Effects of Statistical Particlization and Hadronic Rescattering on Fluctuations and Correlations

Jan Steinheimer

17.01.2019

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[1] J. Steinheimer and V. Koch, Phys. Rev. C **96**, no. 3, 034907 (2017).
J. Steinheimer, V. Vovchenko, J. Aichelin, M. Bleicher and H. Stöcker, Phys. Lett. B **776**, 32 (2018)

Why and how does statistical hadron production influence fluctuation observables?

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- Which correlations and fluctuations you look at.
- In that sense..

Two examples

I: Fluctuations from a phase transition, Cooper-Frye and the curse of small numbers

- Assume we have a macroscopic model for (critical/long-range) fluctuations. fluid-dynamics
- How does the necessity to have a finite number of hadrons change the multiplicity distribution?

Two examples

I: Fluctuations from a phase transition, Cooper-Frye and the curse of small numbers

- Assume we have a macroscopic model for (critical/long-range) fluctuations. fluid-dynamics
- How does the necessity to have a finite number of hadrons change the multiplicity distribution?

II: How hadronic rescattering changes the multiplicity in a fixed acceptance window

- Assuming we produce hadrons according to some multiplicity distribution on some C-F-hypersurface.
- Given a fixed rapidity window how much is the final observed multiplicity related with the initial one?

Ingredient one: Solving Fluid-Dynamics

The equations for ideal relativistic fluid-dynamics are solved numerically on a cartesian-grid of cell size $\Delta x = 0.2$ fm:

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{and} \quad \partial_\mu N^\mu = 0$$

$T^{\mu\nu}$ is the relativistic energy momentum tensor and N^μ the baryon four-current. In ideal fluid-dynamics these can be written as:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \quad \text{and} \quad N^\mu = nu^\mu$$

To close this system of equations one needs the equation of state, of the form $p = p(\epsilon, n)$, as an additional input.

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Remember: Since the cell volume is very small, $V_c = \Delta x^3 = 8 \cdot 10^{-3}$ fm^3, the number of hadrons per cell is also $<< 1$.

Ingredient two: particles from densities

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- Hydro only propagates densities, not specific particle types.
- Experiments measure particles.
- Take into account the effects of resonance decays.
- Take into account the effect of conservation laws.
- For fluctuation or correlation measurements acceptance and efficiency are important.
- The end of hydro is not the end of the collision.

How to make particles from densities

Typically done by use of the Cooper-Frye equation

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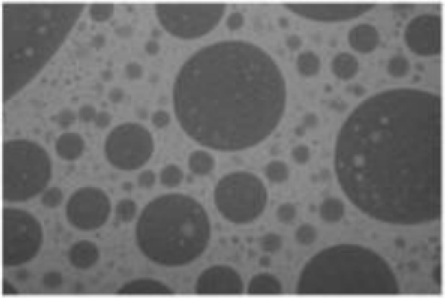
Important

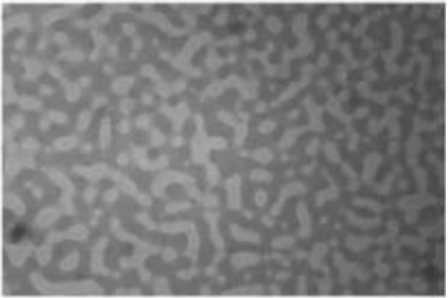
Since the cell volumes are usually small, the particle number in a cell is also $N_p << 1$ and so one assumes it is Poisson distributed!

Why fluctuations at the phase transition?

Nucleation vs. spinodal decomposition : Phase separation

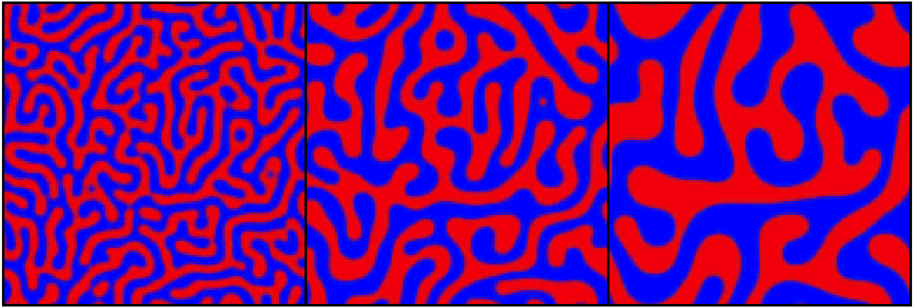
- Nucleation: Thermal fluctuations serve as seeds for bubble formation. (e.g. ice in water). SLOW!
- Spinodal decomposition: System is quenched below separation temperature. Instabilities occur (e.g. hot oil + water). FAST!





