Parity Doubling in QCD Thermodynamics

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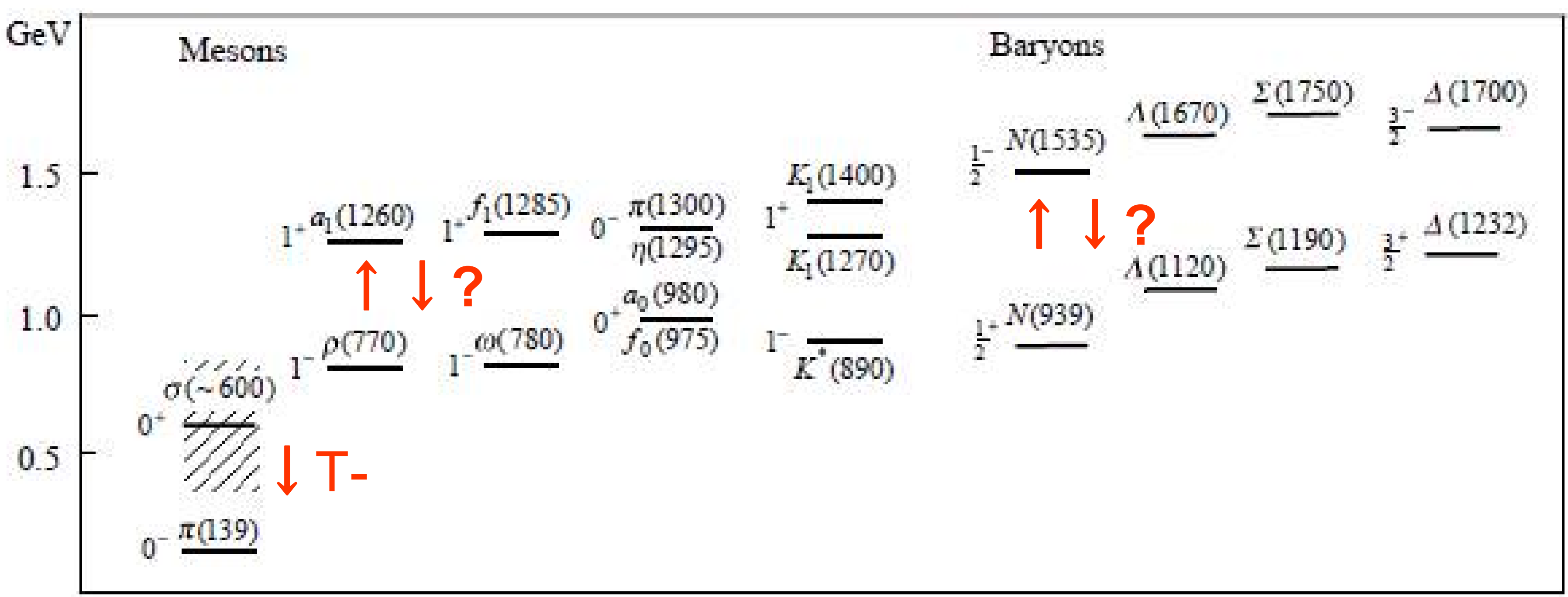
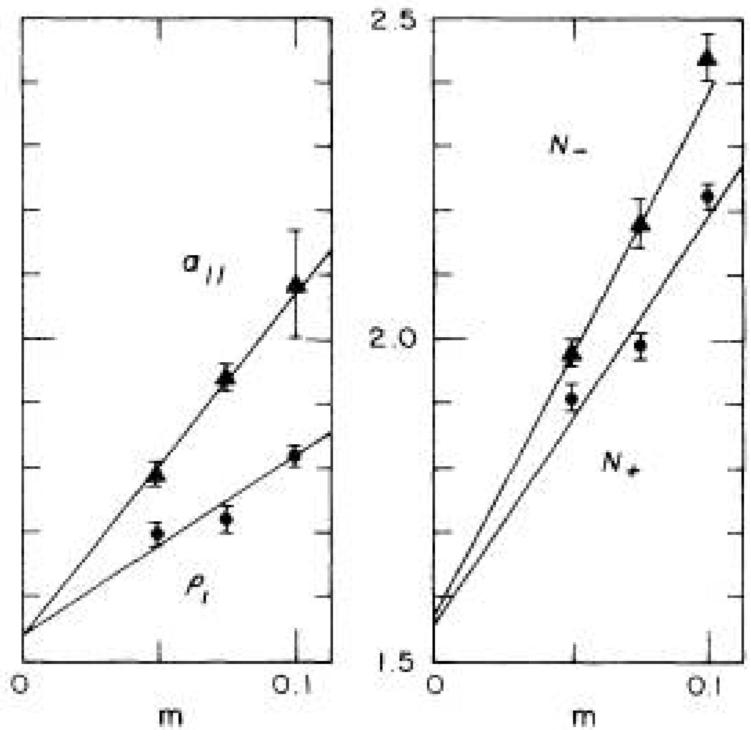
Spectra in a chirally restored world

