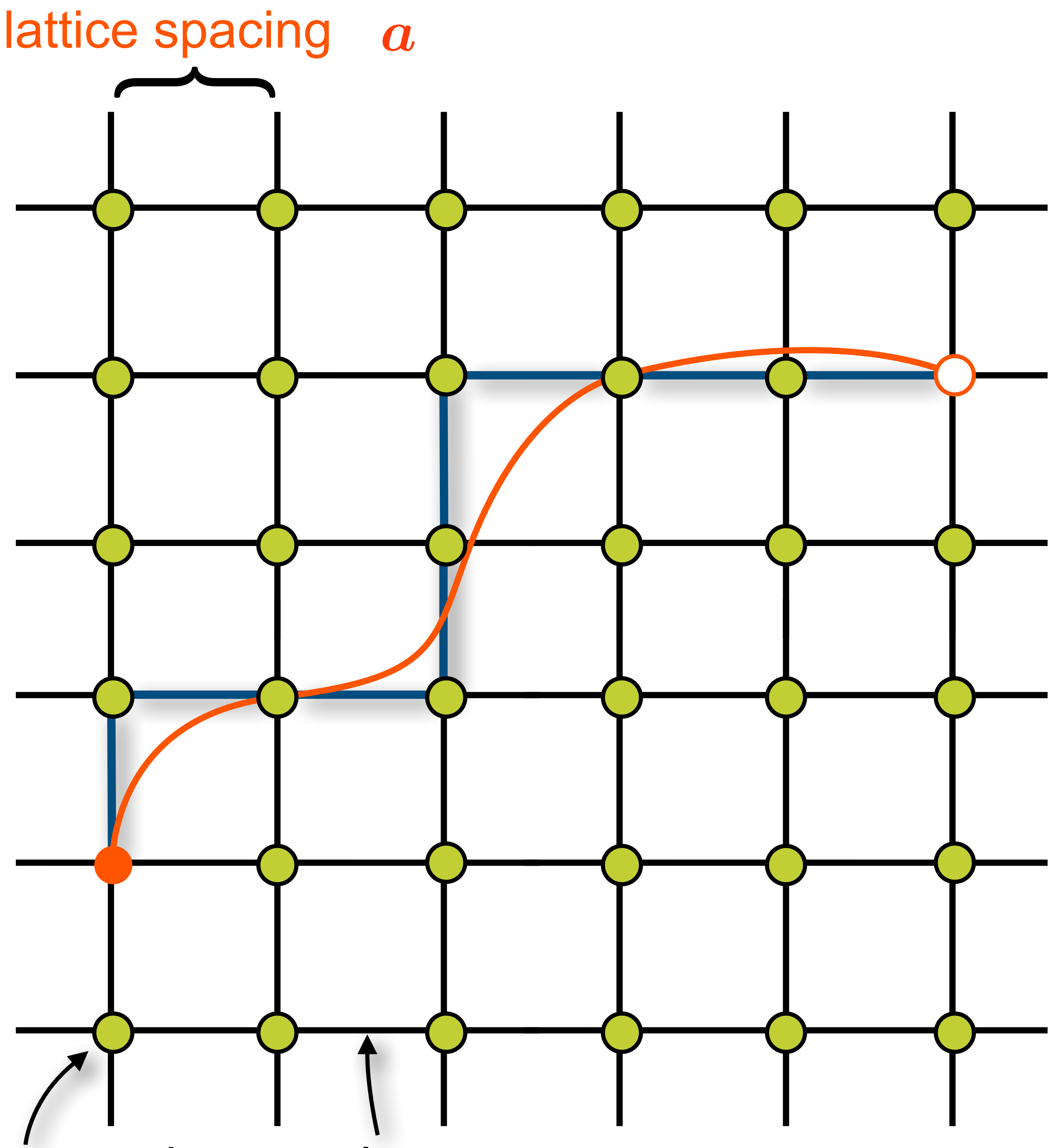
Bazavov et al., Phys. Rev. D95 (2017) 054505.

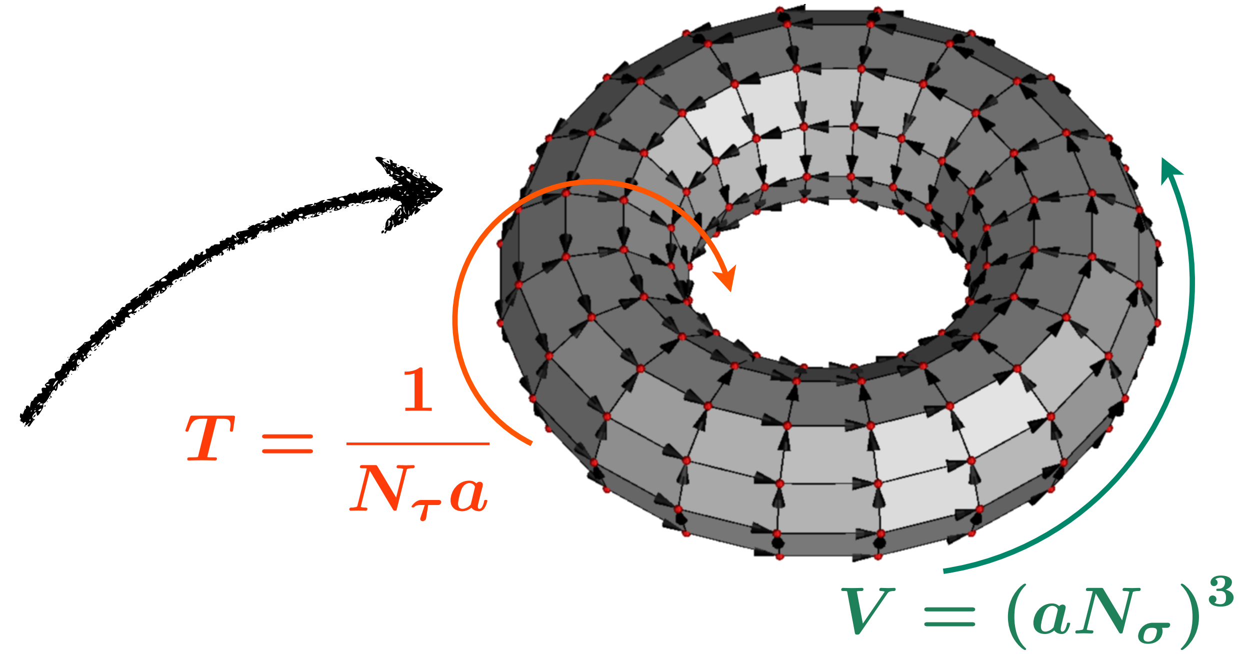
Bazavov et al., Phys. Rev. D96 (2017) 074510.

Bazavov et al., arXiv:1812.08235

Lattice QCD

- introduce a (3+1)-dim. grid with Euclidean signature and periodic boundary conditions (anti-periodic in time), typically 10^6-10^8 grid points





quarks $\psi(x), \bar{\psi}(x)$

gluons $U_\mu(x)$ ➡ SU(3) valued link variables

Consequences:

- Need to take continuum limit $a \to 0$ of all calculations
- momentum cutoff is introduced as $\mathcal{O}(1/a)$
- observables are given in units of a, use physical quantity as input to set the scale

K.G. Wilson, PRD 10 (1974)

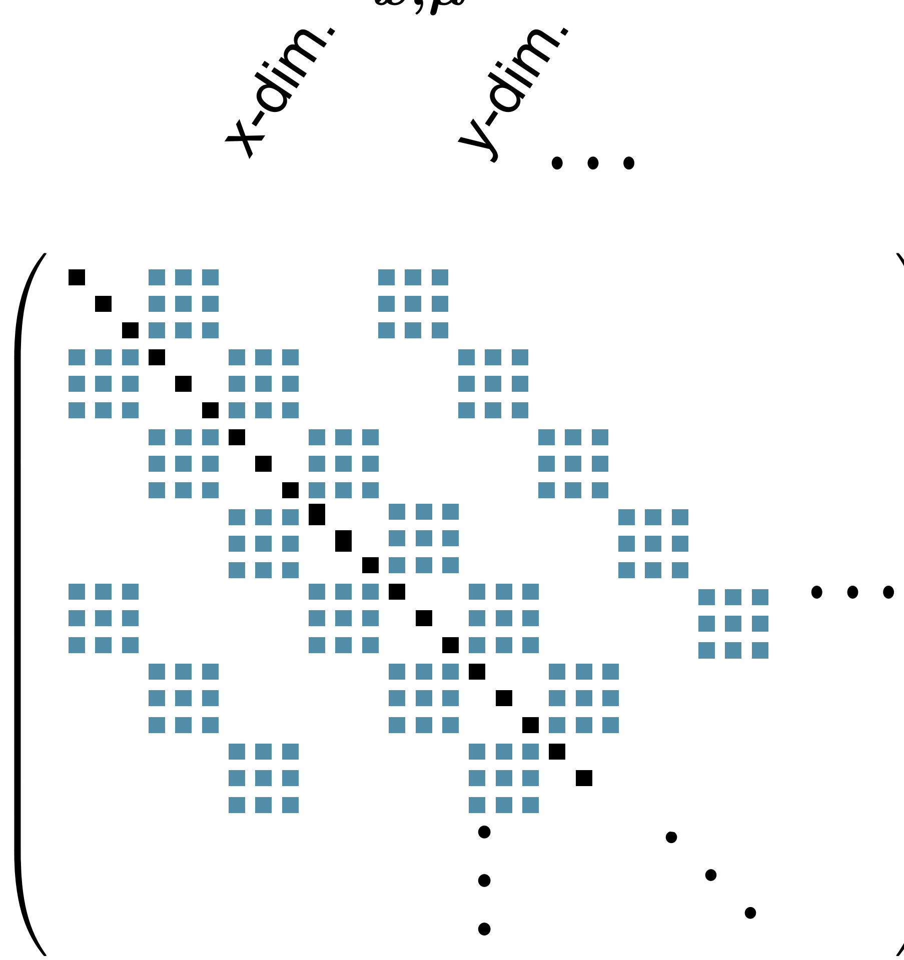
M. Creutz PRD 21 (1980)

- The QCD partition function as path integral:

$$Z(T,V,\mu) = \int \Big(\prod_{x,\mu} \mathrm{d}\bar{\psi}_x \, \mathrm{d}\psi_x \, \mathrm{d}U_{x,\mu}\Big) \, e^{-\bar{\psi}_x M_{xy}(U) \psi_y - \beta S_G(U)}$$

$$= \int \Big(\prod_{x,\mu} \mathrm{d}U_{x,\mu}\Big) \, \det M(U) \, e^{-\beta S_G(U)}$$

$M =$

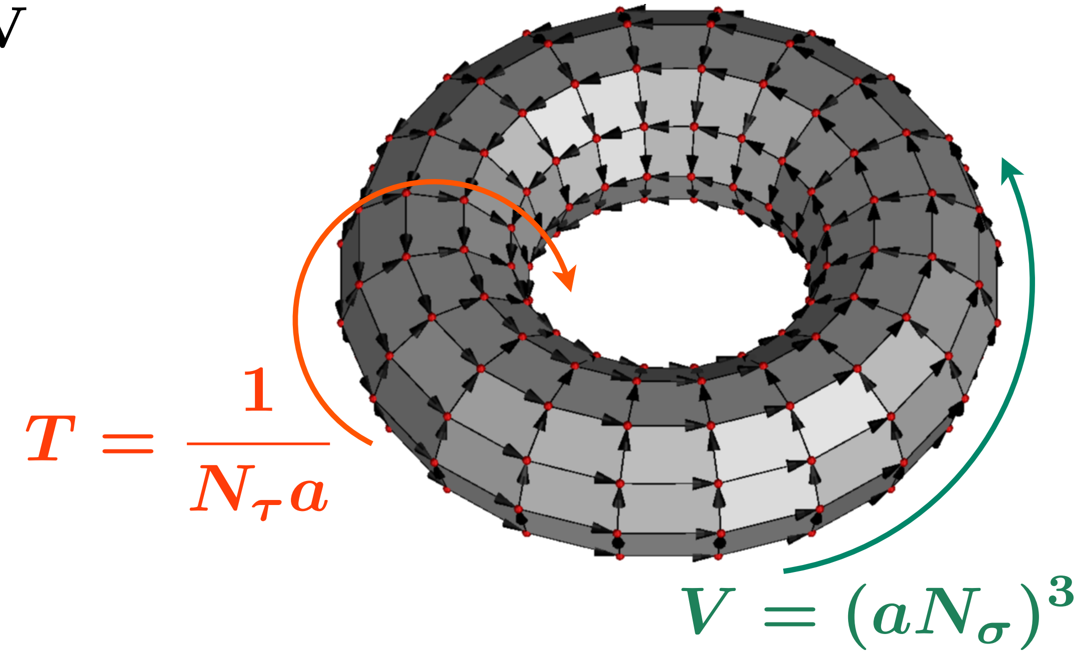


Fermion matrix: large space matrix, band structure due to nearest neighbor interaction, engineered to reduce discretization effects

Monte Carlo integration:
$\approx 10^6$ lattice points,
$\approx 10^8$ degrees of freedom

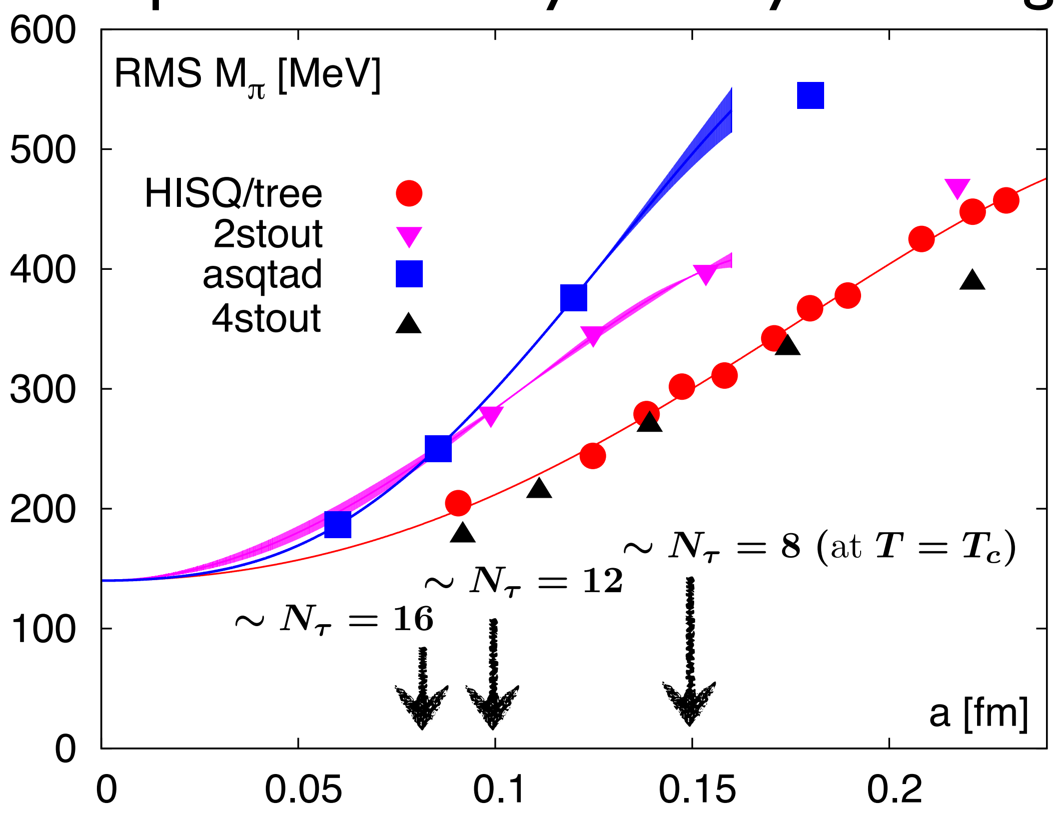
- We use (2+1)-flavor of HISQ fermions at physical quark masses in the range $135\ \mathrm{MeV} \leq T \leq 175\ \mathrm{MeV}$

- Our lattice sizes are $N_\sigma^3 \times N_\tau$, with $N_\sigma = 4N_\tau$ and $N_\tau = 6, 8, 12, 16$

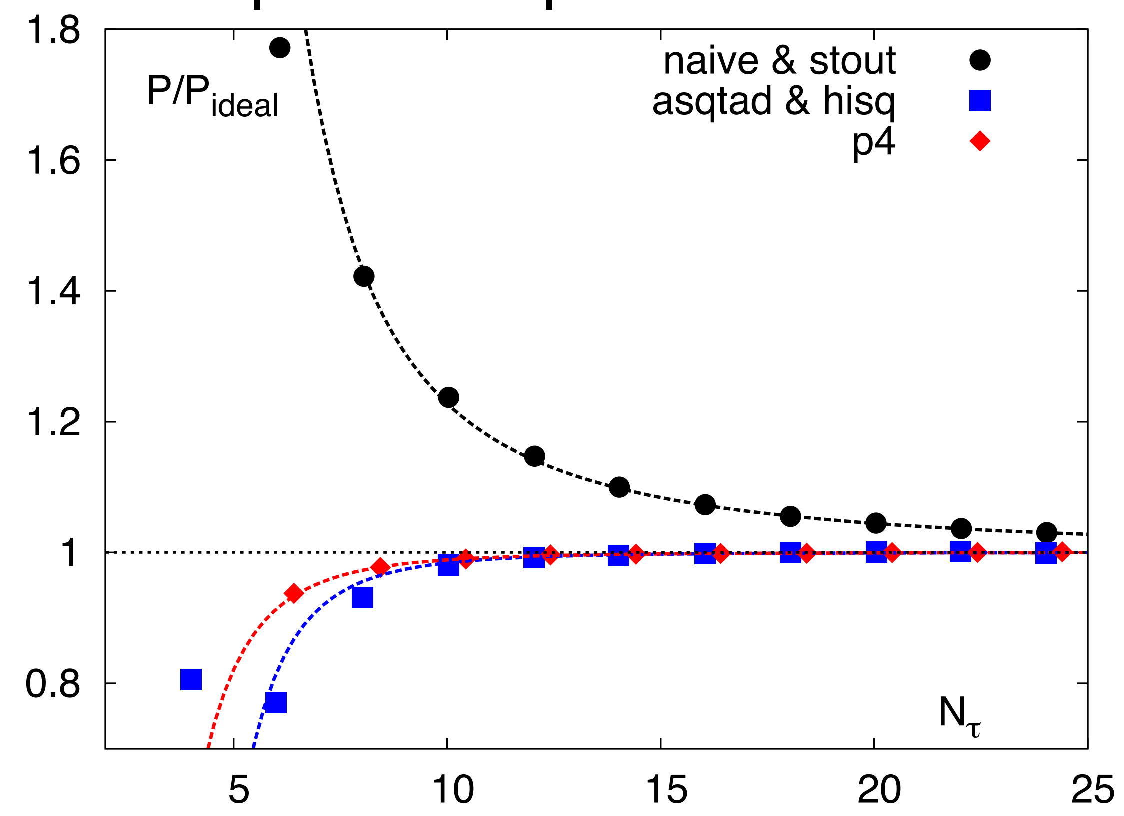


HISQ properties:

improved taste symmetry breaking



improved dispersion relation



Lattice observables

- Equilibrium thermodynamic behavior is determined by the partition function of the system

$$p(T,V,\vec{\mu}) = \frac{T}{V} \ln Z(T,V,\vec{\mu})$$ (thermal EoS)

- Thermodynamic expectation values are calculated numerically by means of Monte Carlo methods (with importance sampling) at $\vec{\mu} \equiv 0$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \Big(\prod_{x,\nu} \mathrm{d}U_{x,\nu} \Big) \, \mathcal{O} \, \underbrace{\det M(U,\vec{m},\vec{\mu}) \; e^{-\beta S_G(U)}}$$

positive-definite weight function at $\vec{\mu} \equiv 0$,

at $|\vec{\mu}| > 0$: $\det(M)$ becomes complex

Sign problem!