Non-Equilibrium Phase Transition

"Spinodal decomposition is essentially a mechanism for the rapid unmixing of a mixture of liquids or solids from one thermodynamic phase, to form two coexisting phases."



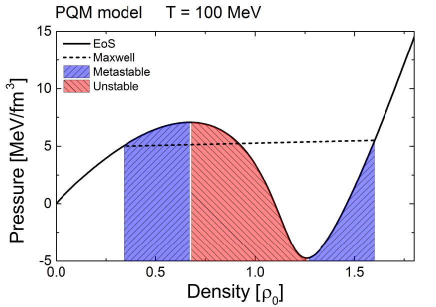
I takes place for example when one quenches a mixture of two substances rapidly below the demixing temperature. Then, the two substances separate locally, giving rise to the complicated structures which can be seen on the leftmost picture.

How to implement in hydro - the EoS.

Implementing an unstable phase is actually straight forward.

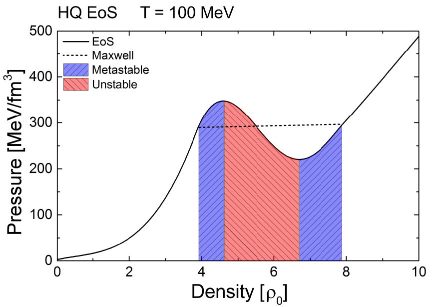
I

Either use a model which has already two phases. E.g. PQM or similar:



II

Use an EoS constructed from a hadronic nucleon-pion model plus a bag model via a construction.

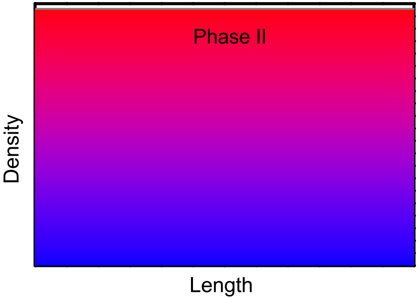


Alternatively a Maxwell-construction is possible as a baseline.

Non-Equilibrium Phase Transition

Equilibrium Phase Transition (Maxwell construction)

As the system dilutes, the phases are always well separated



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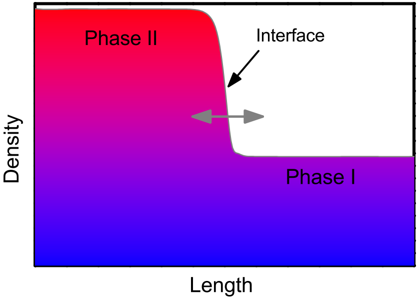

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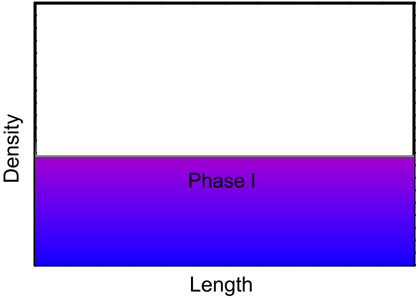

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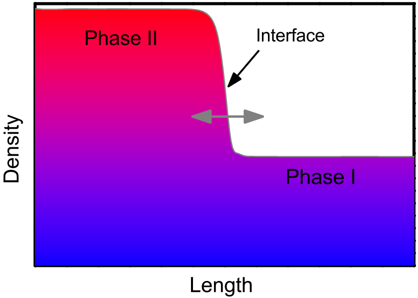
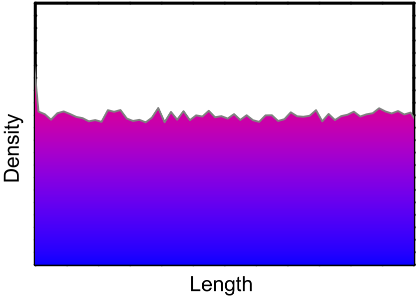
Phase separation is a dynamical process.





