The fate of axial U(1) in 2+1 flavor QCD towards the chiral limit

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The $U_A(1)$ puzzle

- Nature of chiral phase transition for QCD with two light quark flavors is not yet completely resolved.
- Usually symmetries determine the order parameter across a phase transition.
- However, for 2 light quark flavors anomalous $U_A(1)$ may affect the nature of phase transition.
- How severely $U_A(1)$ is broken can be answered non-perturbatively.
- We want to investigate whether or not the anomalous $U_A(1)$ symmetry is effectively restored at $T_c = (130 - 135)$MeV.

Observables sensitive to $U_A(1)$ breaking

- $U_A(1)$ is not an exact symmetry $\Rightarrow$ no unique order parameter!
- Instead, look at n-point correlation functions which become degenerate upon $U_A(1)$ rotation.
  Start with 2-point correlation functions.

$$
\begin{tikzcd}
\chi_{5,con} & \pi : \bar{q}\gamma_5\frac{\tau}{2}q \arrow[r, leftrightarrow, "SU_L(2)\times SU_R(2)"] \arrow[d, leftrightarrow, "U_A(1)"'] & \sigma : \bar{q}q \arrow[d, leftrightarrow, "U_A(1)"] & \chi_{con}+\chi_{disc} \\
\chi_{con} & \delta : \bar{q}\frac{\tau}{2}q \arrow[r, leftrightarrow, "SU_L(2)\times SU_R(2)"'] & \eta : \bar{q}\gamma_5 q & \chi_{5,con}+\chi_{5,disc}
\end{tikzcd}
$$

$U_A(1)$ breaking and QCD eigenvalue density

- Observable of interest is [Shuryak, 1994]

$$\chi_\pi - \chi_\delta = \int d^4x \left[\langle i\pi^+(x) i\pi^-(0) \rangle - \langle i\delta^+(x) i\delta^-(0) \rangle \right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator
[Cohen, 1995, Hatsuda & Lee, 1995]

$$\chi_\pi - \chi_\delta \xrightarrow{V \to \infty} \int_0^\infty d\lambda \frac{4 m_f^2 \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- Possible scenarios:
  - $\lim_{m_f \to 0} \rho(0, m_f) \to 0 \Rightarrow U_A(1)$ trivially restored.
  - $\lim_{\lambda \to 0} \rho(\lambda, m_f) = \delta(\lambda) m_f^\alpha$ with $1 < \alpha < 2 \Rightarrow U_A(1)$ broken.
  - $\lim_{m_f \to 0} \rho(\lambda, m_f) \sim \lambda^3 \Rightarrow U_A(1)$ restored.

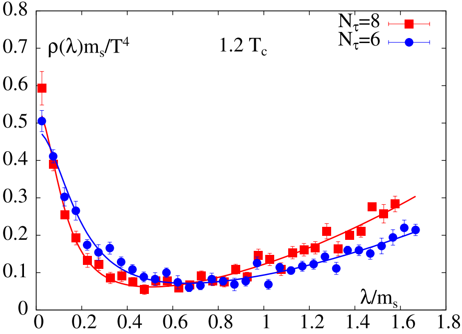
More on the eigenvalue spectrum and $U_A(1)$ breaking

- Looking at higher n-point correlation functions are important!
- $\rho(\lambda, m_f)$ has been investigated analytically using chiral Ward identities of n-point functions of scalar and pseudo-scalar currents, assuming $\rho(\lambda, m_f)$ to be analytic in m_f^2 [Aoki, Fukaya & Taniguchi, 2012].
- It was shown explicitly that $U_A(1)$ breaking is absent in upto six point correlation functions in the same scalar and pseudo-scalar sectors if $\rho(\lambda, m_f \to 0) \sim \lambda^3$.
- They found out that in the chiral limit $\rho(\lambda) \sim \lambda^n$ with $n > 2$.

Summary of the results till now

- Previous studies at almost physical quark masses: Infrared part has both, non-analytic and analytic in λ [Sayantan Sharma et al.,2015].

- $\rho(\lambda, m_f) \sim \lambda^2$ at $T = 1.2\,T_{pc} \Rightarrow U_A(1)$ broken.



[Viktor Dick et al.,2015]

Setup

- Gauge ensembles where generated within the Highly Improved Staggered Quark discretization scheme (HISQ) with 2+1 quark flavor.
- We used the overlap Dirac operator to measure the low-lying eigenvalue spectrum.
- 98 eigenvalues per configuration.

m_l	$N_s^3 \times N_\tau$	β	T/T_{pc}	#conf
$m_s/27$	$32^3 \times 8$	6.390	0.97	69
$m_s/27$	$32^3 \times 8$	6.445	1.03	81
$m_s/27$	$32^3 \times 8$	6.500	1.09	102
$m_s/40$	$32^3 \times 8$	6.390	0.99	45
$m_s/40$	$32^3 \times 8$	6.423	1.03	50
$m_s/40$	$32^3 \times 8$	6.445	1.05	104

Overlap operator

- The overlap operator is given as,

$$D_{ov} = M\left[1 + \gamma_5 \mathrm{sgn}\left(\gamma_5 D_W(-M)\right)\right]$$

$$\mathrm{sgn}\left(\gamma_5 D_W(-M)\right) = \gamma_5 D_W(-M)/\sqrt{D_W^\dagger(-M) D_W(-M)}$$

  where D_W is the Wilson-Dirac operator with a negative mass parameter $M \in [0, 2)$.