- Studie derivatives of $\ln Z$ at $\vec{\mu} \equiv 0$

Chiral condensate

$$\frac{\partial \ln Z}{\partial m} = \langle \mathrm{Tr}[M^{-1}] \rangle = \langle \bar{\psi}\psi \rangle$$

Order-parameter of spontaneous chiral symmetry breaking

Baryon number density

$$\frac{\partial \ln Z}{\partial \mu_B} = \langle \mathrm{Tr}[M^{-1}M'] \rangle$$

$$M' := \frac{\partial M}{\partial \mu}$$

Baryon number fluctuations

$$\frac{\partial^2 \ln Z}{\partial \mu_B^2} = \langle \mathrm{Tr}[M^{-1}M''] \rangle - \langle \mathrm{Tr}[M^{-1}M'M^{-1}M'] \rangle$$

$\rightarrow$ Fermion matrix need to be inverted: computationally demanding

Fluctuations of conserved charges

$$\frac{p(\vec{\mu},T)}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

with $\chi_{i,j,k}^{BQS}(T) = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln Z(\vec{\mu},T)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\vec{\mu}=0}$ and $\hat{\mu} = \mu/T$

Example:

$$\frac{\partial^2 \ln Z}{\partial \mu^2} = \left\langle \mathrm{Tr}\left[M^{-1}M''\right]\right\rangle - \left\langle \mathrm{Tr}\left[M^{-1}M'M^{-1}M'\right]\right\rangle + \left\langle \mathrm{Tr}\left[M^{-1}M'\right]^2\right\rangle$$

$$\simeq \left\langle n^2(x) \bigcirc \right\rangle - \left\langle n(x) \bigcirc n(y) \right\rangle + \left\langle n(x) \bigcirc \; \bigcirc n(y) \right\rangle$$

Lattice		Experiment
$R_{12}^X(T,\mu_B) \equiv \frac{\chi_1^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$\frac{M_X}{\sigma_X^2}$
$R_{32}^X(T,\mu_B) \equiv \frac{\chi_3^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$S_X \sigma_X$
$R_{42}^X(T,\mu_B) \equiv \frac{\chi_4^X(T,\mu_B)}{\chi_2^X(T,\mu_B)}$	$=$	$\kappa_X \sigma_X^2$

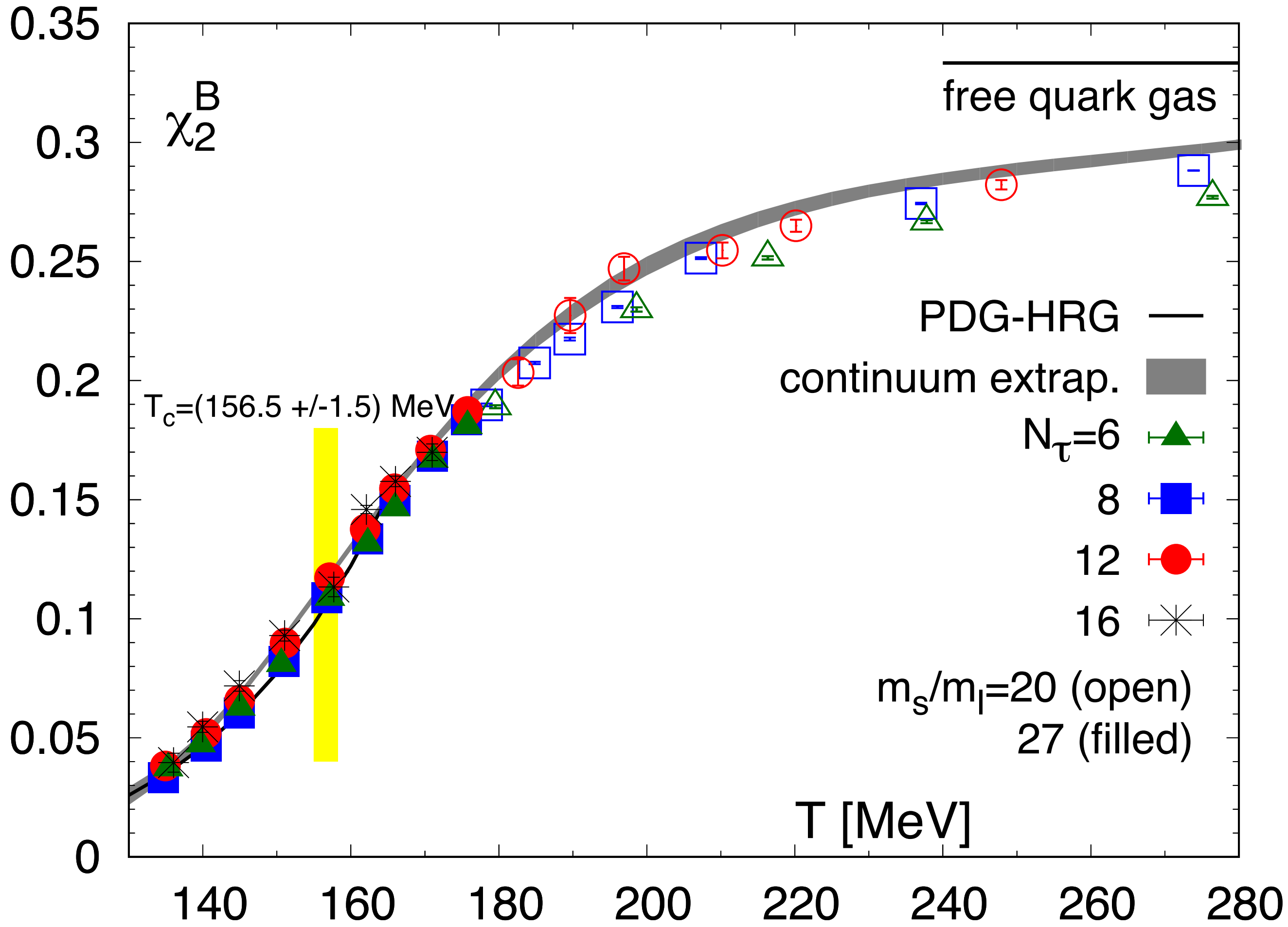
M := mean
σ^2 := variance
S := skewness
κ := kurtosis

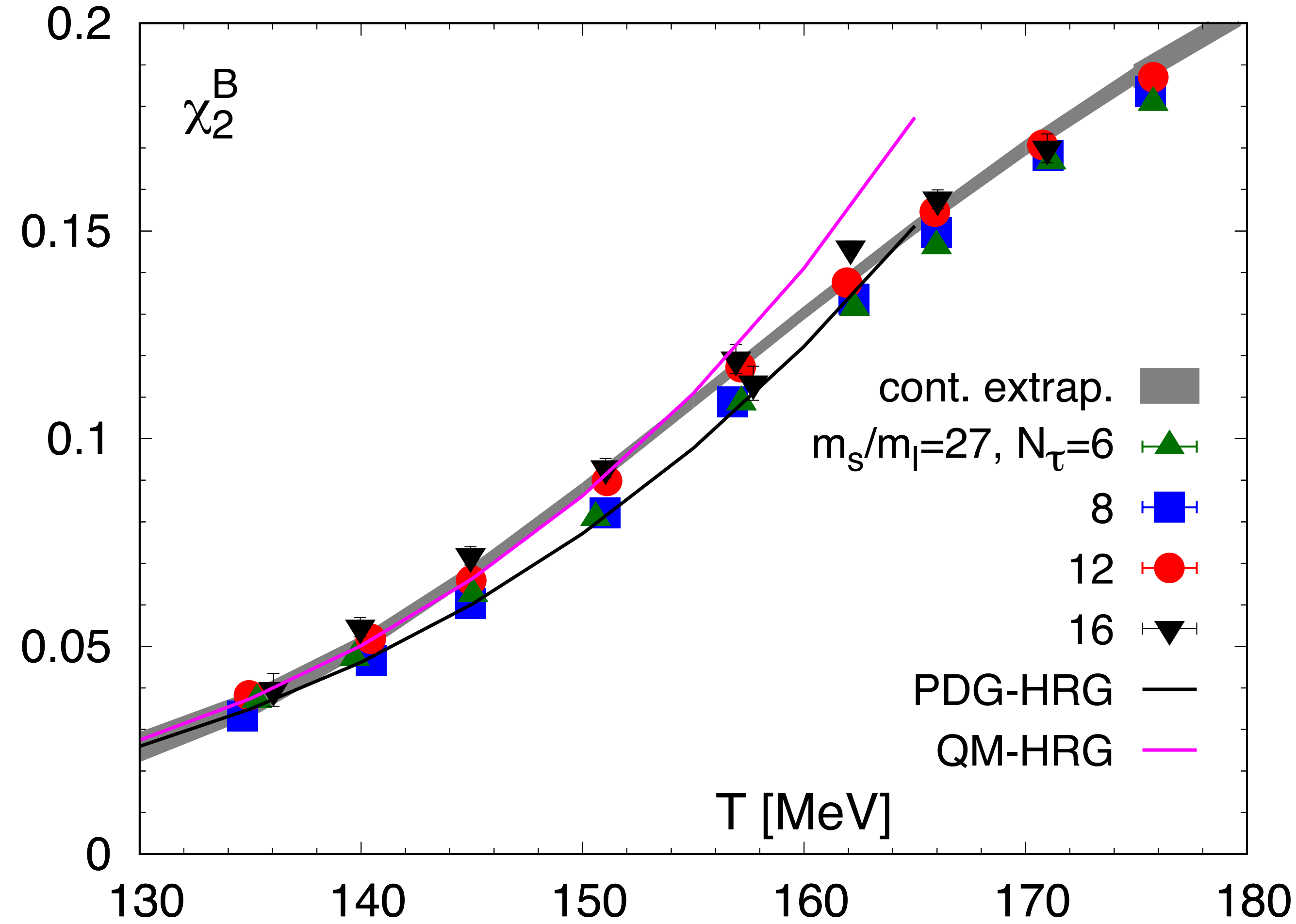
Equation of State at $\mu_B > 0$

pressure correction due to non-vanishing baryon chemical potential:

$$\frac{\Delta p(T,\mu_B)}{T^4} = \frac{p(T,\mu_B) - p(T,0)}{T^4} = \frac{1}{2}\chi_2^B \left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12}\frac{\chi_4^B}{\chi_2}\left(\frac{\mu_B}{T}\right)^2 + \cdots\right)$$

LO: variance of baryon number fluctuation





Equation of State at $\mu_B > 0$

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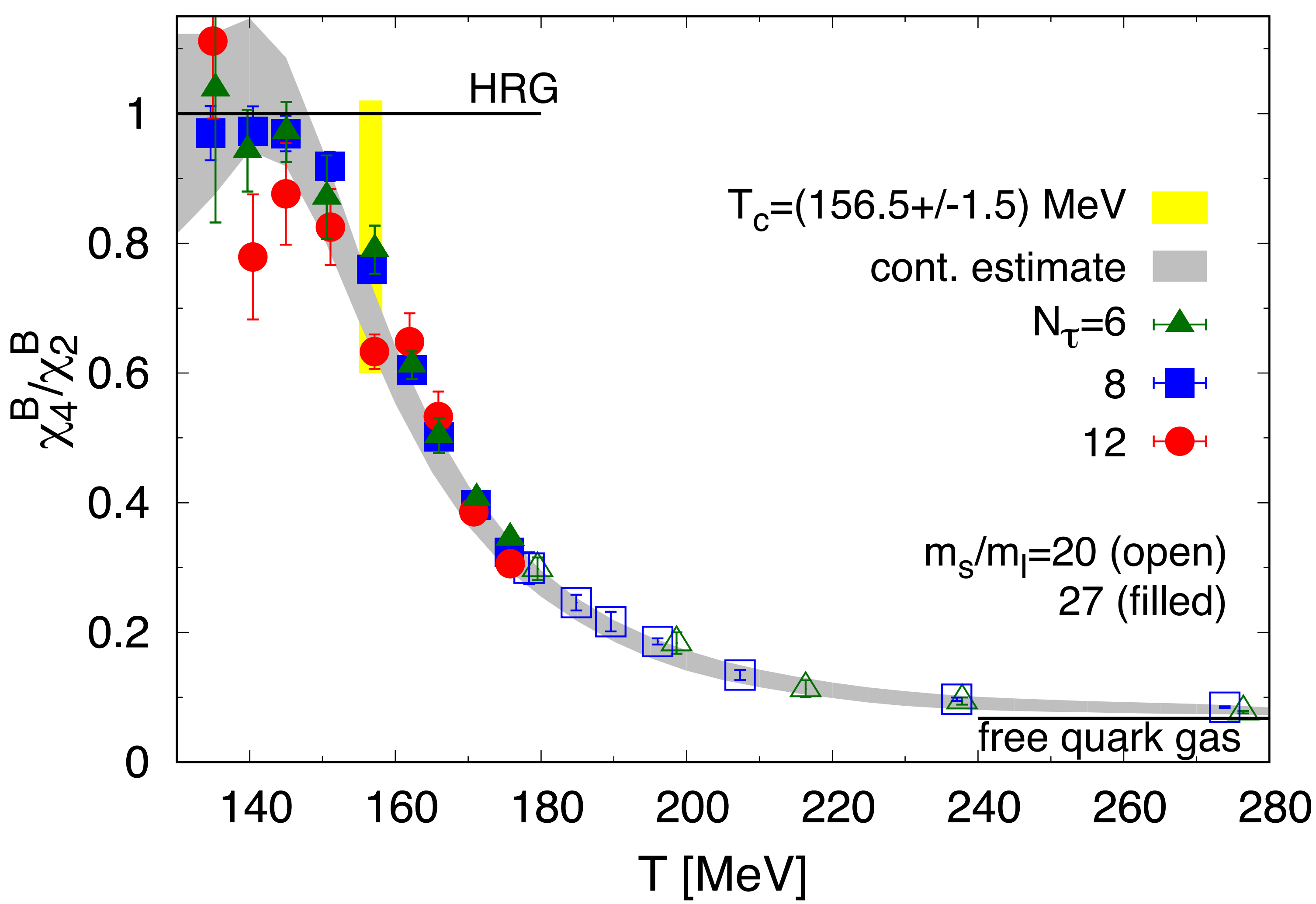
NLO: kurtosis x variance

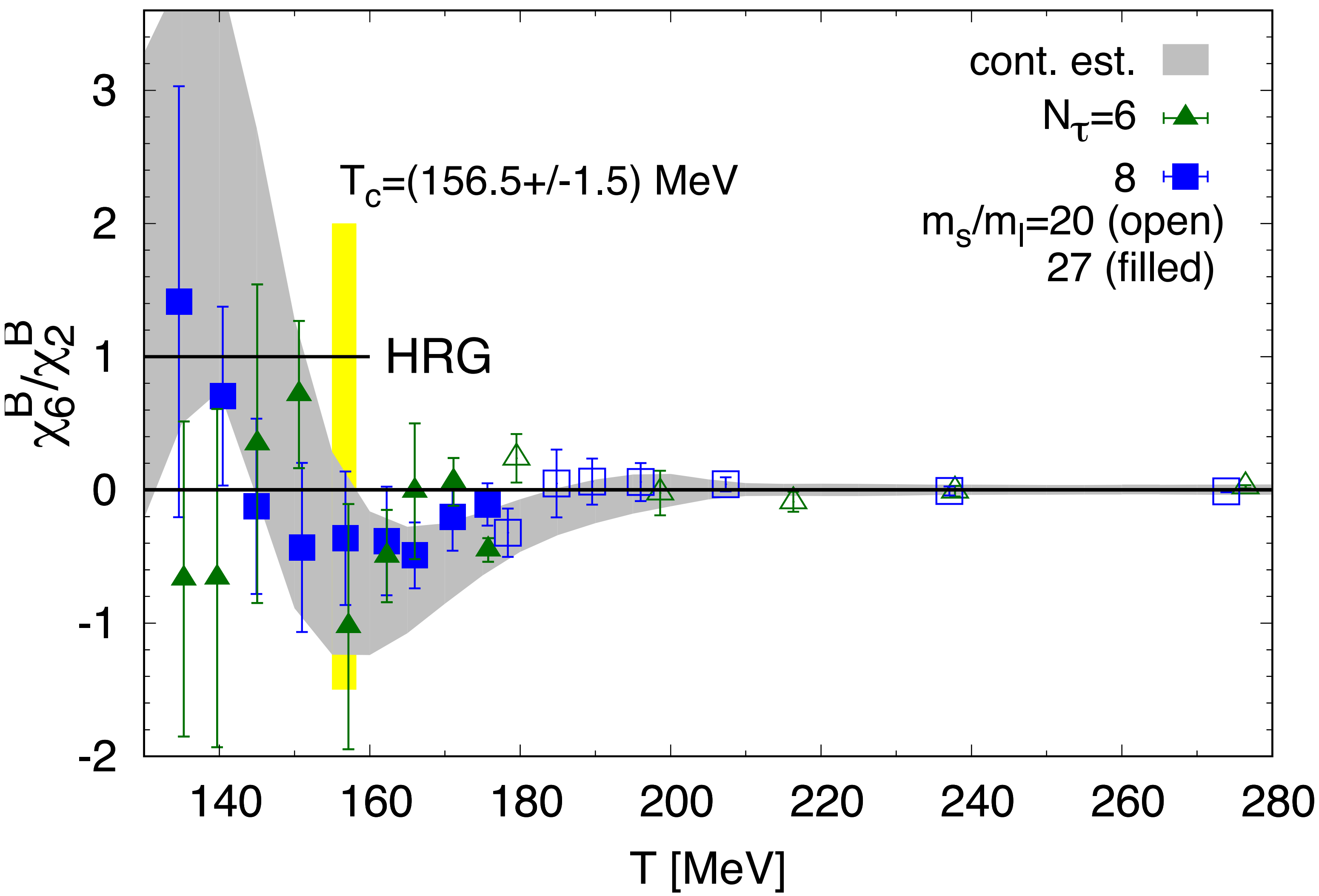


Equation of State at $\mu_B > 0$

pressure correction due to non-vanishing baryon chemical potential:

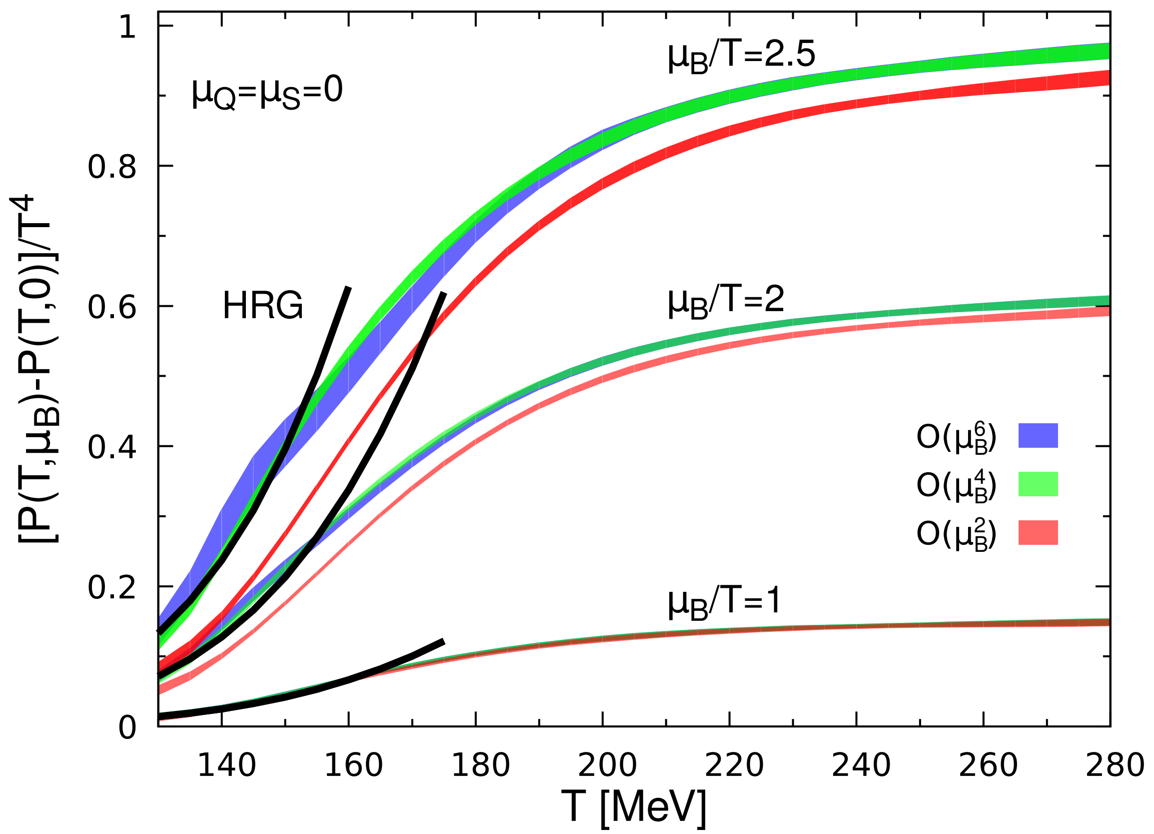
$$\frac{\Delta p(T,\mu_B)}{T^4} = \frac{1}{2}\chi_2^B\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2}\left(\frac{\mu_B}{T}\right)^2+\frac{1}{360}\frac{\chi_6^B}{\chi_2}\left(\frac{\mu_B}{T}\right)^4+\cdots\right)$$