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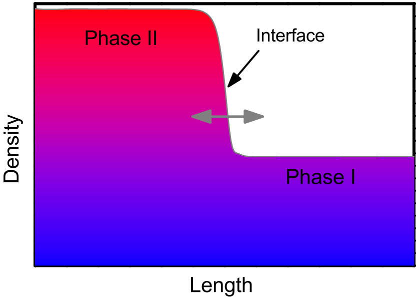

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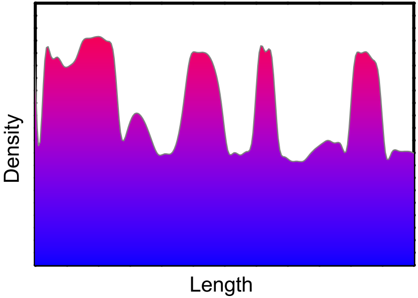

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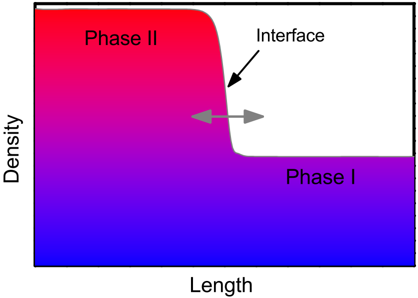
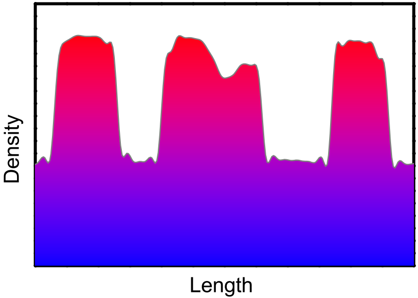
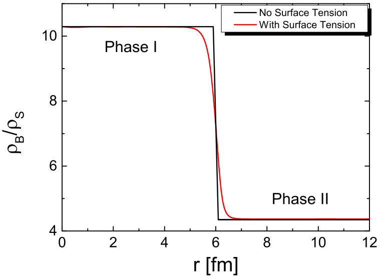




The Gradient Term

A proper description of spinodal decomposition requires that finite-range effects be incorporated.



We rewrite the local pressure as

$$p(\mathbf{r}) = p_0(\varepsilon(\mathbf{r}), \rho(\mathbf{r})) - a^2 \frac{\varepsilon_s}{\rho_s^2} \rho(\mathbf{r}) \nabla^2 \rho(\mathbf{r})$$

The gradient term will cause a diffuse interface to develop when matter of two coexisting phases are brought into physical contact.

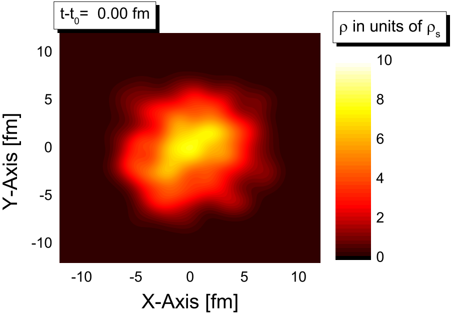
Show Animation

Initialize Random noise in the unstable region and let it evolve.

Initial State And Setup

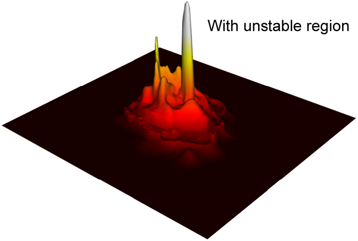
We apply the UrQMD transport model for the initial, non-equilibrium, part of the collision.

- When contracted nuclei have passed through each other,
- energy-, momentum- and baryon densities are mapped onto the computational grid.

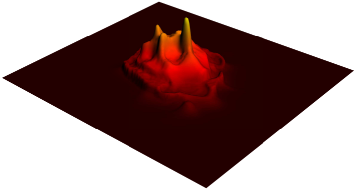


Evolution in Fluid-Dynamics

EoS with unstable phase:



EoS with Maxwell construction:



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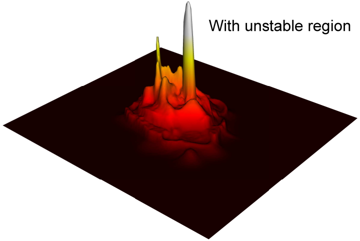

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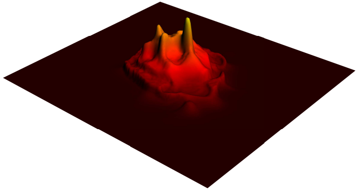
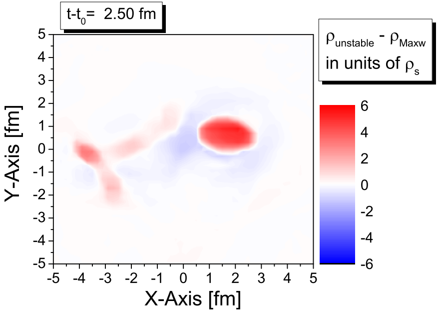




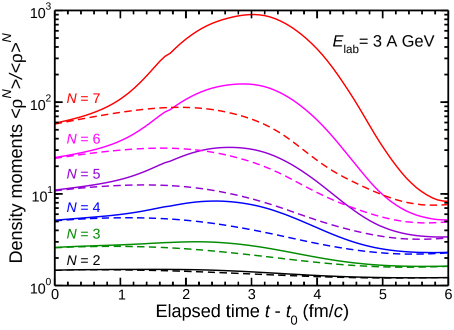
Notable difference observed.
Unstable phase leads to clustering of baryonnumber!