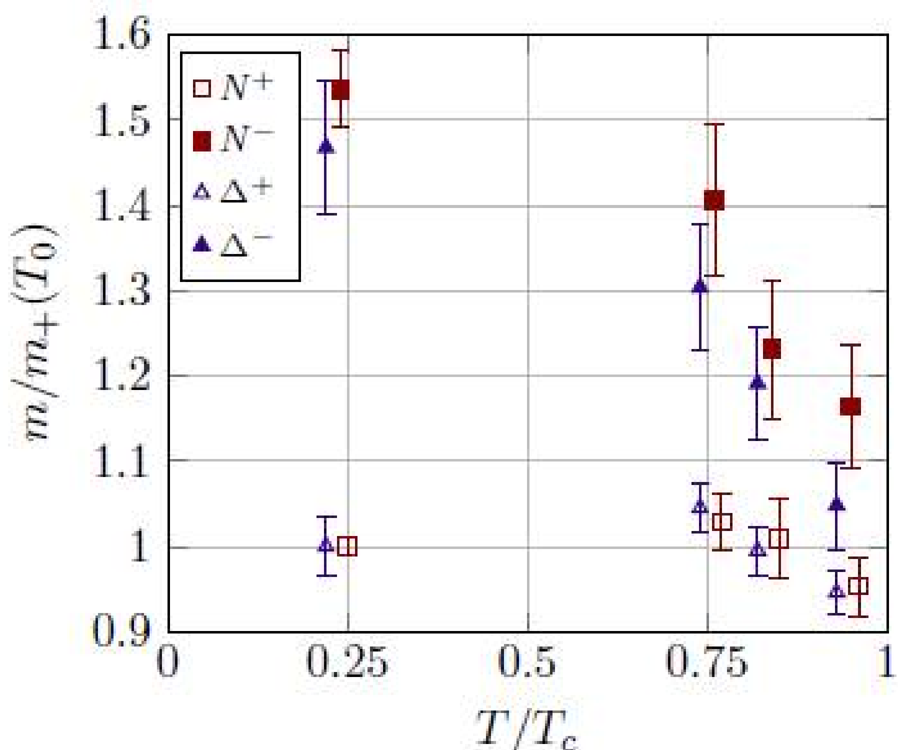
- ❑ Lowest scalar meson → O(4) vector with pion
- ❑ Parity partners degenerate → chiral partners
- ❑ QCD ground-state particles: pions & nucleons



Lattice QCD tells us …

- Spatial correlations [DeTar-Kogut, 1987]
- Temporal correlations [FASTSUM Coll., 2015-17: mpi≈400 MeV, mk≈500 MeV, Wilson fermions, Tch=185 MeV]





M(Tc) = m0 ≠0

Non-SCB mass of nucleons

- How to assign 2 indep. rotation to 2 nucleons?