NNLO

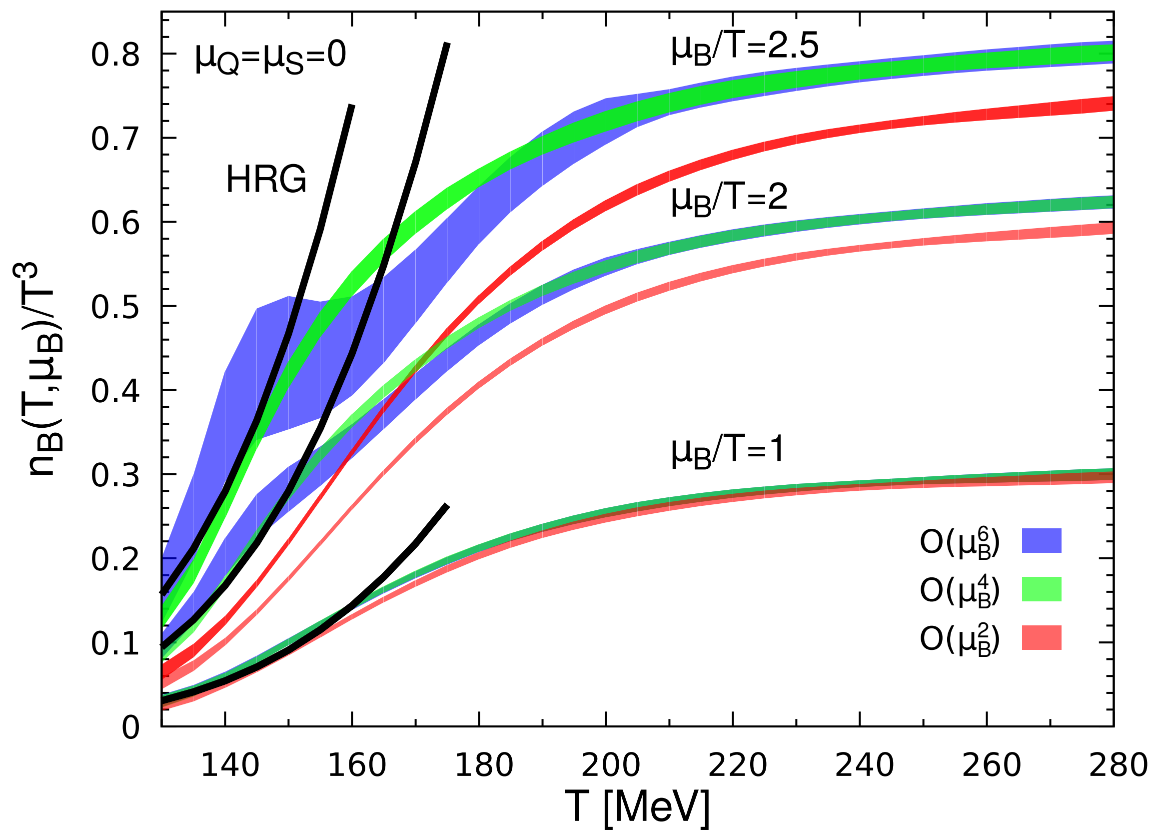


Equation of State at $\mu_B > 0$

pressure correction

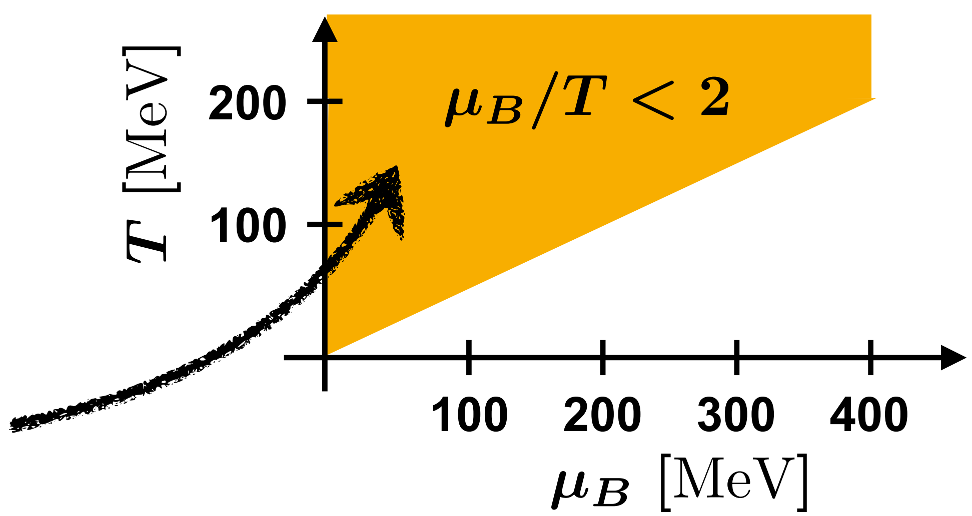


baryon number



Bazavov et al., Phys. Rev. D95 (2017) 054505.

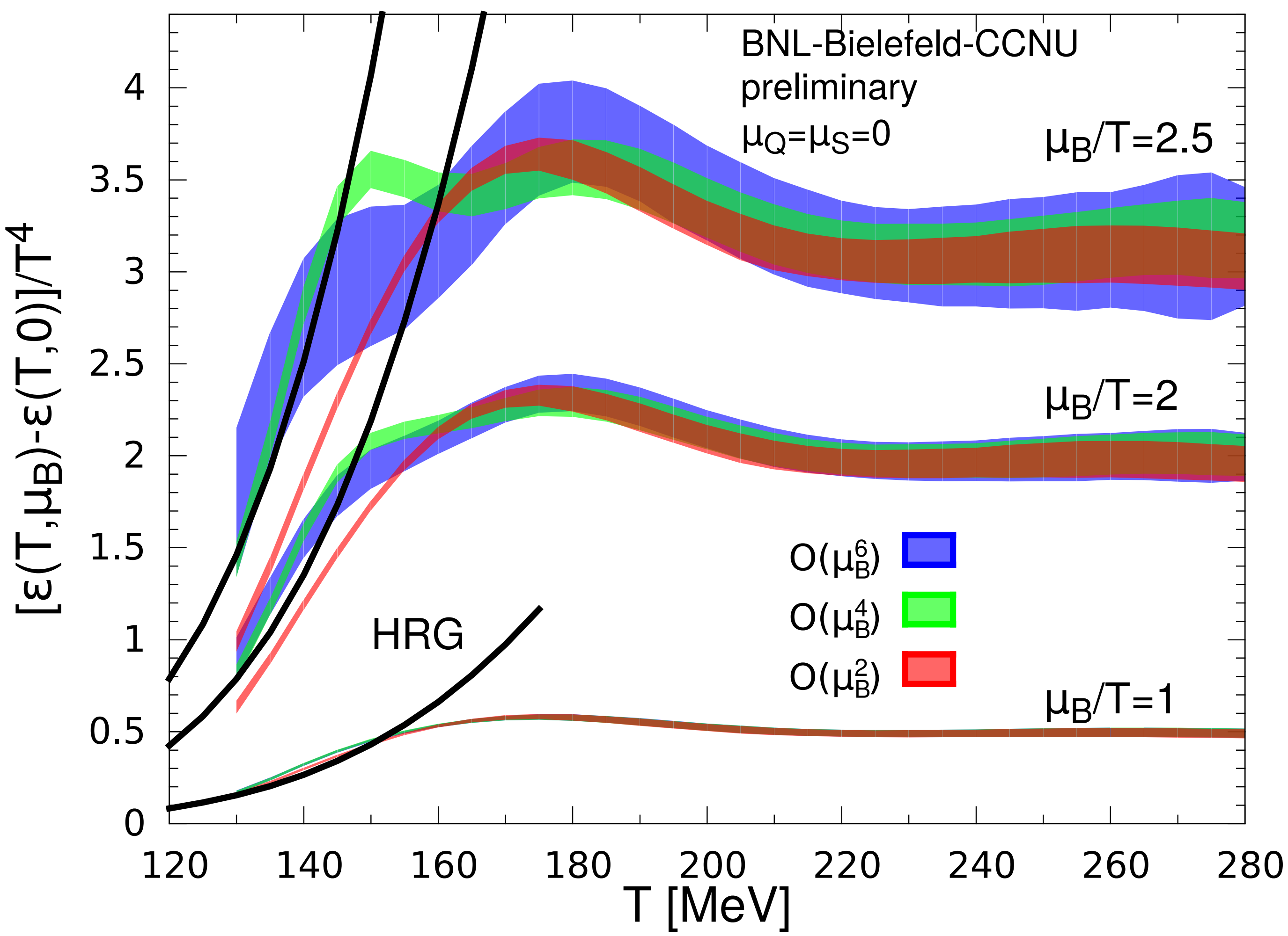
➡ Sixth order contributions are small for $\mu_B/T < 2$



Currently accessible region of the QCD phase diagram (by lattice QCD calculations)

Equation of State at $\mu_B > 0$

energy density correction



- energy coefficients are readily obtained from the pressure coefficients

$$\frac{\Delta\epsilon}{T^4} = \sum_{i=1}^{\infty} \left(3\chi_{2i}^B - \tilde{\chi}_{2i}^B\right) \left(\frac{\mu_B}{T}\right)^{2i}$$

with $\tilde{\chi}_{2i}^B = T \frac{\mathrm{d}\chi_{2i}^B(T)}{\mathrm{d}T}$

- convergence deferred due to additional temperature derivative

Bazavov et al., Phys. Rev. D95 (2017) 054505.

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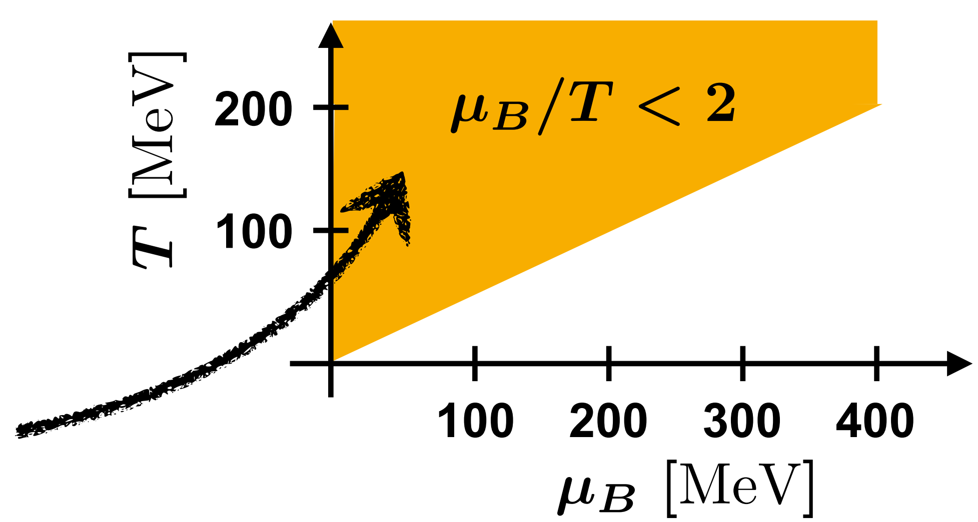
Currently accessible region of the QCD phase diagram (by lattice QCD calculations)



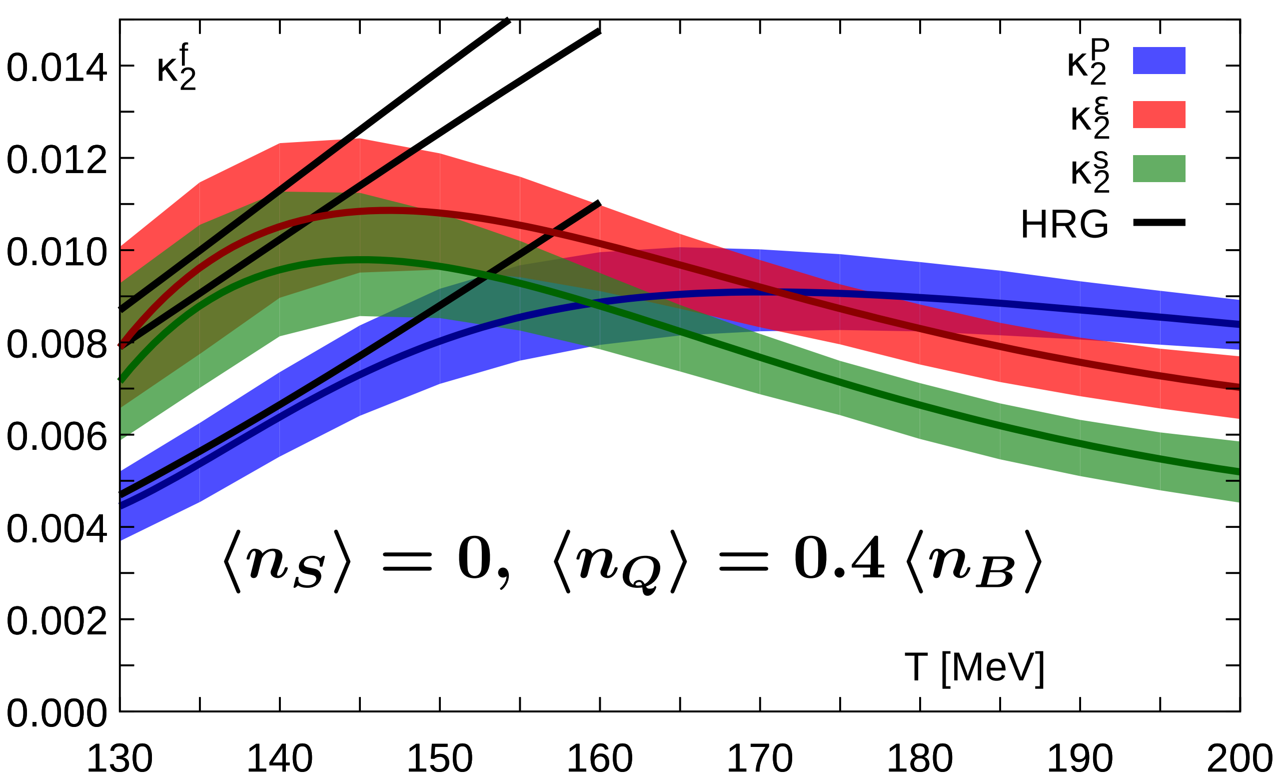
Lines of constant physics

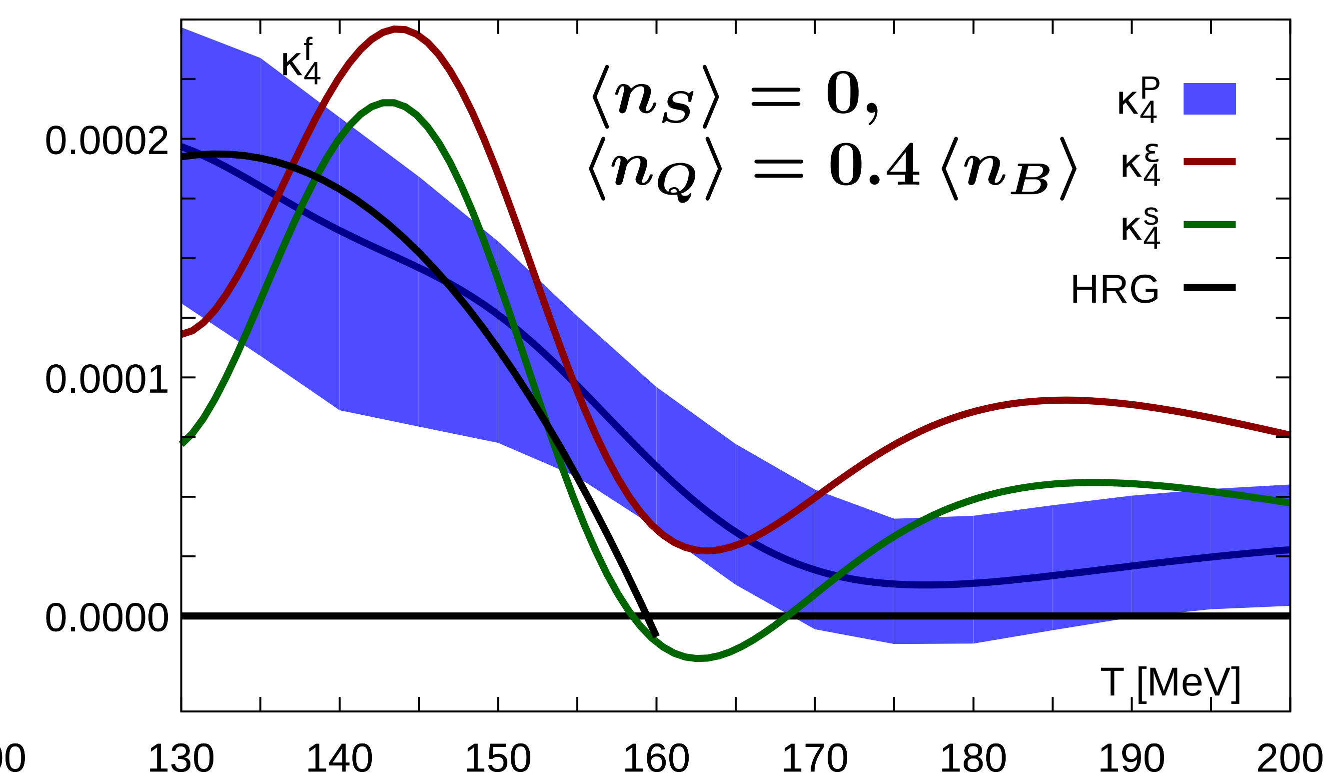
Consider lines of constant $f \equiv p, \varepsilon, s, \ldots$

Ansatz: $$T_f(\mu_B) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_B}{T_0}\right)^4 - \cdots \right)$$

$$\Rightarrow \quad \kappa_2^f = \frac{T_0}{2} \frac{\left.\frac{\partial^2 f(T,\mu_B)}{\partial \mu_B^2}\right|_{(T_0,0)}}{\left.\frac{\partial f(T,\mu_B)}{\partial T}\right|_{(T_0,0)}}$$

Bazavov et al., Phys. Rev. D95 (2017) 054505.