Moments of the Baryon Density

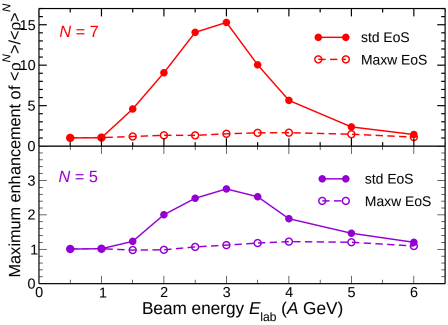
Let's be more quantitative

Define Moments of the net baryon density distribution:

$$\langle \rho^N \rangle \equiv \frac{1}{A} \int \rho(\boldsymbol{r})^N \rho(\mathbf{r})\, d^3\boldsymbol{r}$$



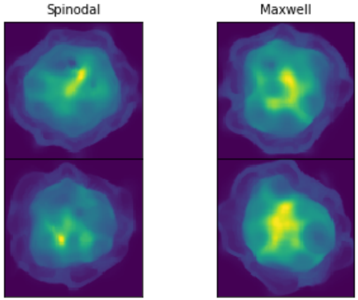
As a function of time

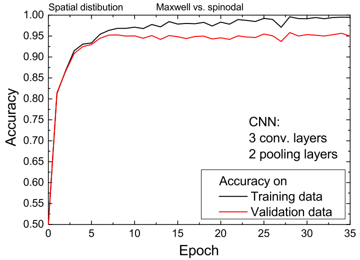


As a function of beam energy

Do we have characteristic features in every event?

We asked a CNN (Convolutional Neural Network) to separete events in Spinodal and Maxwell





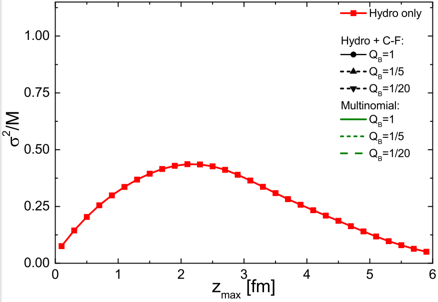
Effect on the cumulants - numeric

Note: The K_i are the cumulants of the baryon number multiplicity distribution. (STAR calls them C_i)

Comparison with numerical results

- Take the hydro results with the spinodal clumping.

K_2/K_1



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- Use the C-F equation and sample baryons, conserving baryon number globally.

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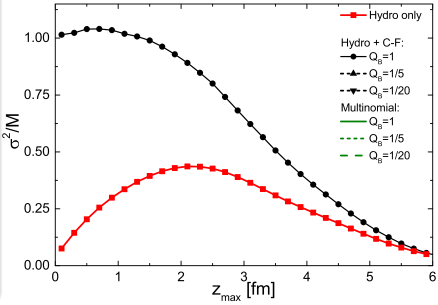

Effect on the cumulants - analytic - multinomial

What happens if one conserves the number of baryon globally?

Multinomial distribution:

$$P(N_1, \ldots, M_M) = \frac{(B_{tot}/Q_B)!}{N_1! \ldots N_M!} p_1^{N_1} \ldots p_M^{N_M} \delta_{\sum_{i=1}^{M} N_i, B_{tot}/Q_B} \quad (2)$$

Effect on the cumulants - analytic - multinomial

What happens if one conserves the number of baryon globally?