$$\psi_{1L} \to g_l \, \psi_{1L} \,, \quad \psi_{1R} \to g_r \, \psi_{1R} \ \sim \ \psi_{1L} : (1/2, 0) \quad \psi_{1R} : (0, 1/2)$$

$$\psi_{2L} \to g_r \, \psi_{2L} \,, \quad \psi_{2R} \to g_l \, \psi_{2R} \ \sim \ \psi_{2L} : (0, 1/2) \quad \psi_{2R} : (1/2, 0)$$

$$\mathcal{L}_m = m_0 \left(\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2 \right) \ \Rightarrow \ m_{N_\pm} = \frac{1}{2} \left[\sqrt{c_1 \sigma^2 + 4 m_0^2} \mp c_2 \sigma \right]$$

[DeTar-Kunihiro, 1989]

- SU(3): mass relatoins for octet & decuplet
  - Gell-Mann—Okubo mass formula
  - Gell-Mann’s equal spacing rule
  - Comparison to FASTSUM’s results → strong mpi dep., Ω_- mass? [CS, 2017]

1. Parity doubling of nucleons

How to model dense QCD?

- Lattice simulations invalid → model analyses
- Good model must possess
  - Correct properties of nuclear ground state
    - ✓Saturation density, binding energy, compressibility
    - ✓Rather big chiral-inv. mass m0 ≈500-800 MeV favored

      [Zschiesche et al. (07), Gallas et al. (11)]
  - Correct degrees of freedom
    - ✓Nucleons at low density/quarks at high density

→ How to realize the 2nd property?

[Benic et al. 2015]

Quark-nucleon hybrid model

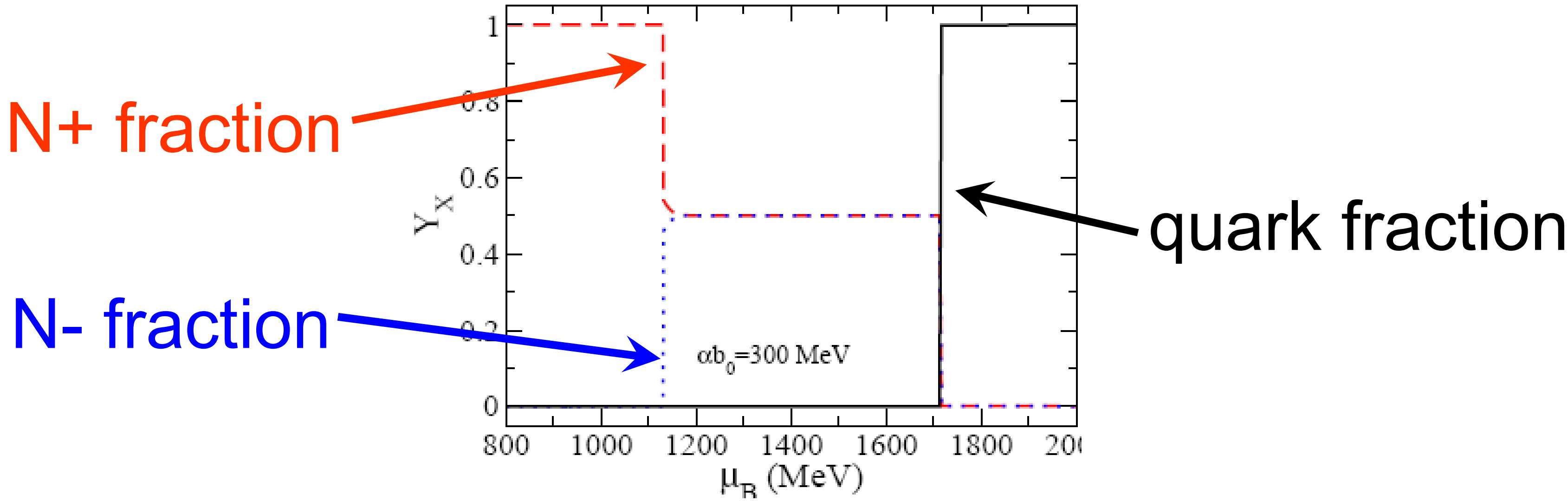
- How to suppress quarks at low density?
  - IR/UV cutoff "b" in Fermi dist. functions
  - from const. "b" to a VEV of a scalar field b
- Chiral & deconf. p.t. in a single framework