➡ Small κ_2^f

➡ $\kappa_2^p, \kappa_2^\varepsilon, \kappa_2^s$ agree for $T \in [145, 165]$

➡ κ_2^f is O(10²) smaller than κ_4^f

Lines of constant physics

Consider lines of constant $f \equiv p, \varepsilon, s, \ldots$

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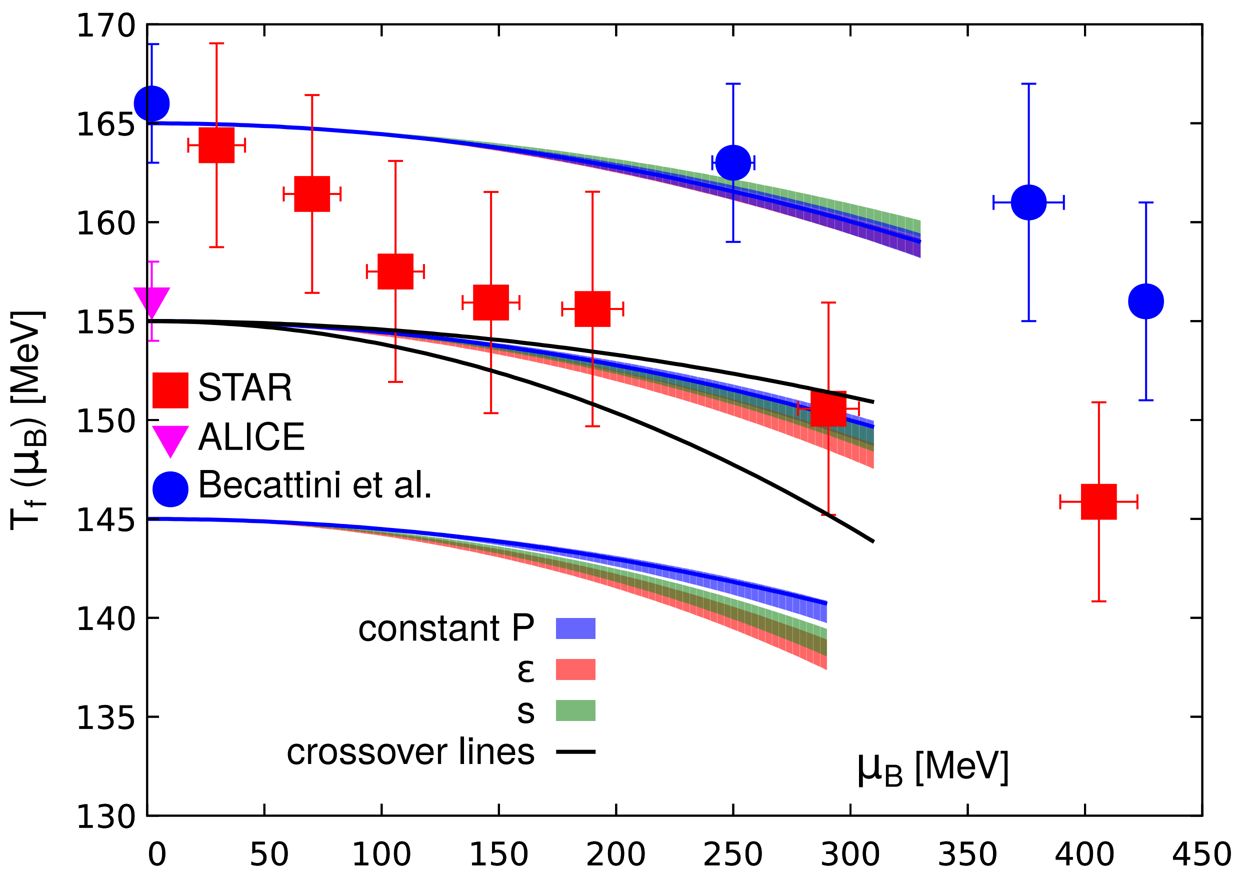
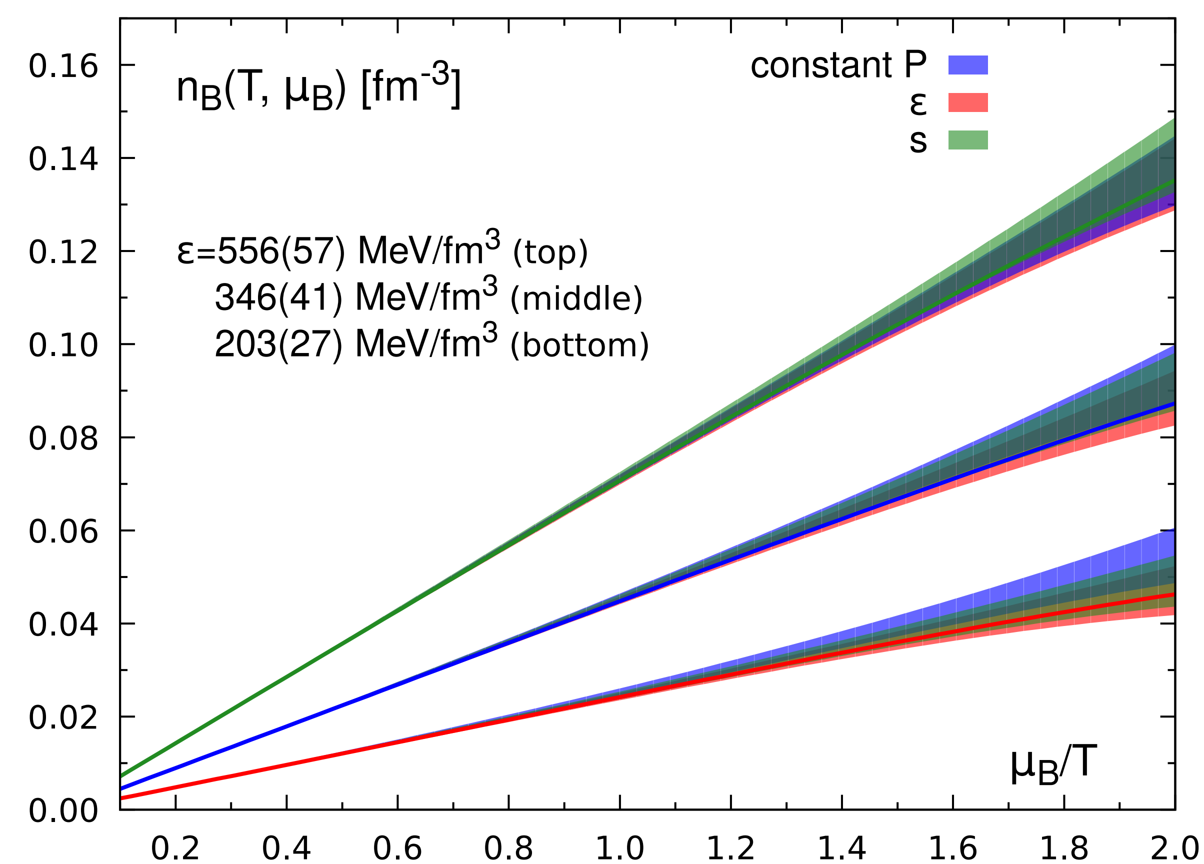




$$\langle n_S \rangle = 0, \ \langle n_Q \rangle = 0.4 \langle n_B \rangle$$

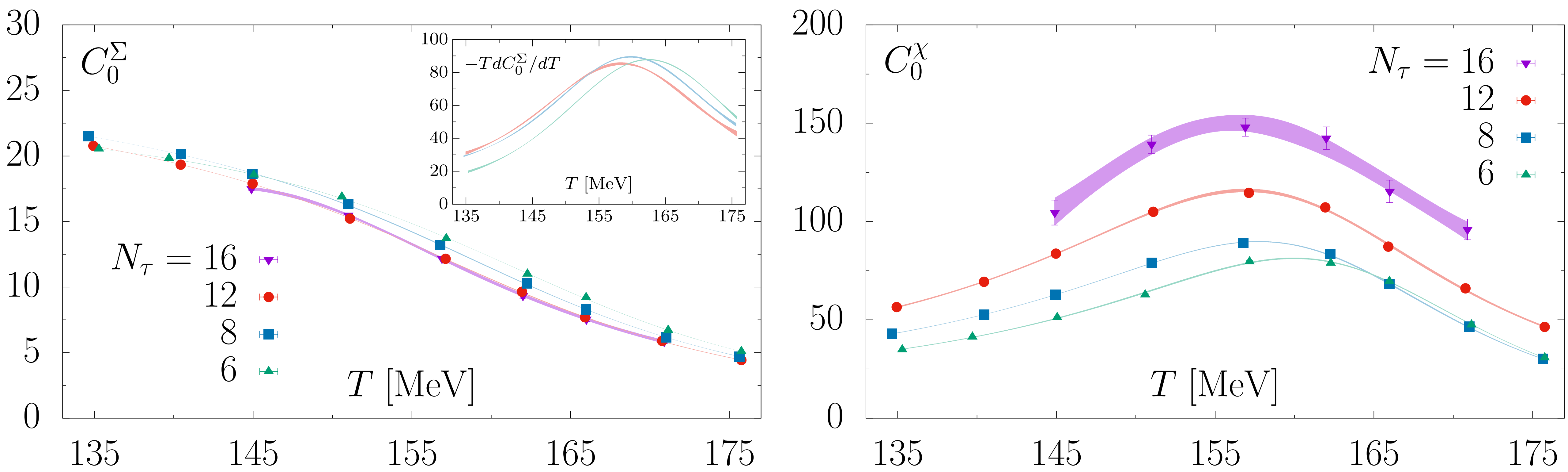
- Chiral observables:

$$\Sigma = \frac{1}{f_K}\left[m_s \langle \bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle \bar{s}s\rangle\right]$$

$$\chi = \frac{m_s^2}{f_K}\left[m_s \langle (\bar{u}u + \bar{d}d)^2\rangle - \left(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle\right)^2\right]$$

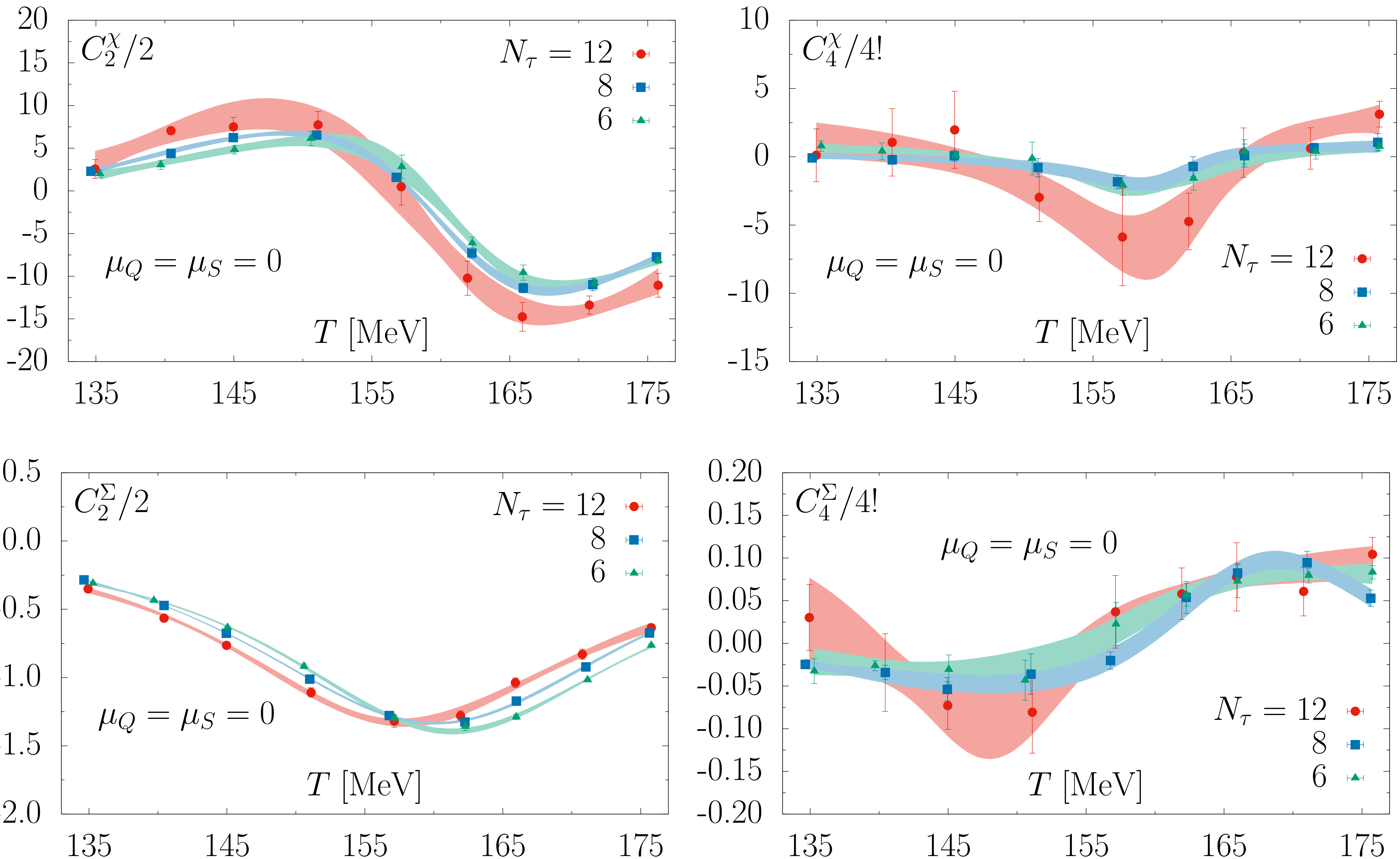
- Taylor expansion in $\mu_X,\ X = B, Q, S, I, \ldots$, *e.g.*

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad \text{with} \qquad C_{2n}^{\Sigma}(T) = \left.\frac{\partial^{2n}\Sigma}{\partial(\mu_X/T)^{2n}}\right|_{\mu_X=0}$$



Bazavov et al., arXiv:1812.08235

The chiral crossover line



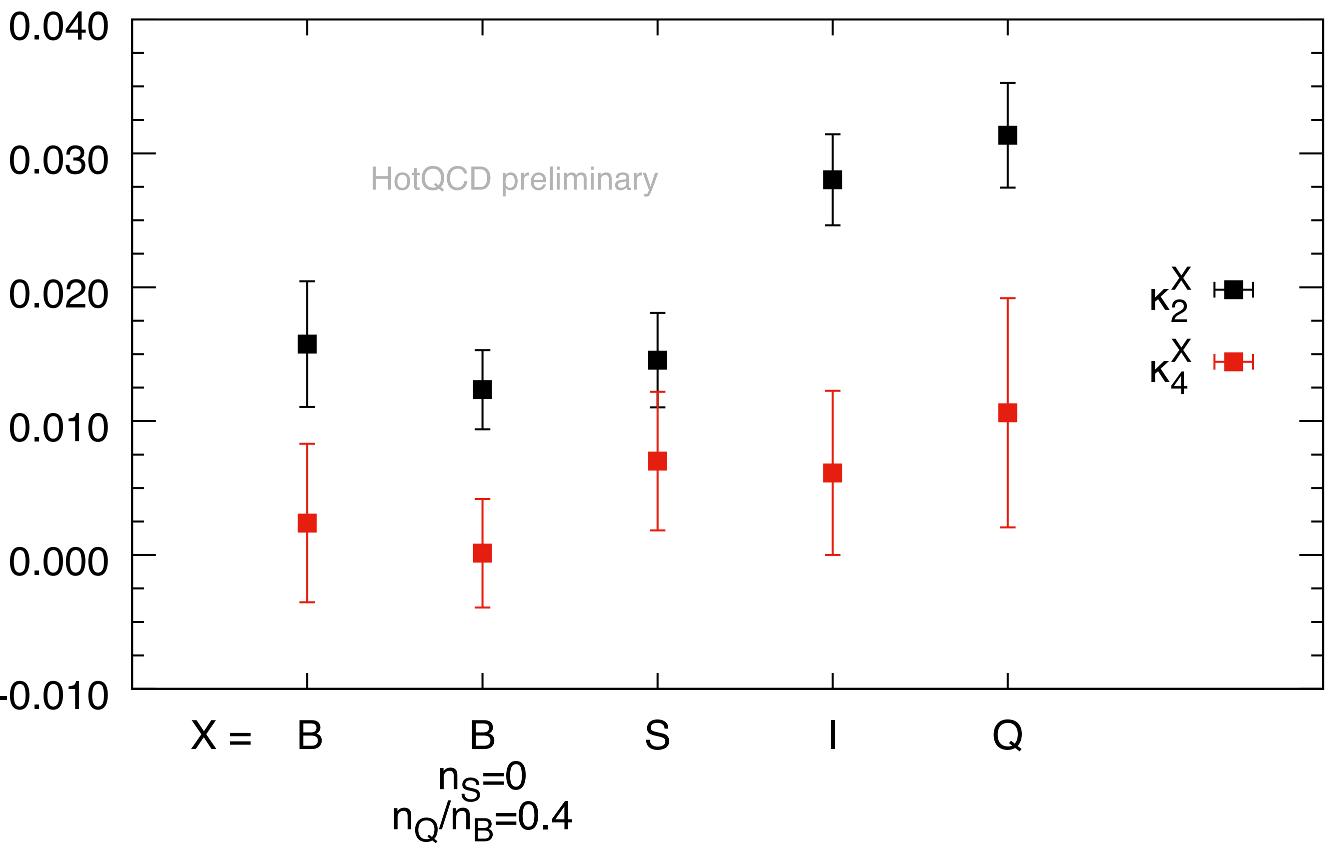
Bazavov et al., arXiv:1812.08235

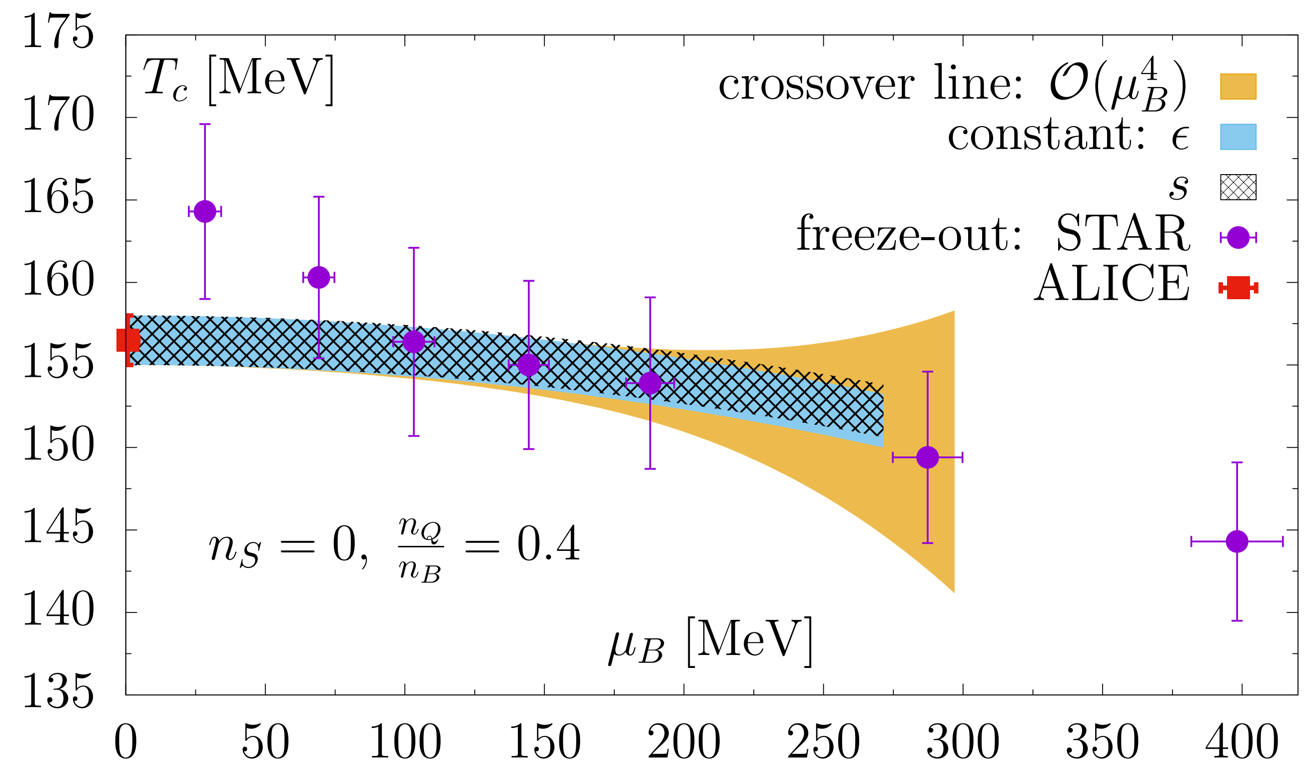
The chiral crossover line

- Calculate lines of constant: $\partial_T^2 \Sigma(T, \mu_X)\big|_{\mu_X} = 0$

$$\partial_T \chi(T, \mu_X)\big|_{\mu_X} = 0$$

	κ_2^B	κ_4^B	κ_2^S	κ_4^S	κ_2^Q	κ_4^Q	κ_2^I	κ_4^I	$\kappa_2^{B,f}$	$\kappa_4^{B,f}$
Σ	0.015(4)	-0.001(3)	0.018(3)	0.001(3)	0.027(4)	0.004(5)	0.023(3)	0.004(4)	0.012(2)	0.000(2)
χ	0.016(5)	0.002(6)	0.015(4)	0.007(5)	0.031(4)	0.011(9)	0.028(3)	0.006(6)	0.012(3)	0.000(4)
Average	0.016(6)	0.001(7)	0.017(5)	0.004(6)	0.029(6)	0.008(1)	0.026(4)	0.005(7)	0.012(4)	0.000(4)