The resulting cumulants:

$$
\begin{aligned}
K_1^{B,CF,multi} &= \langle B \rangle = K_1^B \\
K_2^{B,CF,multi} &= K_2^B + Q_B \left(K_1^B - \frac{{K_1^B}^2 + K_2}{B_{tot}} \right) \\
K_3^{B,CF,multi} &= K_3^B + 3Q_B \left(K_2^B - \frac{2K_1^B K_2^B + K_3^B}{B_{tot}} \right) \\
&\quad + Q_B^2 \left(K_1^B - 3\frac{{K_1^B}^2 + K_2^B}{B_{tot}} \right. \\
&\quad \left. + 2\frac{{K_1^B}^3 + 3K_1^B K_2^B + K_3^B}{B_{tot}^2} \right)
\end{aligned}
\tag{3}
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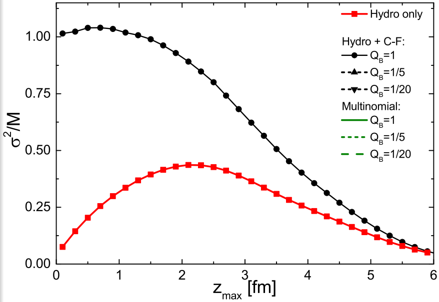

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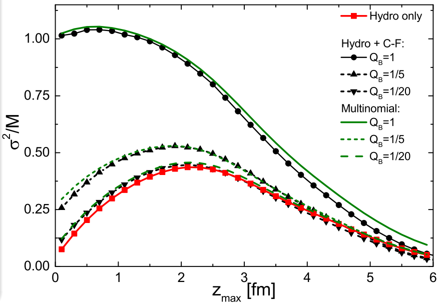

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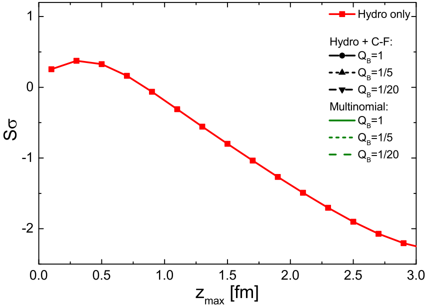

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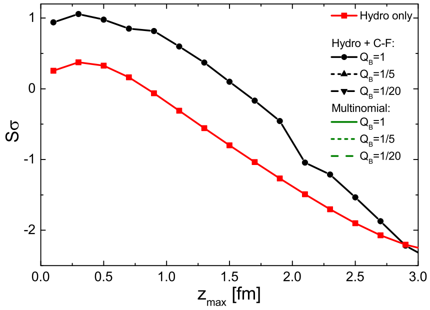
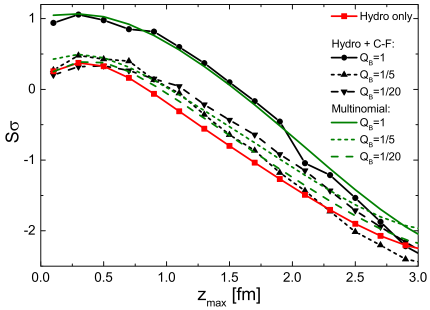


Effect on the cumulants - numerical results

Comparison with numerical results

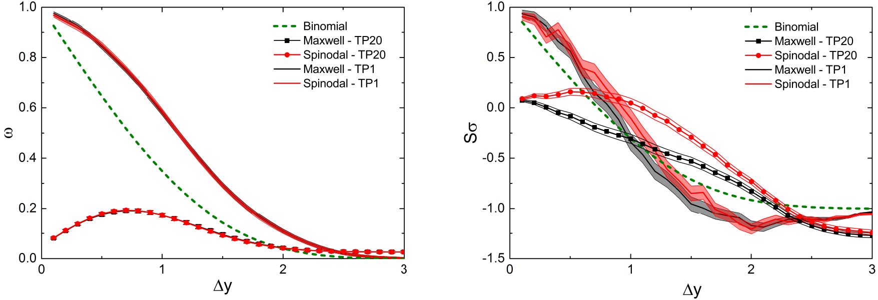
- Take the hydro results with the spinodal clumping.
- Calculate the baryon number fluctuations in a spatial volume directly
- Use the C-F equation and sample baryons, conserving baryon number globally.
- Analytic and numerical results agree well.

K_3/K_2



Effect on the cumulants - momentum space

Comparison of scaled variance and skewness in rapidity space

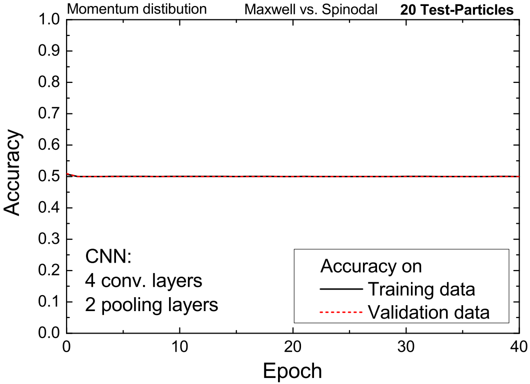


Preliminary: small difference in Skewness seems visible.