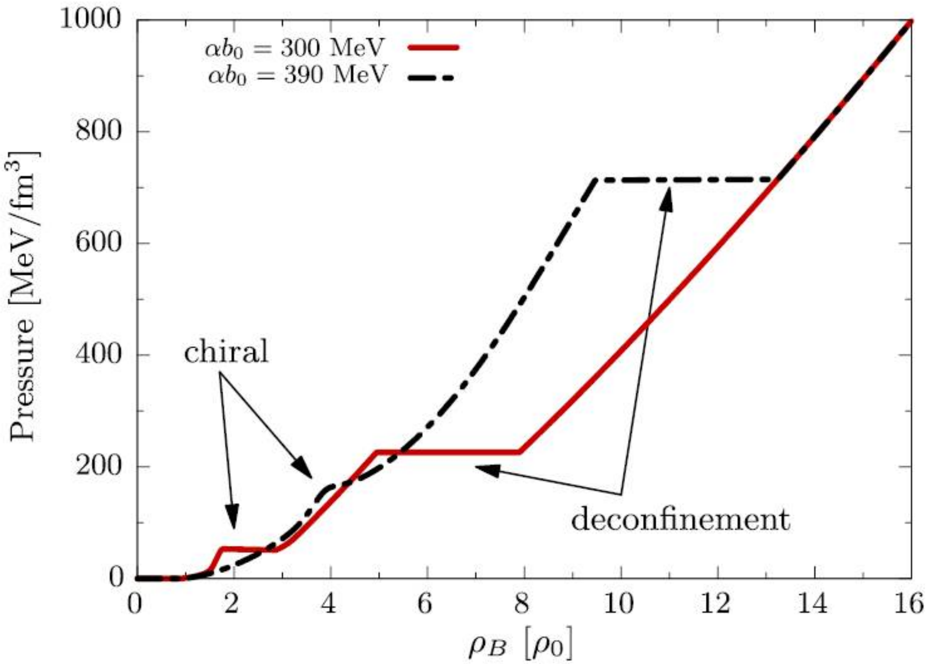
$$\int_0^{\langle b\rangle} dp\, f_N(p;T,\mu) \to \int_0^0 dp\, f_N(p;T,\mu) = 0$$

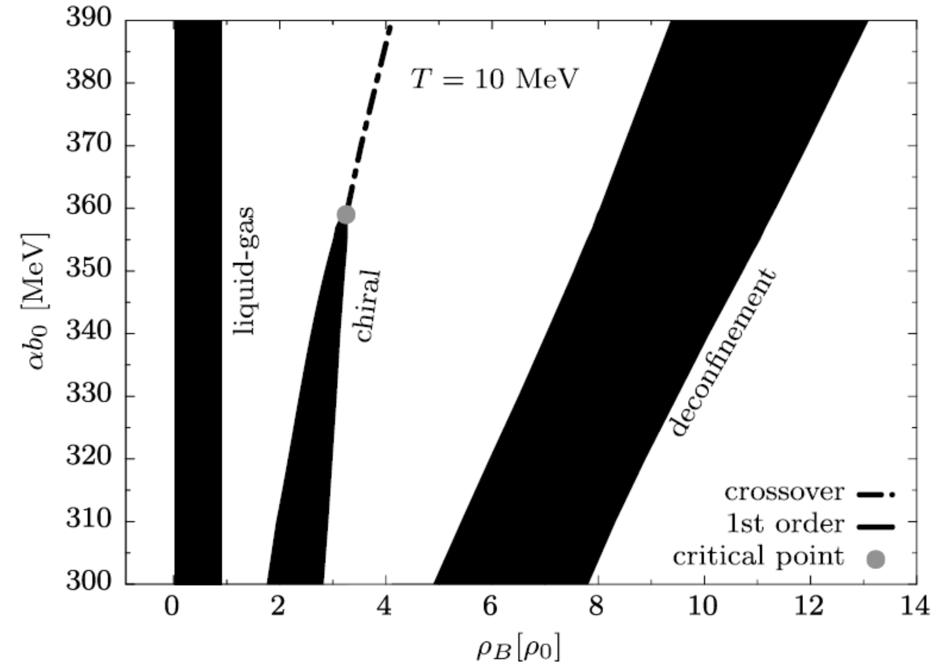
$$\int_{\langle b\rangle}^{\infty} dp\, f_Q(p;T,\mu) \to \int_0^{\infty} dp\, f_Q(p;T,\mu)$$