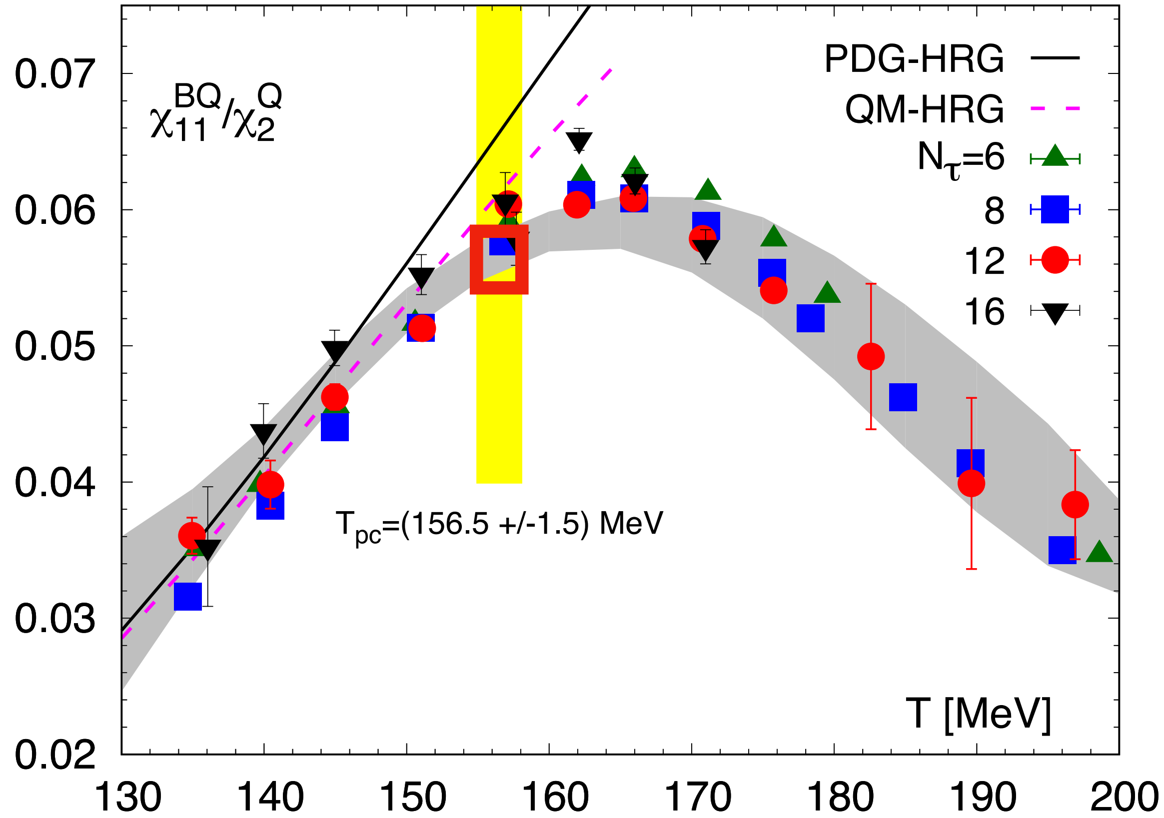
➡ Slight dependence on the type of conserved charge

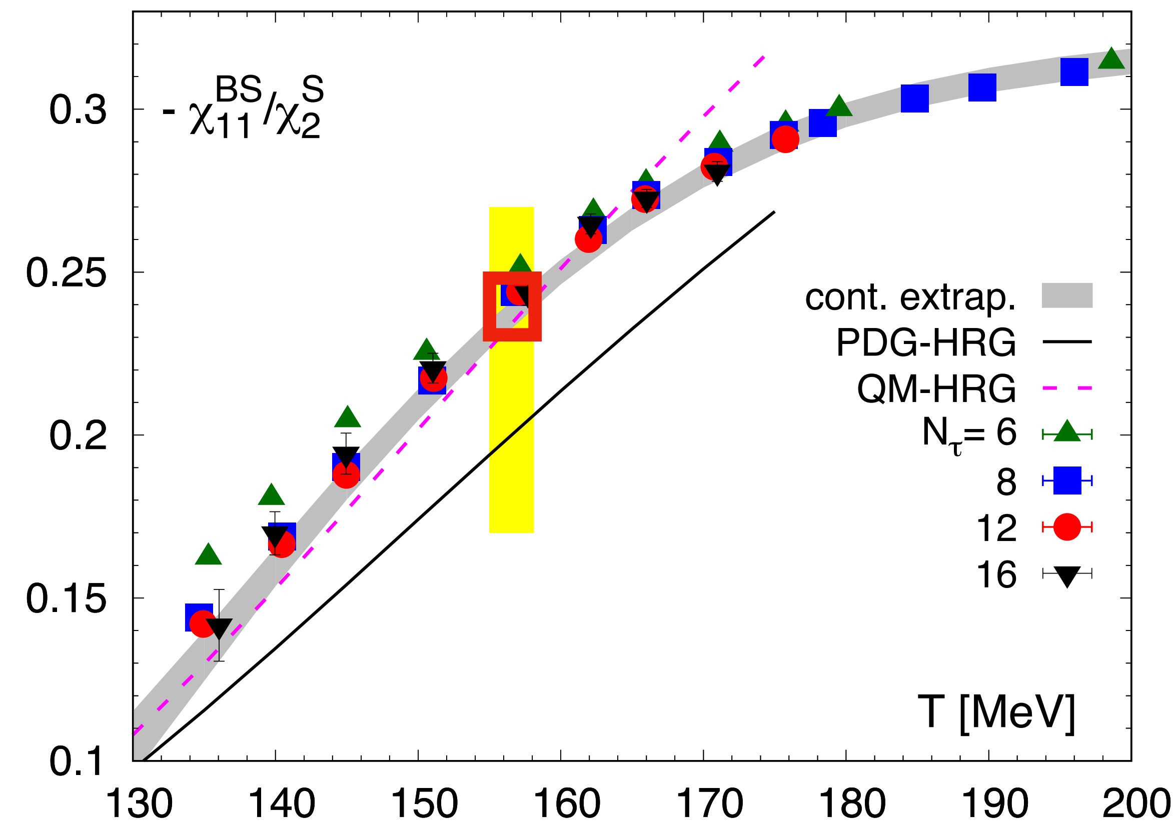
➡ Agreement with the lines of constant physics

Bazavov et al., arXiv:1812.08235

Fluctuations of conserved charges at LHC

- 2nd order fluctuations: BQ and BS correlations





At ALICE: freeze-out temperature

== chiral crossover temperature

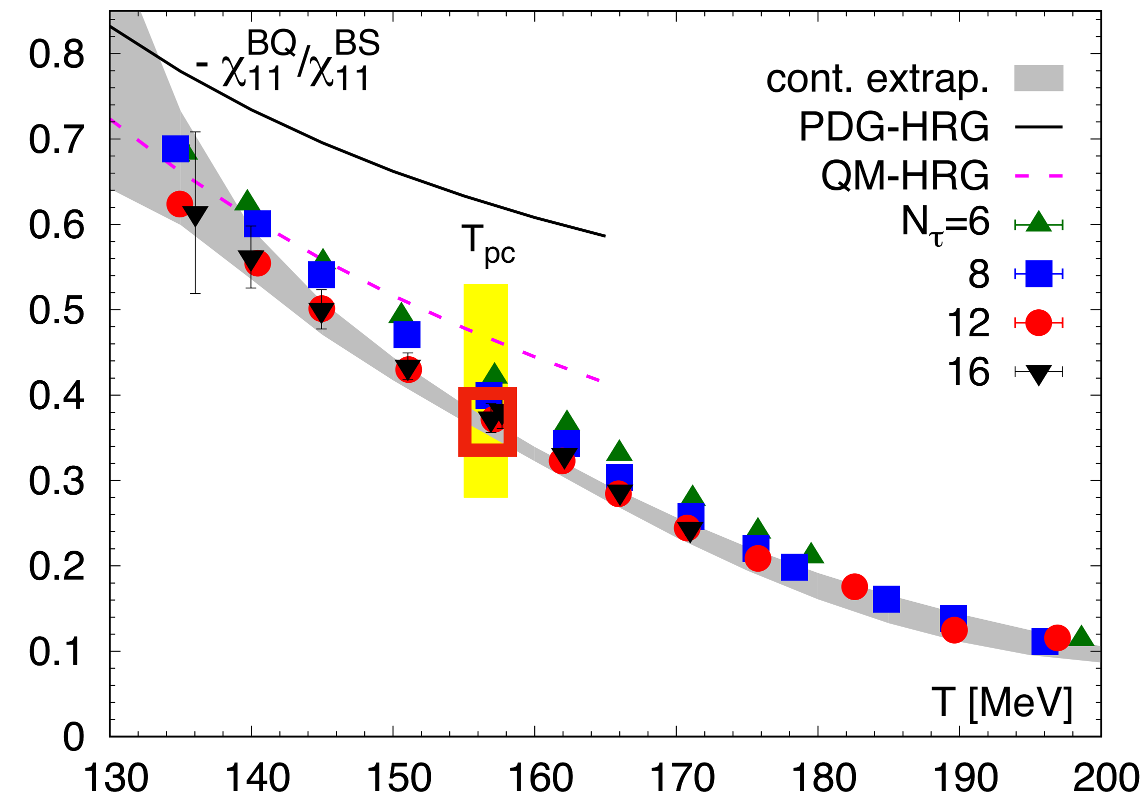
$$T_{f0} = 156.5(1.5)\ \mathrm{MeV}$$

$$\chi_{11}^{BQ}/\chi_2^Q = 0.058(2)$$

$$\chi_{11}^{BS}/\chi_2^S = -0.235(15)$$

$$\chi_{11}^{BQ}/\chi_{11}^{BS} = -0.37(3)$$

➡Evidence for not yet observed hadrons?



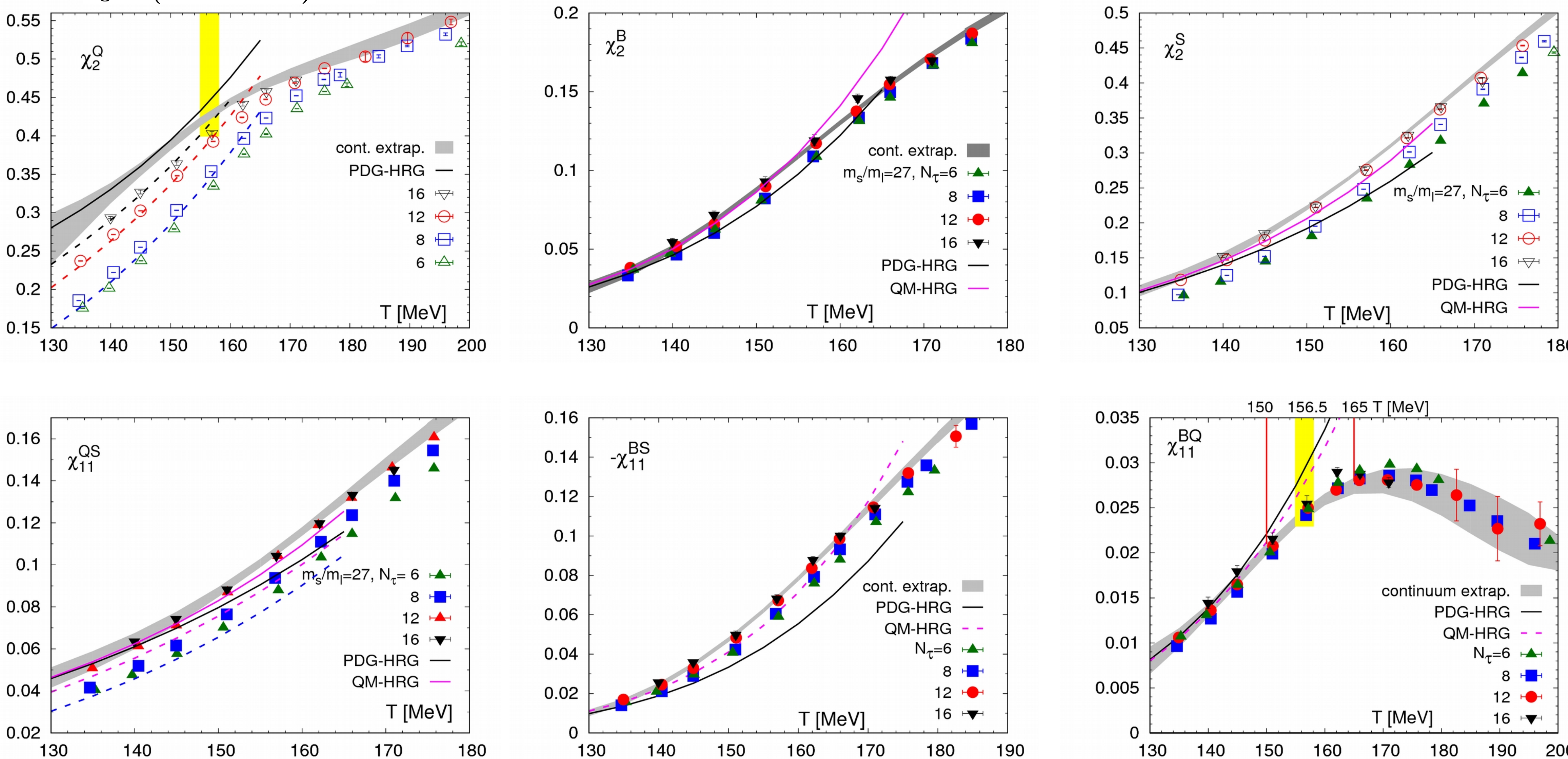
Fluctuations of conserved charges at LHC

- Continuum results for all 2nd order observables

$T_c = (156.5 \pm 1.5)\mathrm{MeV}$

$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$

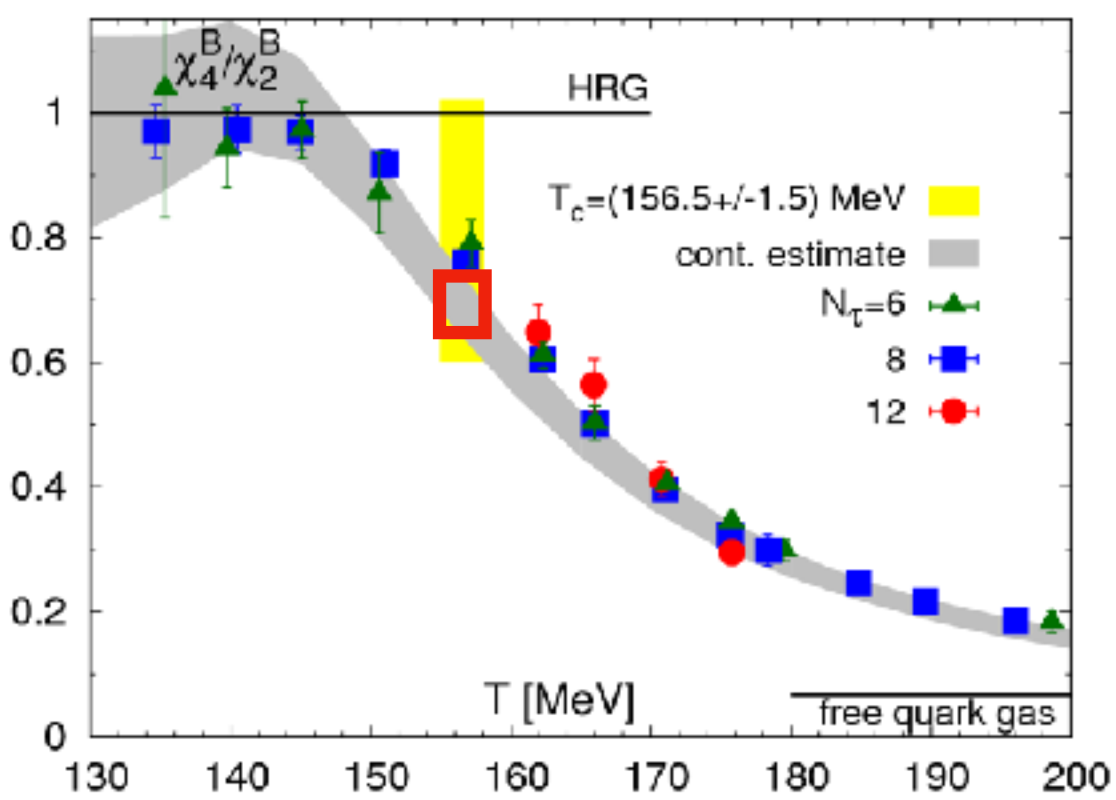
$\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$

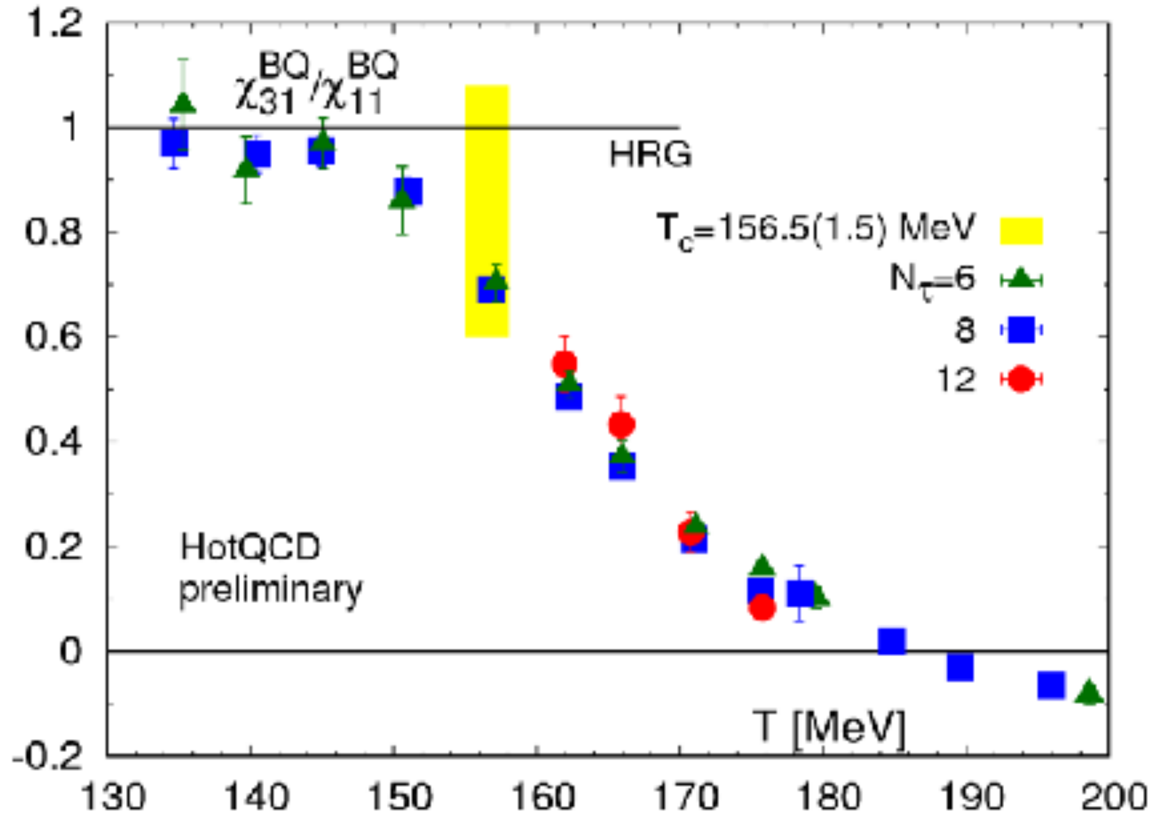


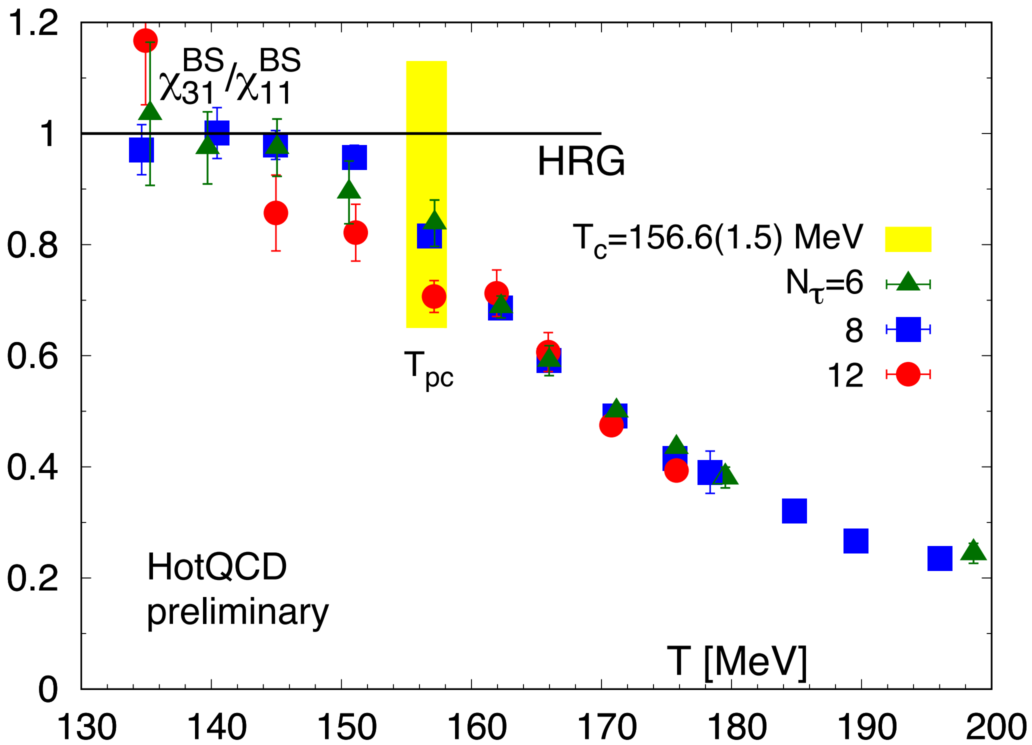
➡Only 4 out of 6 observables are independent