Analytic case not so trivial.

IS it also in each event in momentum space?

We repeated the CNN analysis with 20 test-particles in momentum space.



Slight change of topic: What happens after particlization?

What to expect from hadronic rescattering

Elastic and pseudo-elastic rescattering

- CHANGES: spectra
- CHANGES: correlations and fluctuations

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The idea of a hadronic phase was/is motivated by:

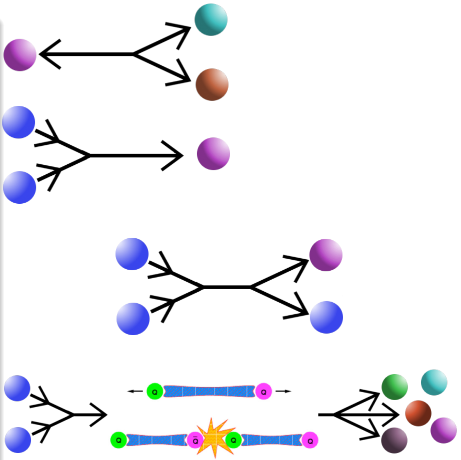
- Particle spectra have a temperature much smaller than the "chemical" Temperature.
- Resonance yields are not consistent with thermal fits
- I will NOT talk about annihilation.

The UrQMD model

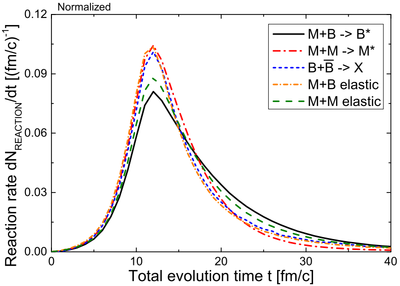
Use UrQMD for the hadronic phase. Of course the result will depend on the model ingredients. We just include, and constrain it, as much as we can.

What is UrQMD

- Microscopic model based on geometric interpretation of cross sections.
- $2 \to n$ particle scattering according to measured cross sections
- Resonance decays plus string excitations at $\sqrt{s} > 3$ GeV.
- Newest version: Strangeness exchange $\overline{K} + N \leftrightarrow \pi + Y$ and $Y + Y \leftrightarrow \Xi + N$



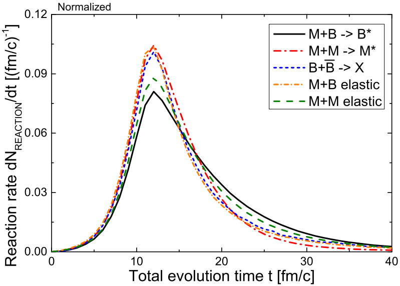
A general view on reaction rates at RHIC



Reaction rates $|y| < 0.5$

- Normalized reaction rates are similar for all different processes.

A general view on reaction rates at RHIC



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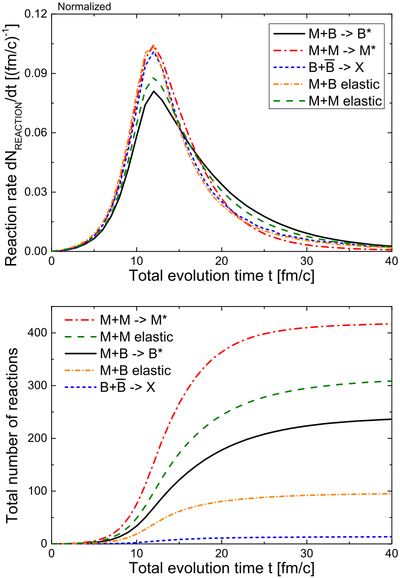

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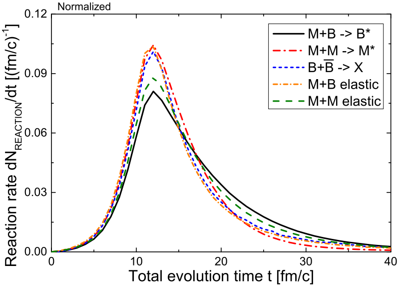

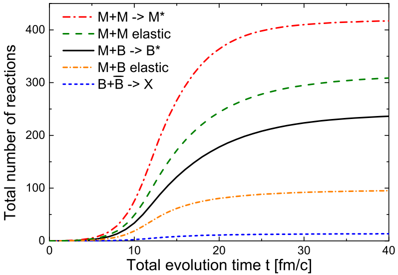

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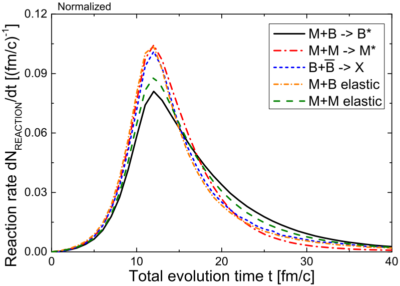