[Marczenko, CS, 2017]

Onset of different fermions





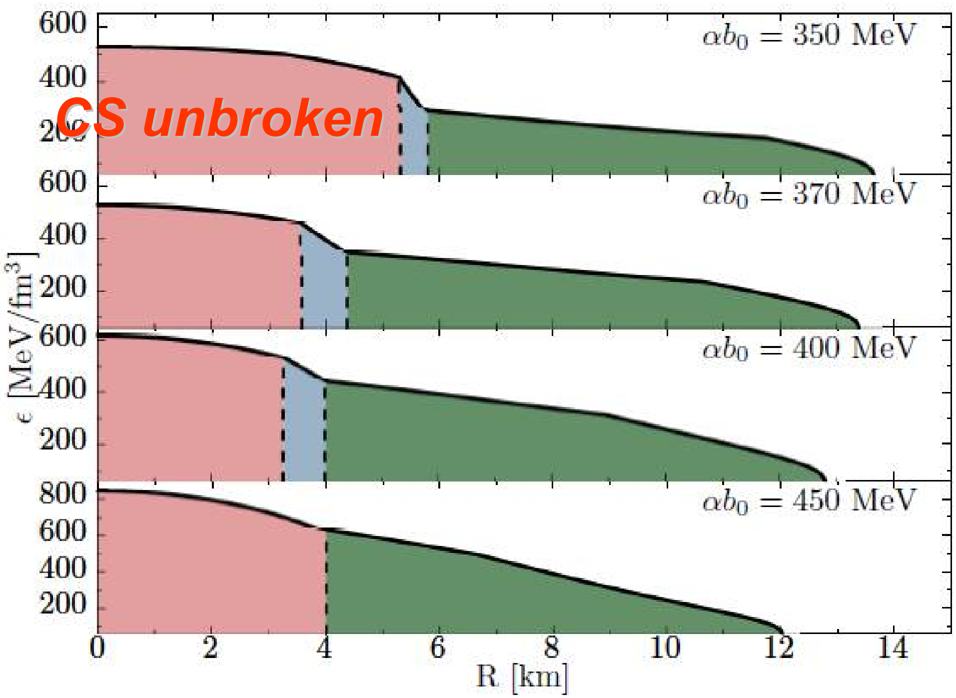


[Marczenko, Blaschke, Redlich, CS, 2018]