Fluctuations of conserved charges at LHC

- Some 4th order fluctuations







➡ Differ from HRG for T>145 MeV

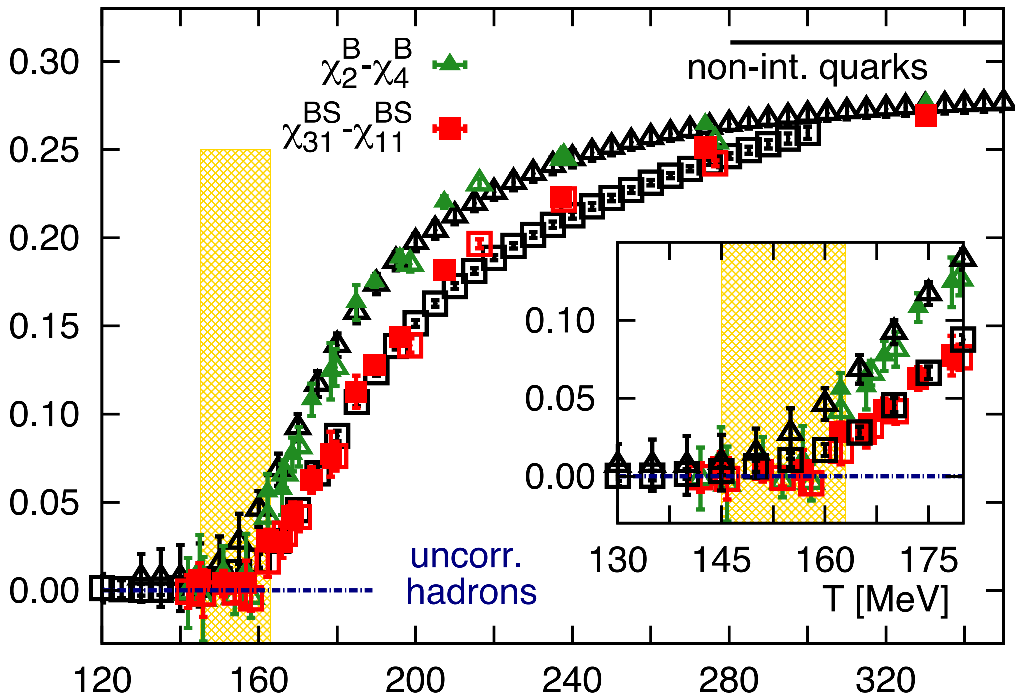
➡ Change (20-40)% percent in the crossover region

➡Sensitive probes for freeze-out

The effective degrees of freedom

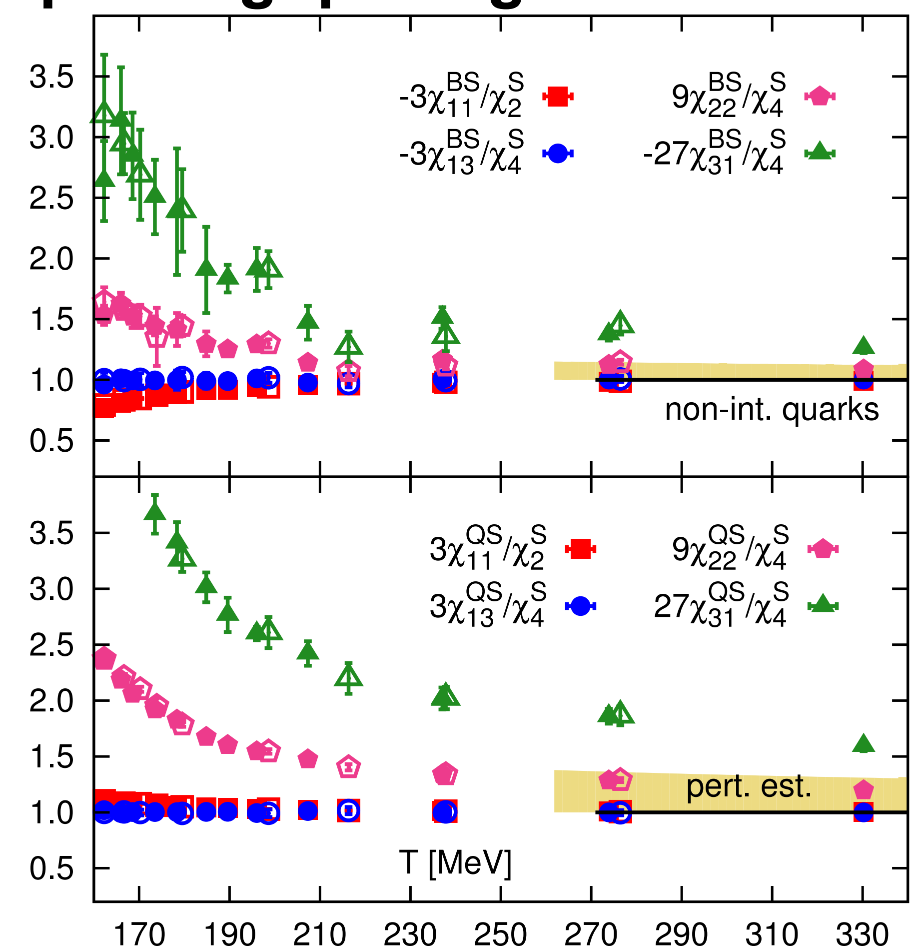
- **cumulants are sensitive to effective charges:** compare cumulants from non-perturbative (lattice) QCD calculations to other scenarios such as an uncorrelated gas of hadrons (HRG) or perturbative QCD

probing HRG DoFs:



Bazavov et al., PRL 111 (2013)
Bellwied et al., PRL 111 (2013)

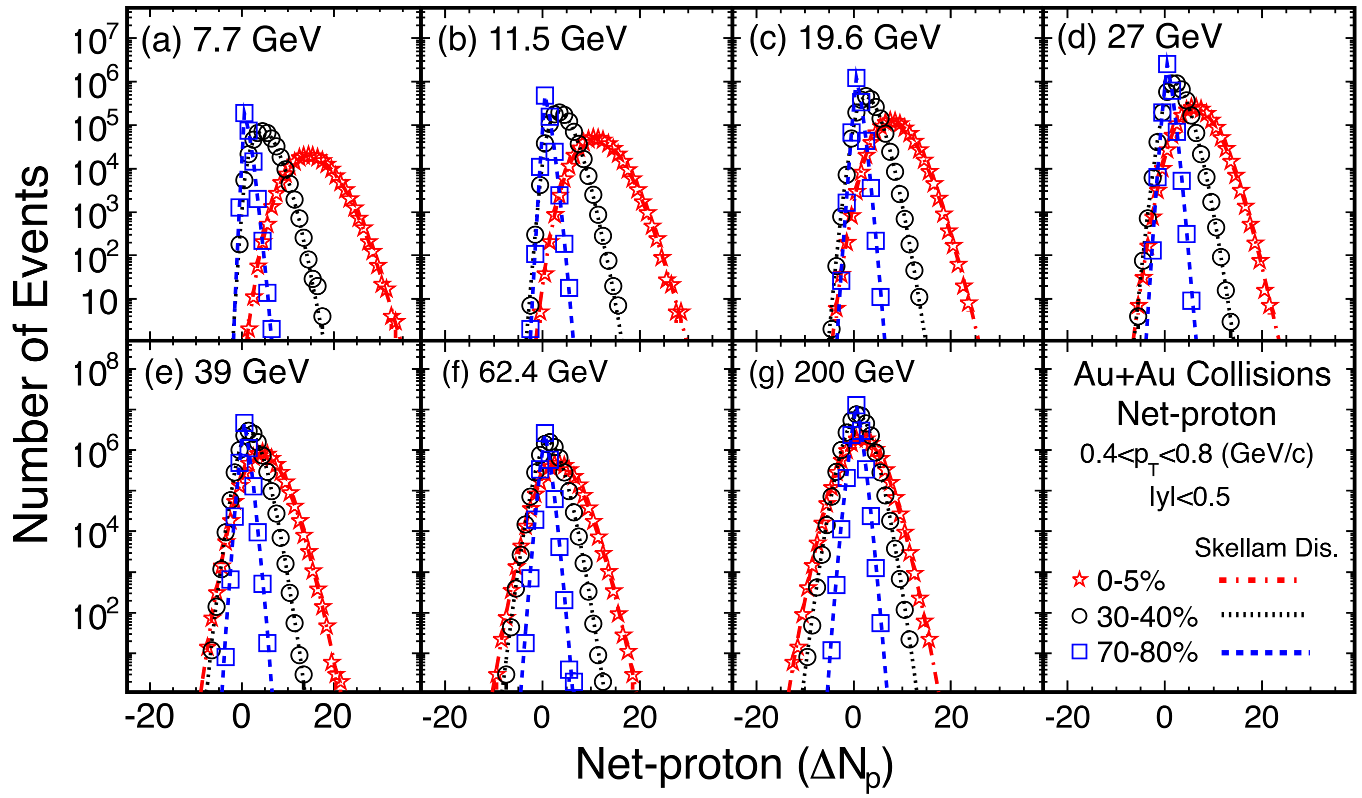
probing quark gas DoFs:



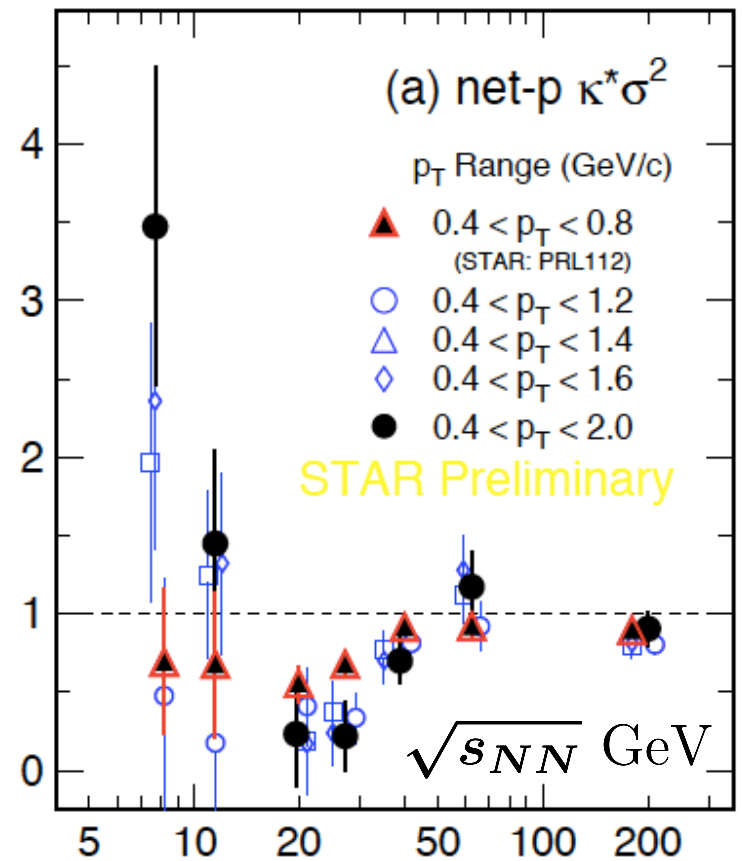
$\Rightarrow$ relevant degrees of freedom are hadronic and uncorrelated for $T \lesssim T_c$

$\Rightarrow$ relevant degrees of freedom are that of a weakly interact. quark gas for $T \gtrsim 2T_c$

STAR data for net-proton number fluctuations



STAR, PRL 112 (2014)

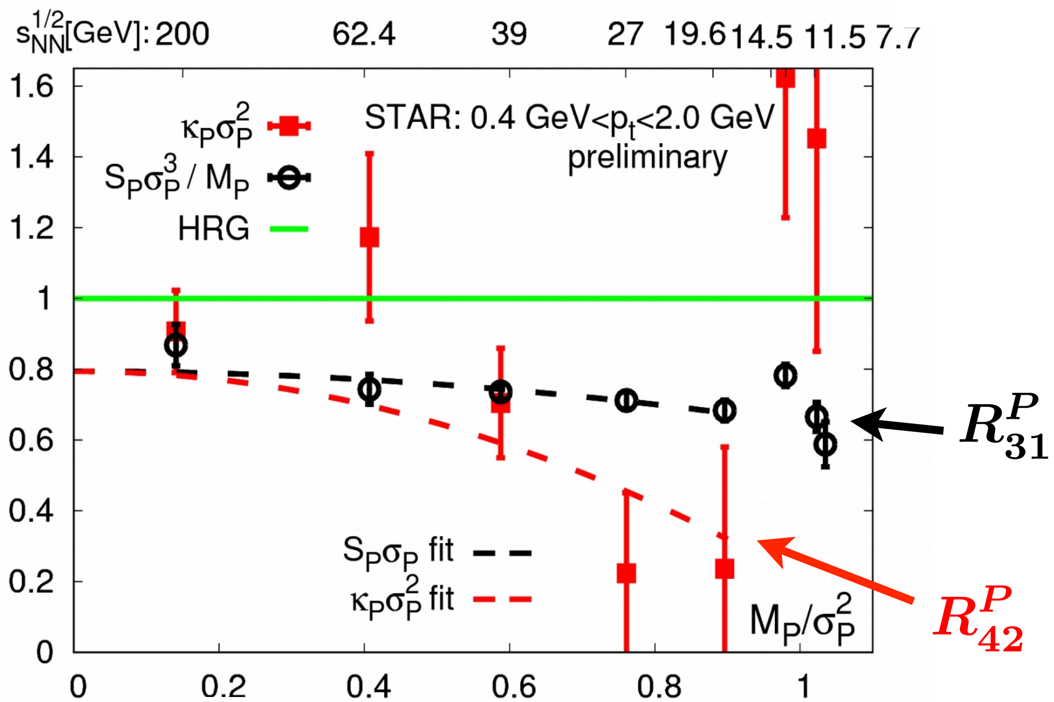


X. Luo, CPOD'14

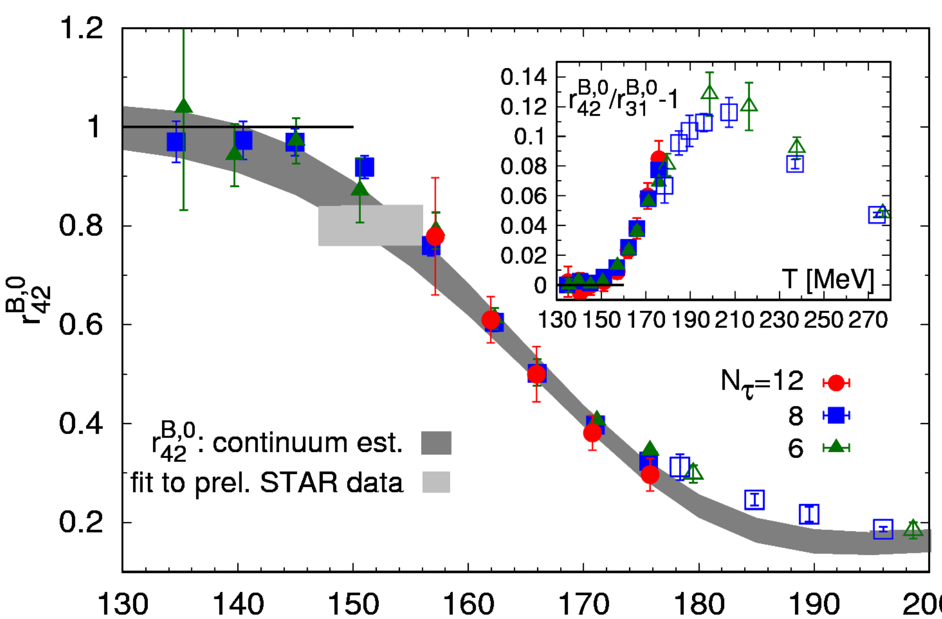
- STAR has measured the event-by-event fluctuations of the net-proton number.
- STAR found non-Gaussian fluctuations (none-vanishing skewness and kurtosis).
- Indication for a critical point?
- Can the data be explained by equilibrium thermodynamics?

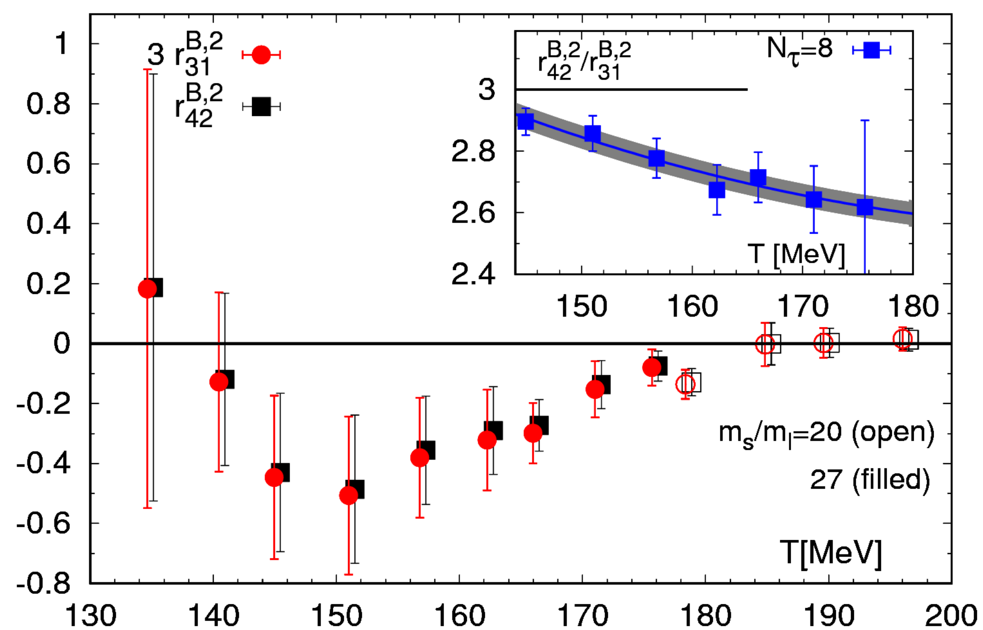
STAR data for net-proton number fluctuations

- How can we compare STAR data to the lattice results without using a model dependent parametrization of the freeze-out line?
  - ➡ Consider only ratios of cumulants to eliminate the freeze-out volume.
  - ➡ Eliminate also μ_B in favor of the ratio $R_{12}^B \equiv \chi_1^B / \chi_2^B$.



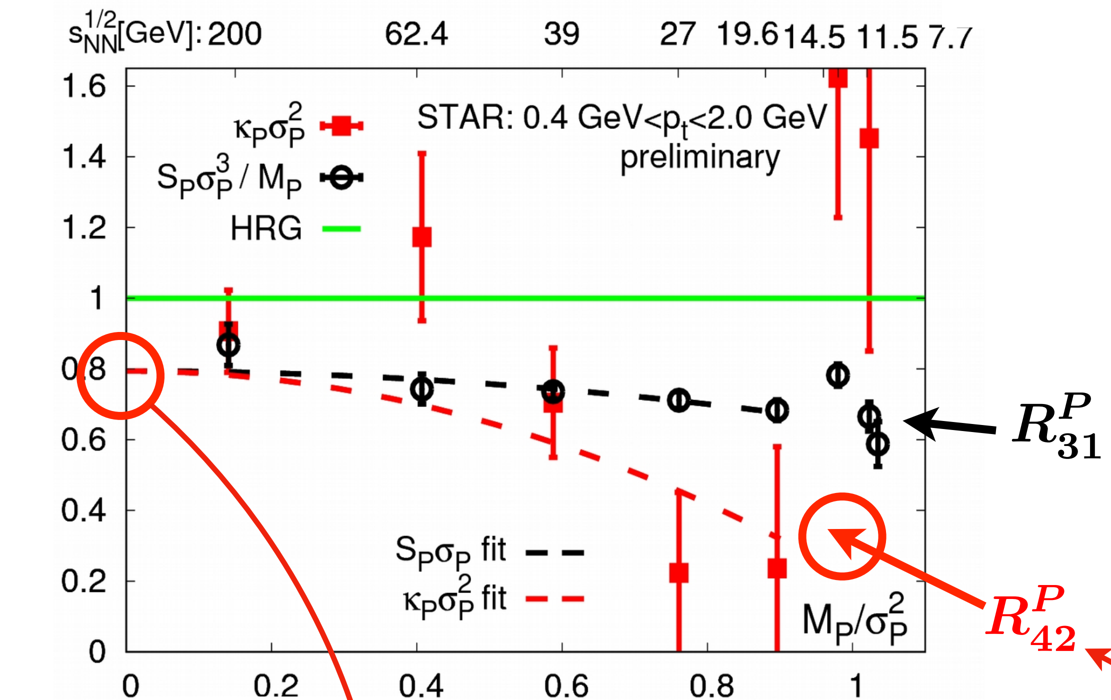
Bazavov et al., Phys. Rev. D96 (2017) 074510.