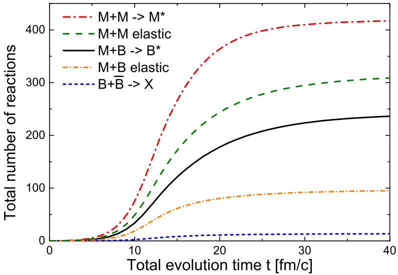

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- Few annihilations.
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Effects on particle fluctuations I

Even more than changing particle yields the interactions will change correlations and fluctuations.

How well are fluctuations remembered?

$$r_{\mathrm{IF}}(t) = \frac{\sum_n (I_n(t) - \overline{I}(t))(F_n - \overline{F})}{\sqrt{\sum_n (I_n(t) - \overline{I}(t))^2 \sum_n (F_n - \overline{F})^2}} \qquad (4)$$

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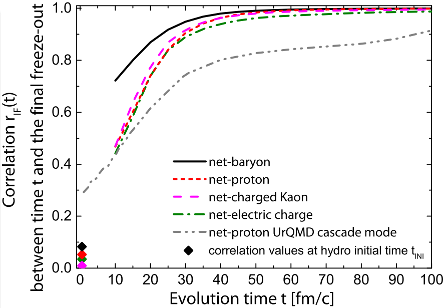
Different charges in STAR experimental acceptance.



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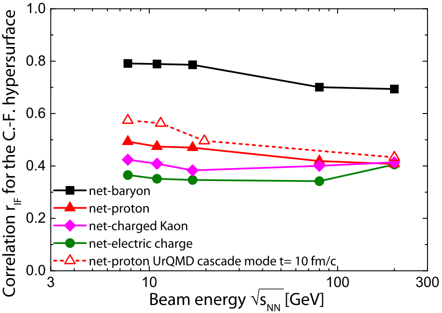

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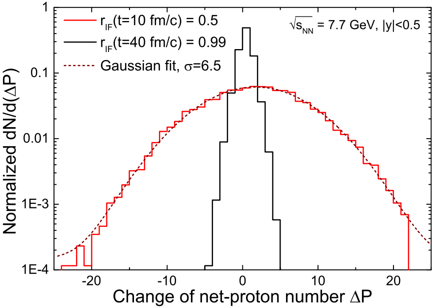
Effects on particle fluctuations II



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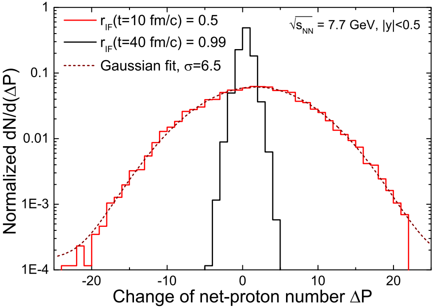

- Rescattering corresponds to a Gaussian smearing of the multiplicity distribution
- Width of the Gaussian ca be as broad as the entire distribution.

Hadronic rescattering

Important!

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- Only if the cumulants at C-F are dominated by correlations then rescattering should change the cumulants!
- At the same time: Measuring cumulants which are dominated by the mean does not mean there haven't been any correlations.
- If all correlations are washed out be the rescattering then Chemical fit $\approx$ Cumulant fit ($+$ conservation laws).

Discussion

Backup

Effects of Viscosity

In actual systems there is some degree of physical dissipation which gives rise to both viscosity and heat conduction.

To leading order, the viscosity reduces the growth rate by

$$\approx \frac{1}{2}[\frac{4}{3}\eta + \zeta]k^2/h \tag{5}$$

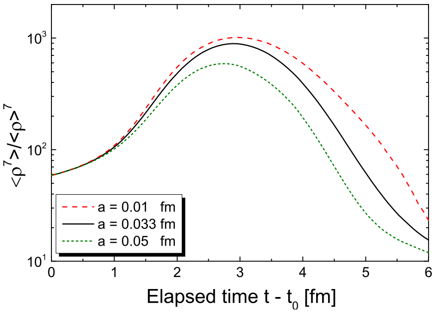
where η and ζ are the shear and bulk viscosity coefficients, respectively.