- Zero-modes were not measured. Only eigenvalues with positive or negative chirality have been measured.

- Valence overlap quark mass has been tuned to the HISQ sea quark masses by matching the renormalized quantity

$$\Delta = \frac{m_s \langle \bar{\Psi}\Psi \rangle_l - m_l \langle \bar{\Psi}\Psi \rangle_s}{T^4}$$

Topological charge

- From left/right-handed fermion zero modes:

$$Q = n_+ - n_- .$$

- Gluonic definition of topological charge:

$$Q = \int d^4x \, q(x),$$

- with the topological charge density

$$q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr} \left\{ F_{\mu\nu}(x) F_{\rho\sigma}(x) \right\} .$$

The Gradient Flow

- All gluonic definitions valid only on smooth configurations.
- Introduce extra coordinate t (flow-time) and define a t-dependent gauge field $B_\mu(x,t)$ [Lüscher,2014]

$$\frac{d}{dt} B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t),$$

$$D_\mu = \partial_\mu + [B_\mu(x,t),\ \cdot\ ],$$

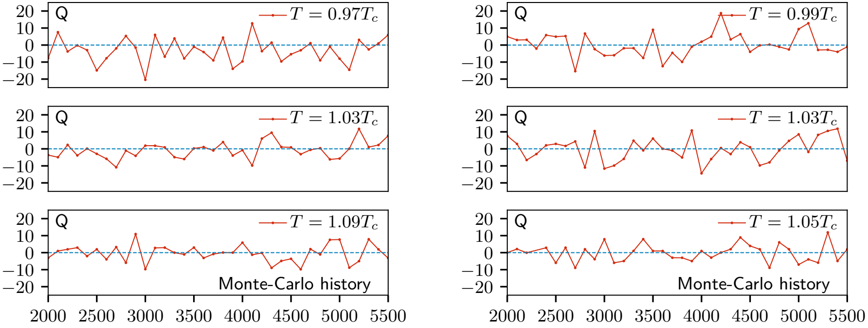
$$G_{\mu\nu}(x,t) = \partial_\mu B_\nu(x,t) - \partial_\nu B_\mu(x,t) + [B_\mu(x,t), B_\nu(x,t)]$$

- with the initial condition: $B_\mu(x,t)|_{t=0} = A_\mu(x)$
- The flow smoothens the fields over a region of radius $\sqrt{8t}$

Is topological tunneling sufficient?

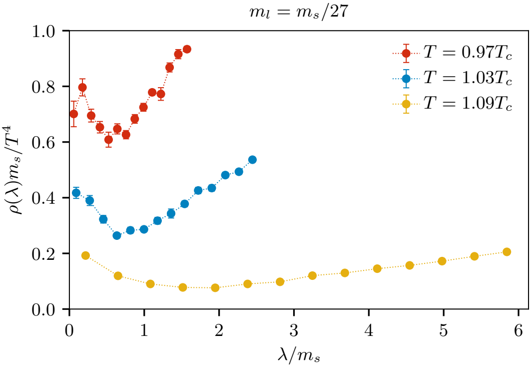
- No freezing is seen in the trajectories of the topological charge of the lattices.

$m_l = m_s/27$: $m_l = m_s/40$:



Dirac eigenvalue spectrum

- Analytic part goes as λ^γ with $\gamma \geq 1$.
- Non-analytic part reduces with temperature.
- Going towards smaller quark masses these features survive.
- Results are preliminary!



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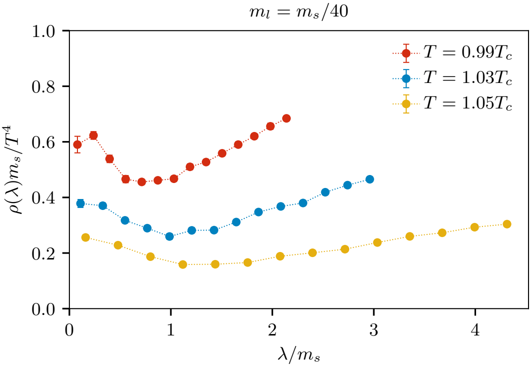
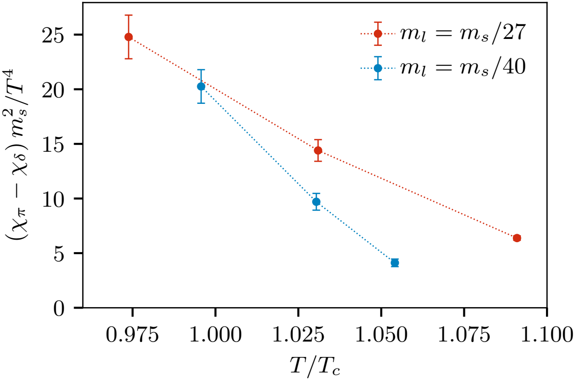


Renormalized $U_A(1)$ breaking parameter as a function of quark mass

- $\chi_\pi - \chi_\delta$ reduces when m_f reduces from $m_s/27$ to $m_s/40$ for $T/T_{pc} < 1.1$.
- Results are preliminary!



Conclusion

- $\rho(\lambda, m_f) \sim \lambda$ even when m_f reduces from $m_s/27$ to $m_s/40$ for $T/T_{pc} < 1.1$.
- Non-analytic part also survives when we reduce the mass.
- Both of them contribute to the breaking of $U_A(1)$ above T_c after proper retuning of the valence quark masses and looking at renormalized quantities.
- We are looking at even smaller quark masses to check whether our conclusions survive in the chiral limit.