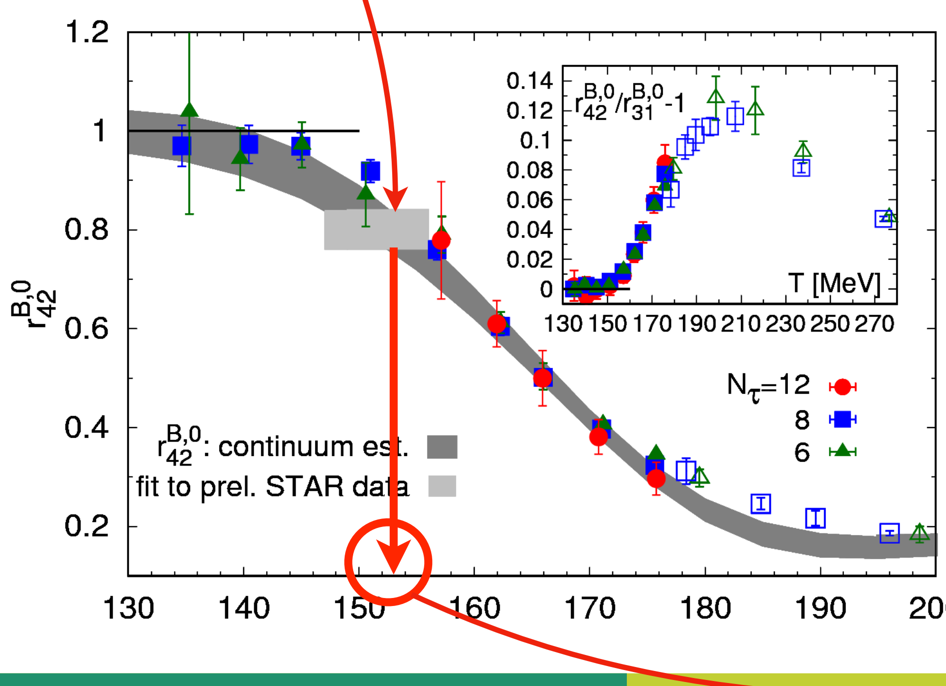
STAR data for net-proton number fluctuations

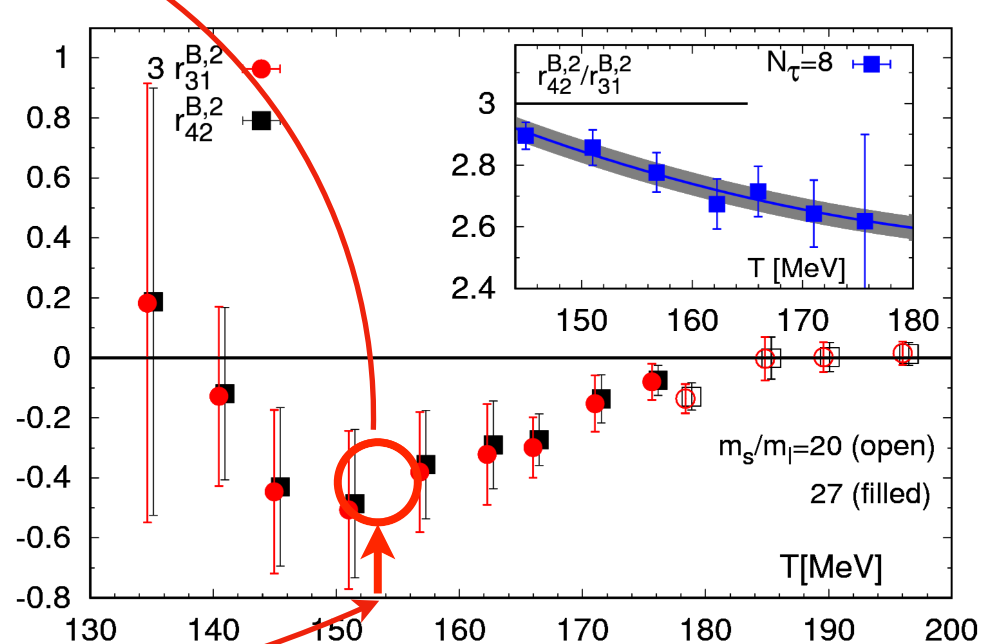
- How can we compare STAR data to the lattice results without using a model dependent parametrization of the freeze-out line?
  - ➡ Consider only ratios of cumulants to eliminate the freeze-out volume.
  - ➡ Eliminate also μ_B in favor of the ratio $R_{12}^B \equiv \chi_1^B/\chi_2^B$.



Comparison works astonishingly well, even though many effects need to be understood better.

Bazavov et al., Phys. Rev. D96 (2017) 074510.





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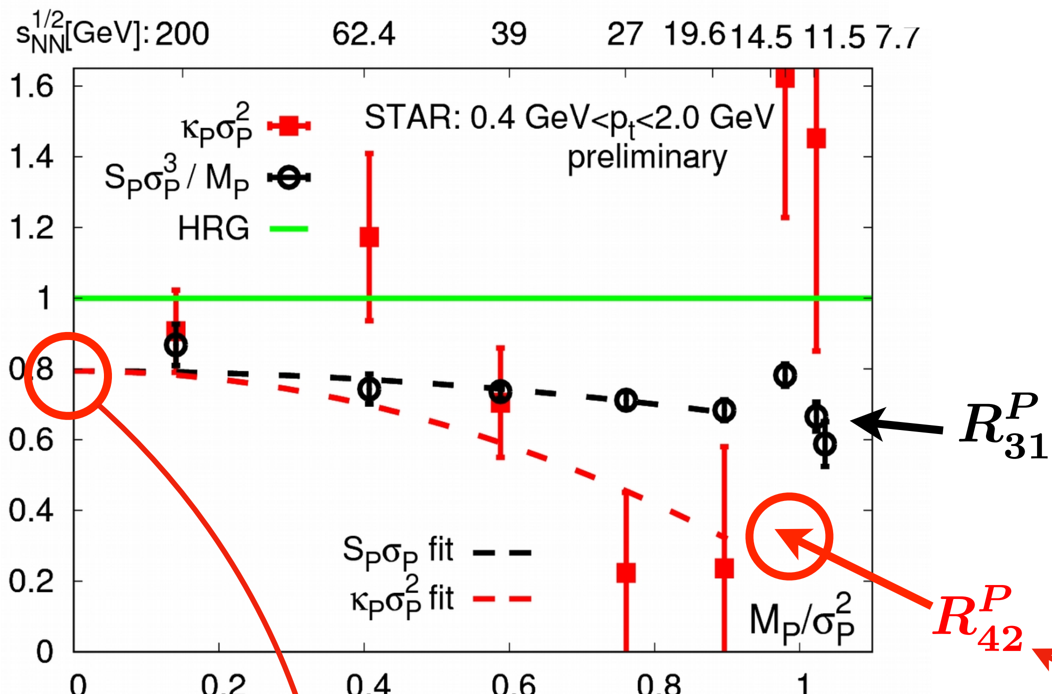
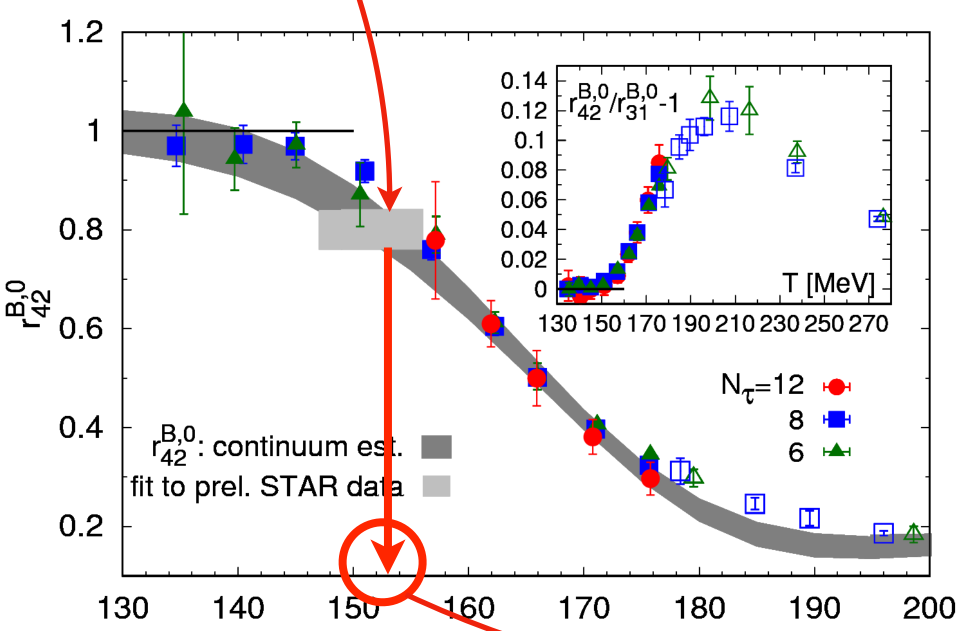
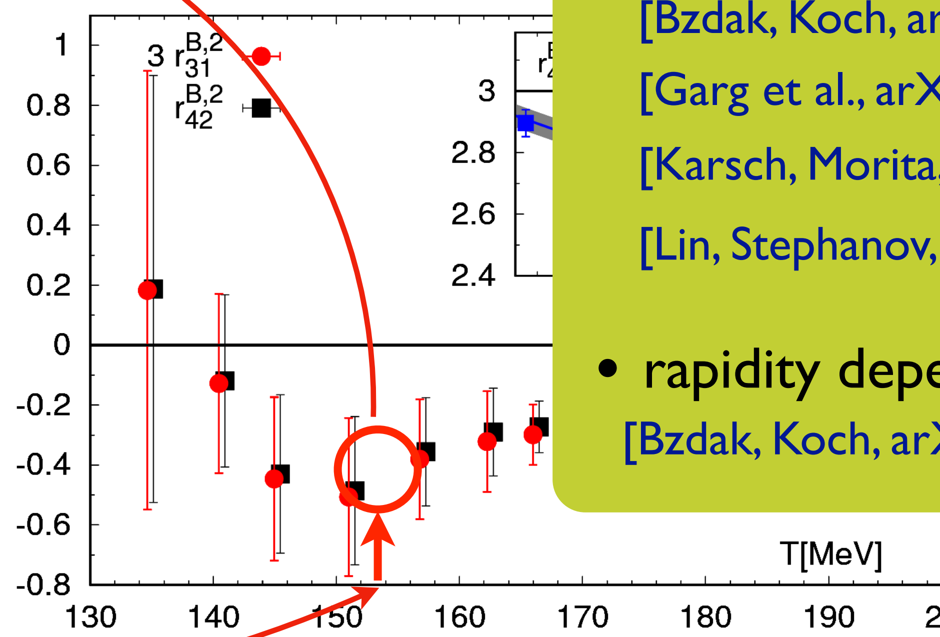




Bazavov et a

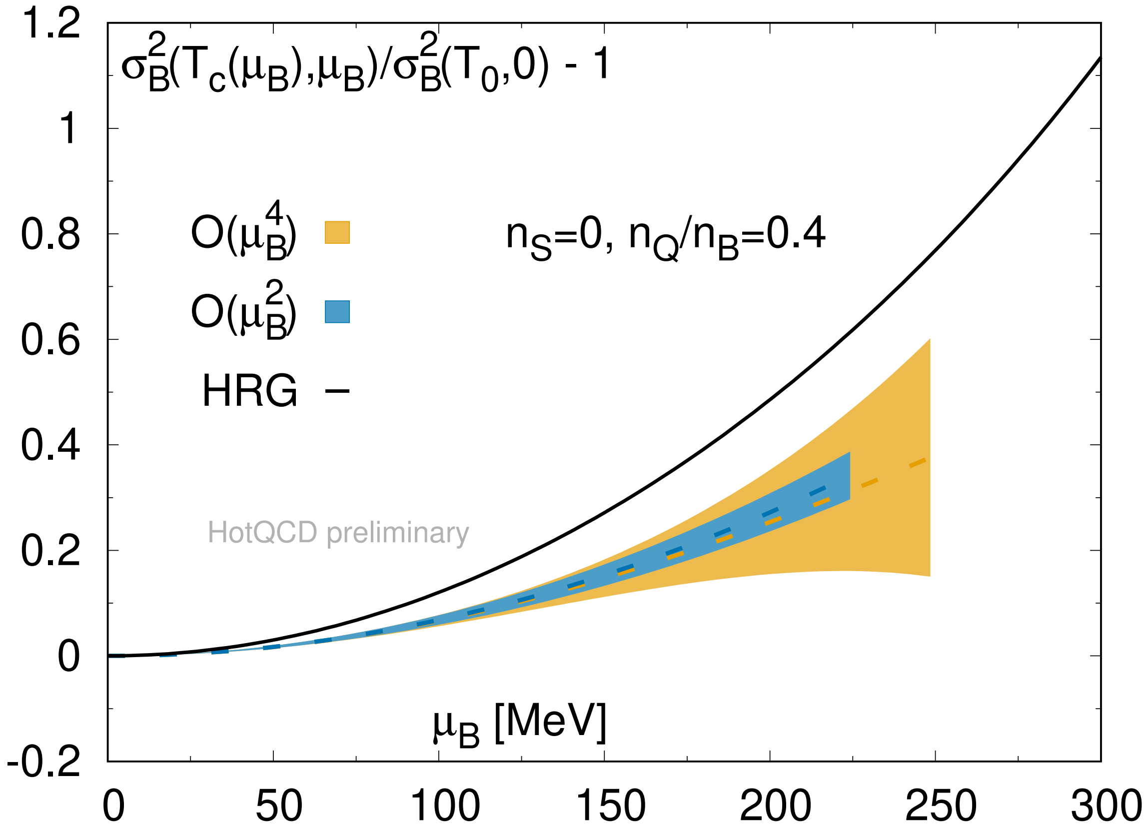


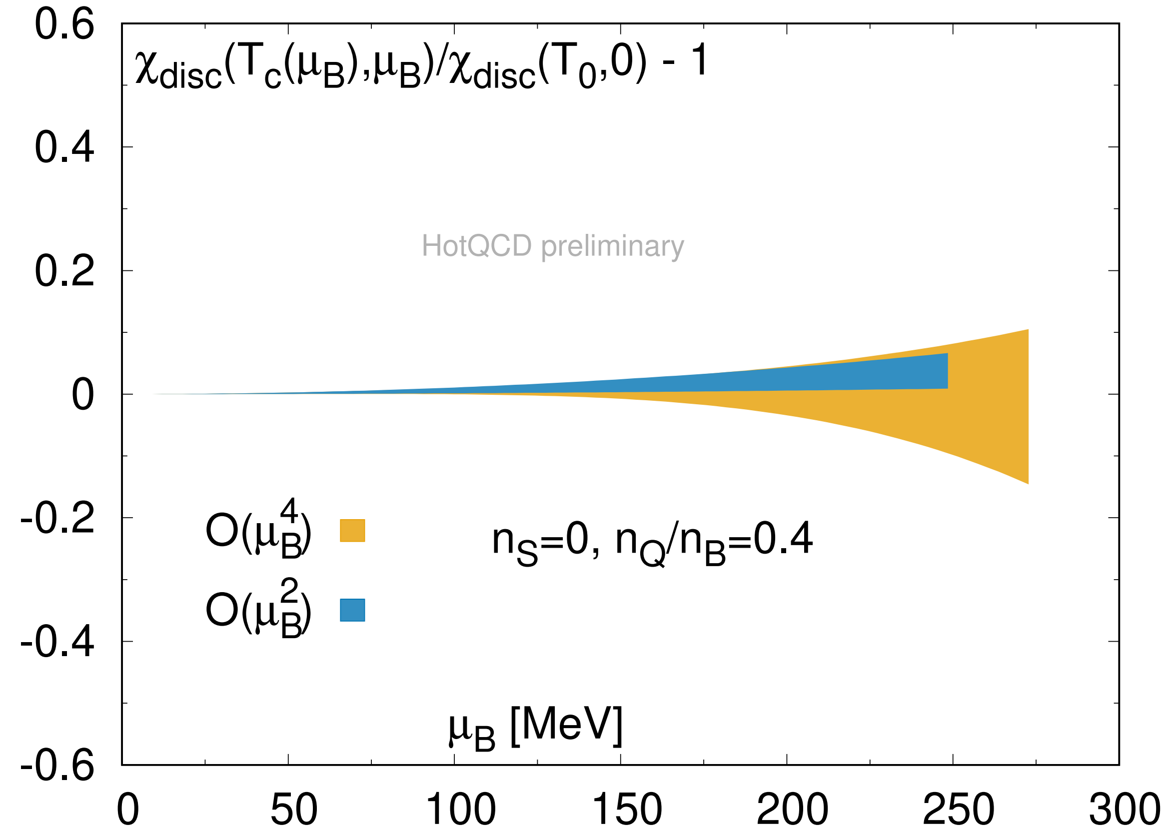
need to understand:

- non-equilibrium effects
  [Berdnikov, Rajagopal, hep-ph/9912274]
  [Mukherjee et al., arXiv:1506.00645, arXiv:1605.09341]
- proton vs. baryon number distributions
  [Kitazawa et al. arXiv:1205.3292, arXiv:1303.3338]
- acceptance and pt-cuts
  [Bzdak, Koch, arXiv:1206.4286]
  [Garg et al., arXiv:1304.7133]
  [Karsch, Morita, Redlich, arXiv:1508.02614]
  [Lin, Stephanov, arXiv:1512.09125]
- rapidity dependence
  [Bzdak, Koch, arXiv:1707.02640]

Fluctuations and critical behavior

- Consider baryon number fluctuations and fluctuations of the chiral condensate along the crossover line





➡ No sign of critical fluctuations / critical point
➡ No sign that crossover gets stronger

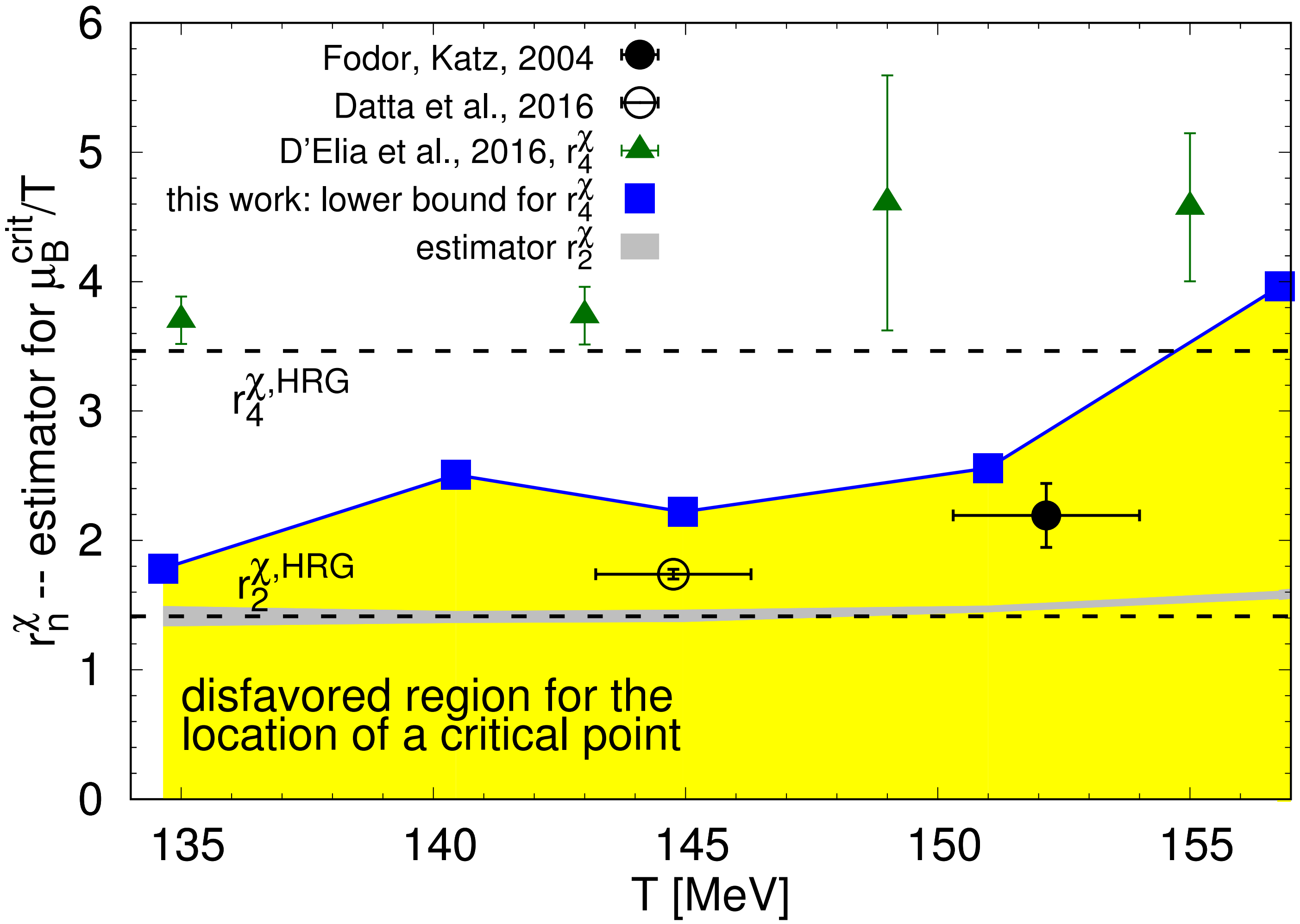
Will a critical point ever be seen by a Taylor expansion?