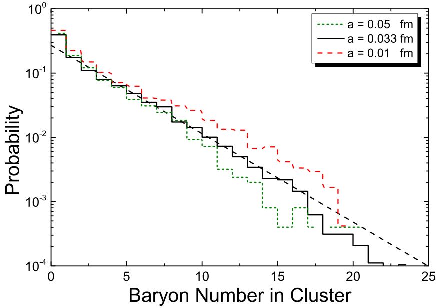
A heat conductivity generally increases the growth rate because the speed of sound will be closer to the iso-thermal v_T instead of the isentropic, increasing the size of the unstable region (and also decreasing the squared speed of sound).

Changing the Surface Tension

Parameter Dependencies

We change the value of the surface tension by varying a from 0.01 to 0.05.



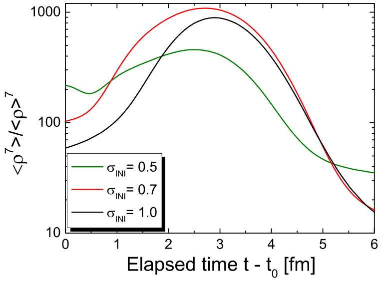


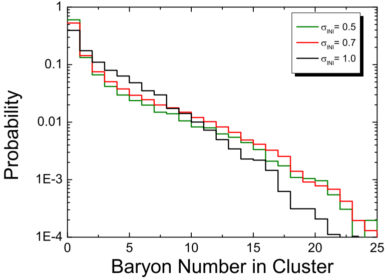
Quantitative results depend on choice of a.
Cluster formation is stronger for small values of a, as expected.

Changing the Initial Fluctuations

Parameter Dependencies

We change the value of the initial fluctuation width from $\sigma_{ini} = 0.5$ to 1.0 fm.





Strongest clustering observed for intermediate width!

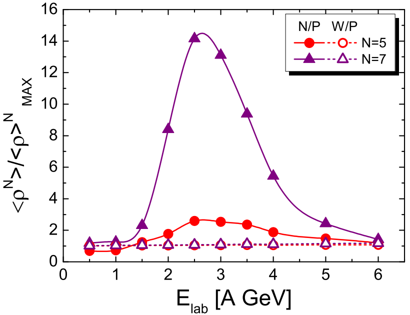
Changing the Initial State

Model Dependencies?

For the initial state we can apply a version of the UrQMD model that includes nuclear interactions. [a]

[a] Q. -f. Li, Z. -x. Li, S. Soff, M. Bleicher and H. Stoecker, J. Phys. G **32**, 407 (2006)

The densities achieved in the UrQMD+Potentials calculations are considerably smaller than in those without (close to the geometrical overlap values).
This is mainly due to the repulsive interaction.

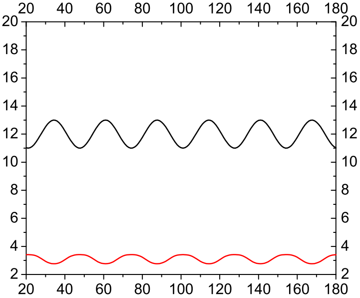


Growth Rates in Numerical Fluid-Dynamics

Calculation in a box with periodic boundaries

The amplitude of a density undulation should grow exponentially within the unstable region

$$A(t) = A_{t=0}(e^{\gamma_k t} + e^{-\gamma_k t})$$



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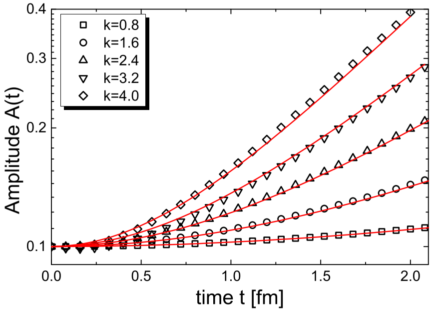

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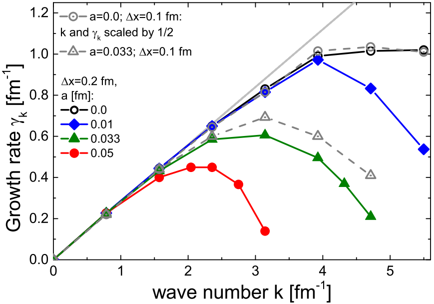

- Numerical viscosity cuts off large wave number growth
- The gradient term modifies the growth rates:

$$\gamma_k^2 = |v_s|^2 k^2 - a^2(\varepsilon_s/h)(\rho/\rho_s)^2 k^4$$

- Depending on a certain wave numbers are favored