Theoretical Method: estimate the radius of convergence from successive expansion coefficient. So far, not conclusive.

P. Steinbrecher, QM 2018, [1807.05607]

Radius of convergence

$$\frac{p(T,\mu_B)-p(T,0)}{T^4} = \frac{\chi_2^B}{2!}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B}{4!}\left(\frac{\mu_B}{T}\right)^4 + \frac{\chi_6^B}{6!}\left(\frac{\mu_B}{T}\right)^6 + \cdots$$



Bazavov et al., Phys. Rev. D95 (2017) 054505.

possible definitions of estimators:

$$r_{2n}^{P} = \left| \frac{(2n+2)(2n+1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}$$

$$r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}$$

true radius of convergence:

$$\rho(T) = \lim_{n\to\infty} r_{2n}^{P}(T) = \lim_{n\to\infty} r_{2n}^{\chi}(T)$$

- the radius of convergence only corresponds to a critical point if all expansion coefficients are positive
- HRG: all ratios $\chi_{2n}^B/\chi_{2n+2}^B$ are unity.

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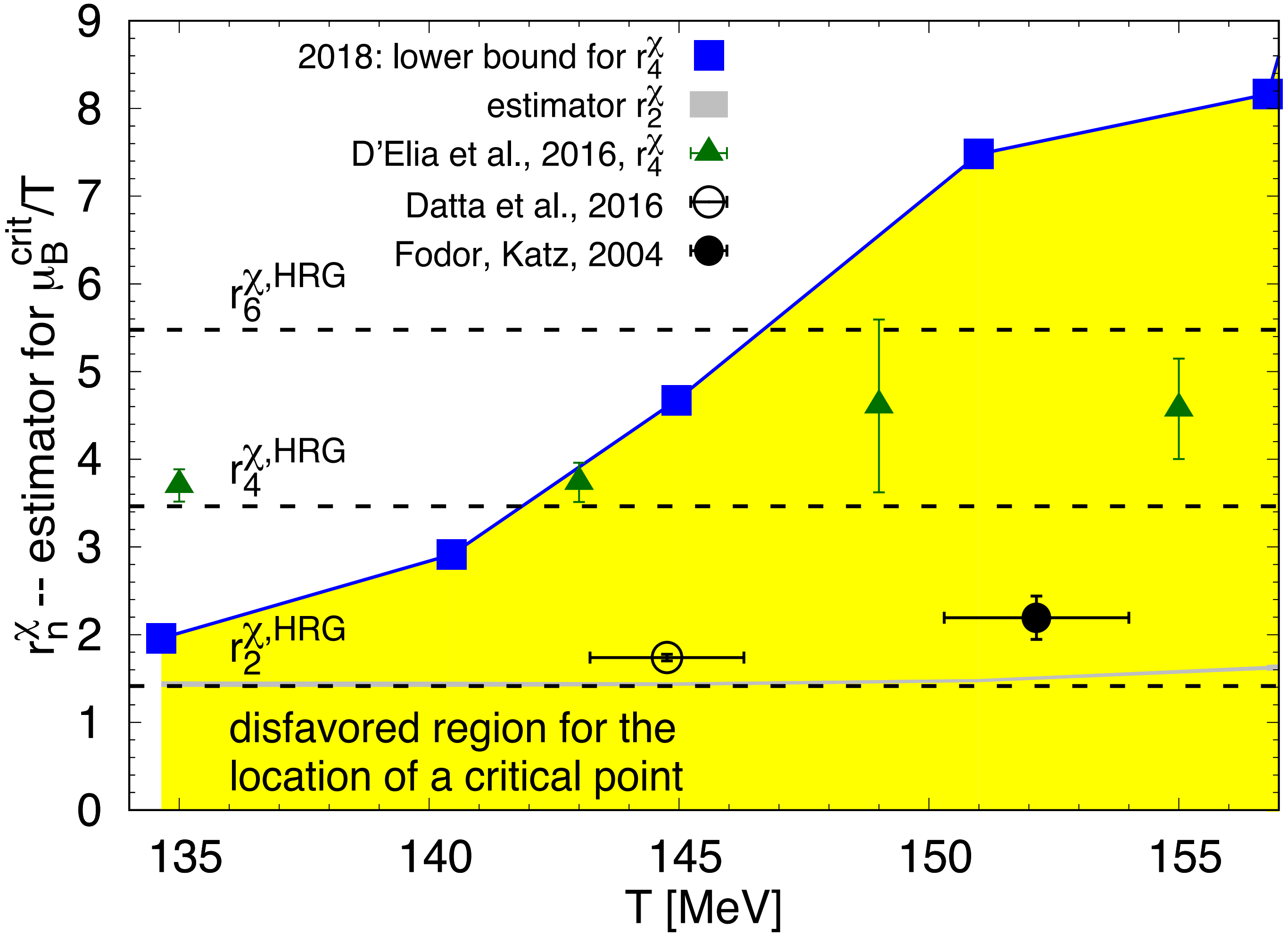

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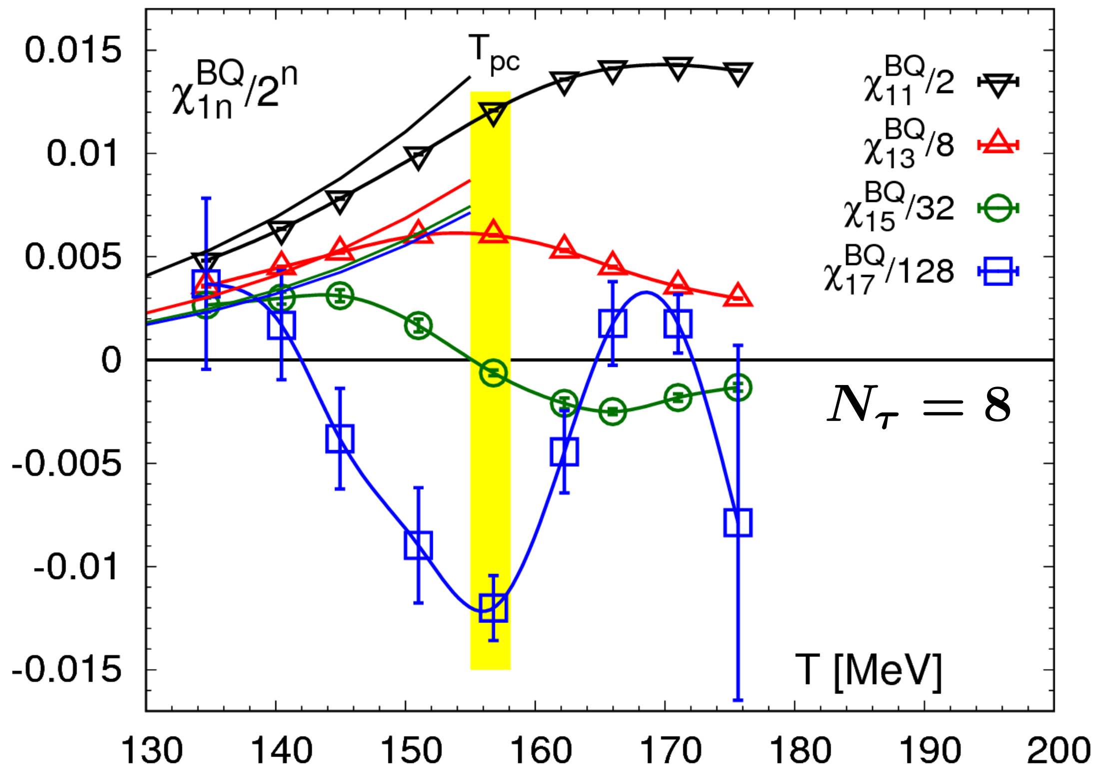
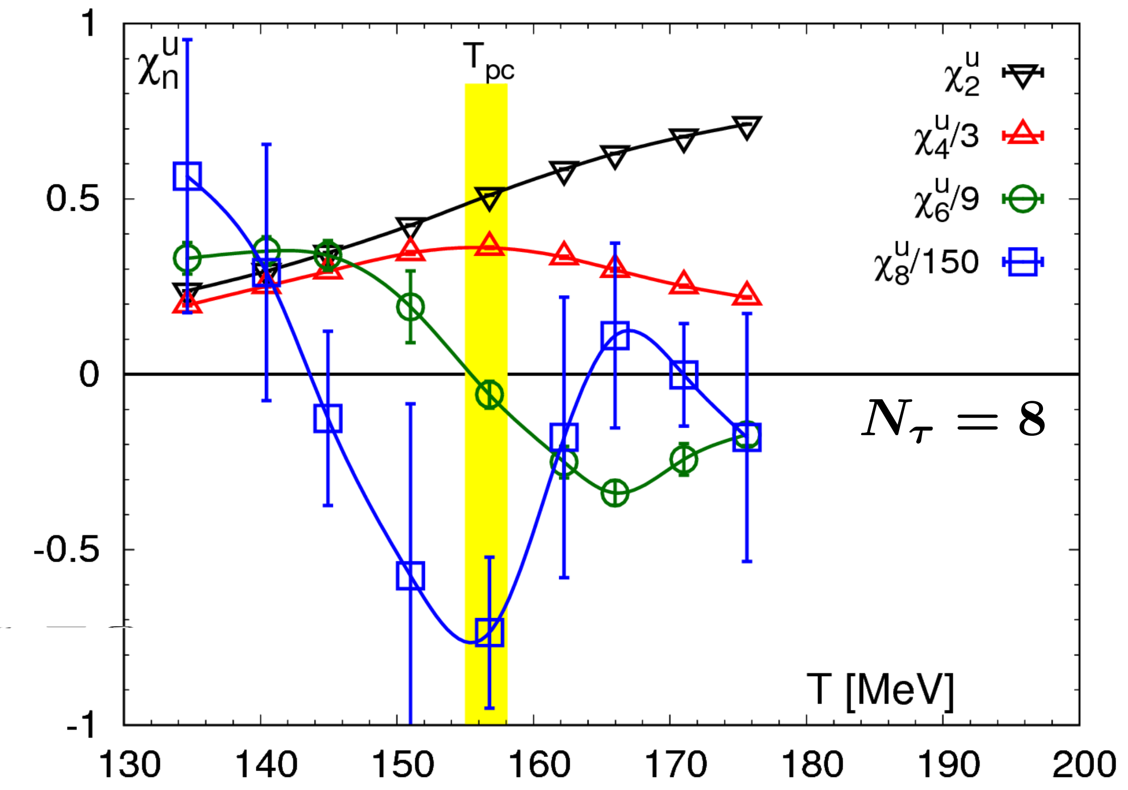
Where is the critical point?

- All expansion coefficients need to be positive (for $n > n_c$)

$$\chi^{BQ}_{1n} = \left.\frac{\partial^{n+1} P/T^4}{\partial\hat{\mu}_B \partial\hat{\mu}_Q^n}\right|_{\mu_{B,Q,S}=0} \qquad \chi^u_n = \left.\frac{\partial^n P/T^4}{\partial\hat{\mu}_u^n}\right|_{\mu_{u,d,s}=0}$$





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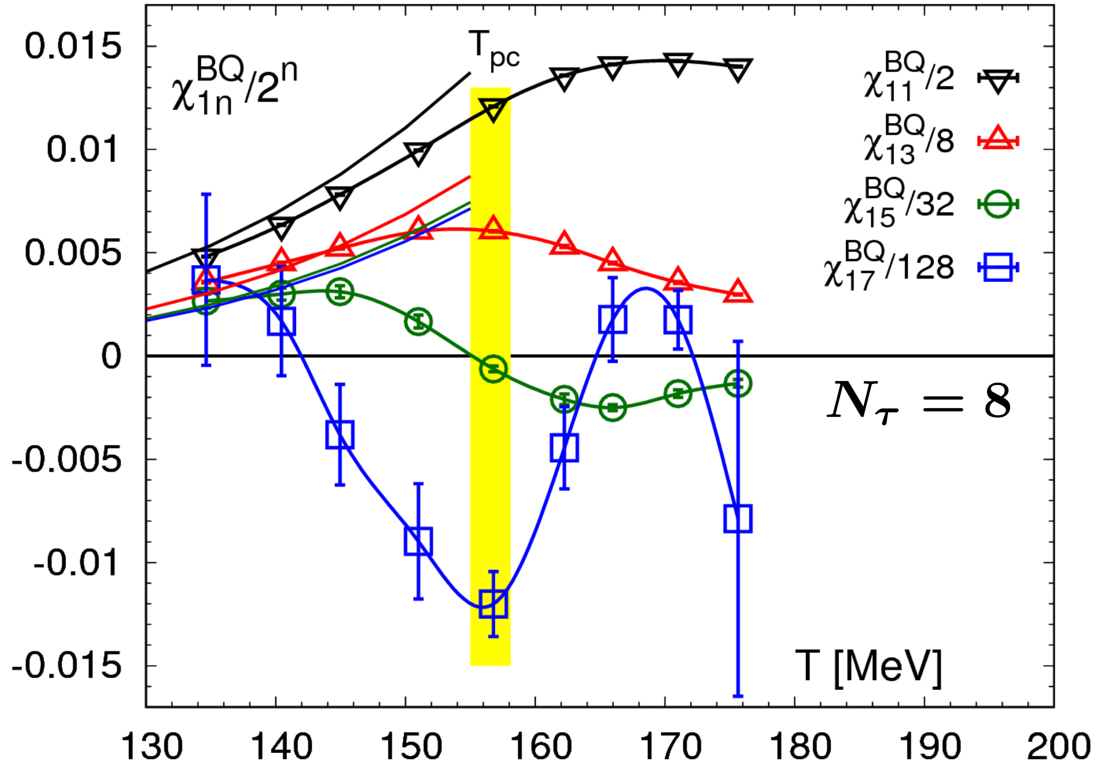
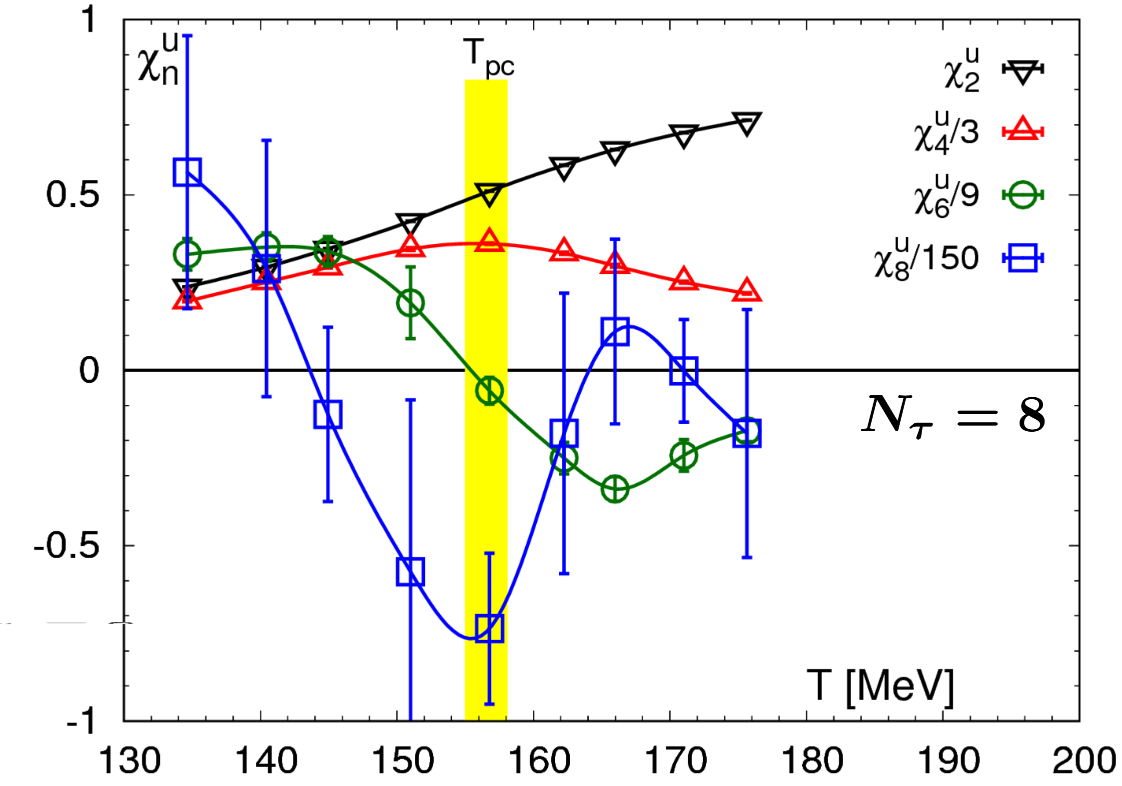
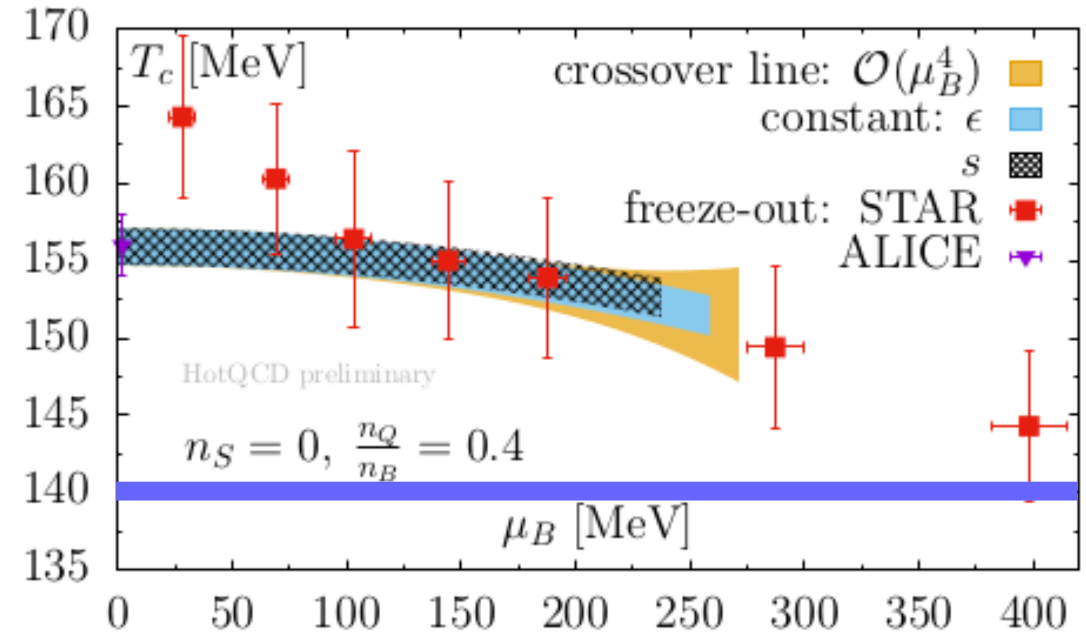




➡ Temperature of CEP is likely below 140 MeV

➡ μ_B^{CEP} likely above 400 MeV



Summary

➡ New precise transition temperature: $T_{pc} = 156.5 \pm 1.5$ MeV

➡ Equation of state (phase diagram) accessible to $\mu_B/T < 2$

➡ Curvature of the crossover line is small ….

$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

$$\kappa_2 = 0.012(2)$$

$$\kappa_4 = 0.000(2)$$

➡ At T_c: Higher order cumulants show large deviations from (non-interacting) HRG

➡ Above T_c: quark number cumulants provide evidence for liberated quark degrees of freedom.

➡ No indication for critical point, limit: $\mu_B^{CEP} > 400\mathrm{MeV}$