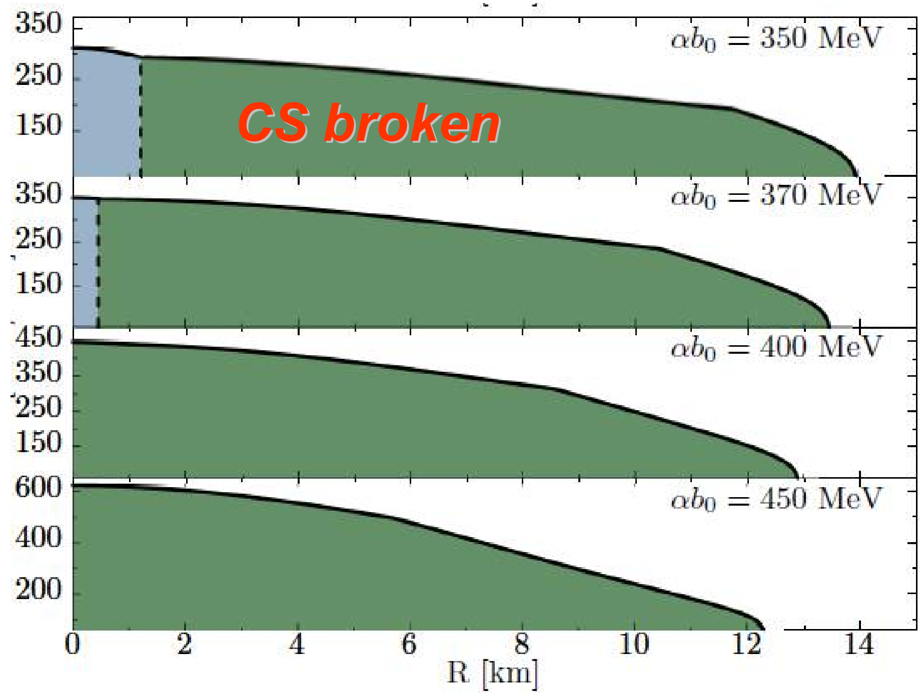
Neutron stars

- ❑ β -equilibrium and charge neutrality
- ❑ Constraints on the mass and compactness of a star → hadronic scenario w/o deconf.quarks

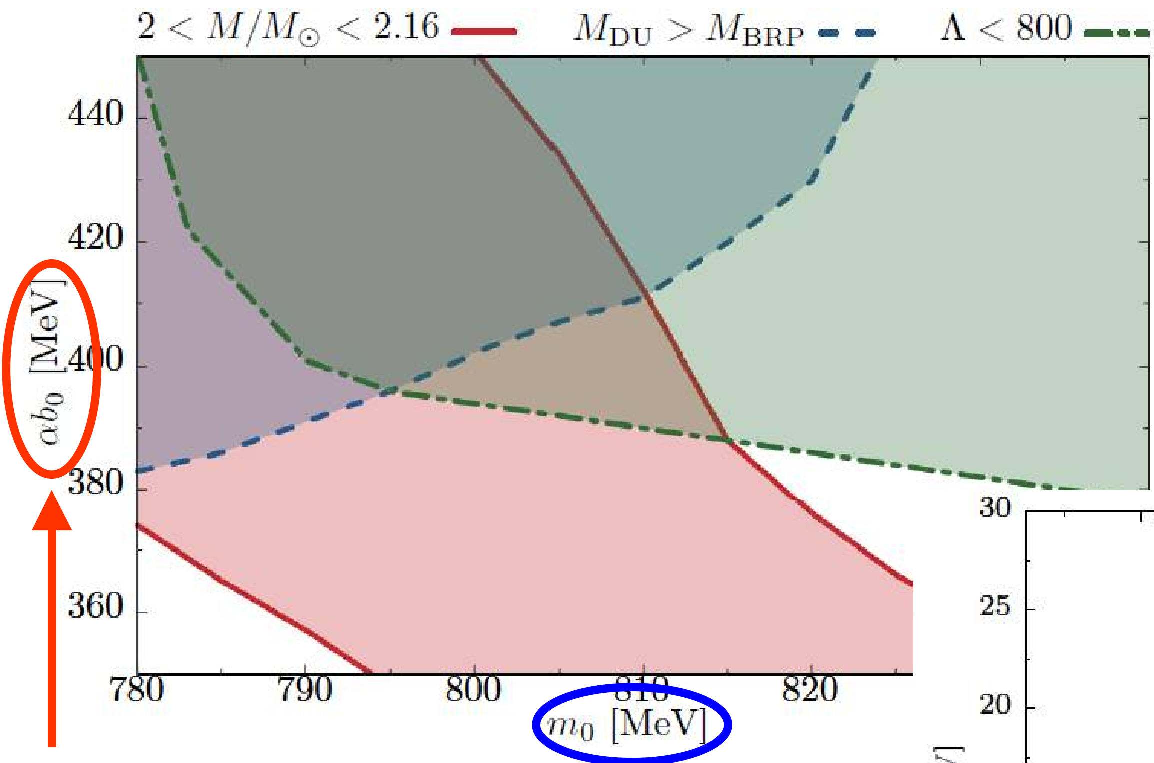
2.05*SM NS



1.97*SM NS



Toward the QCD phase diagram

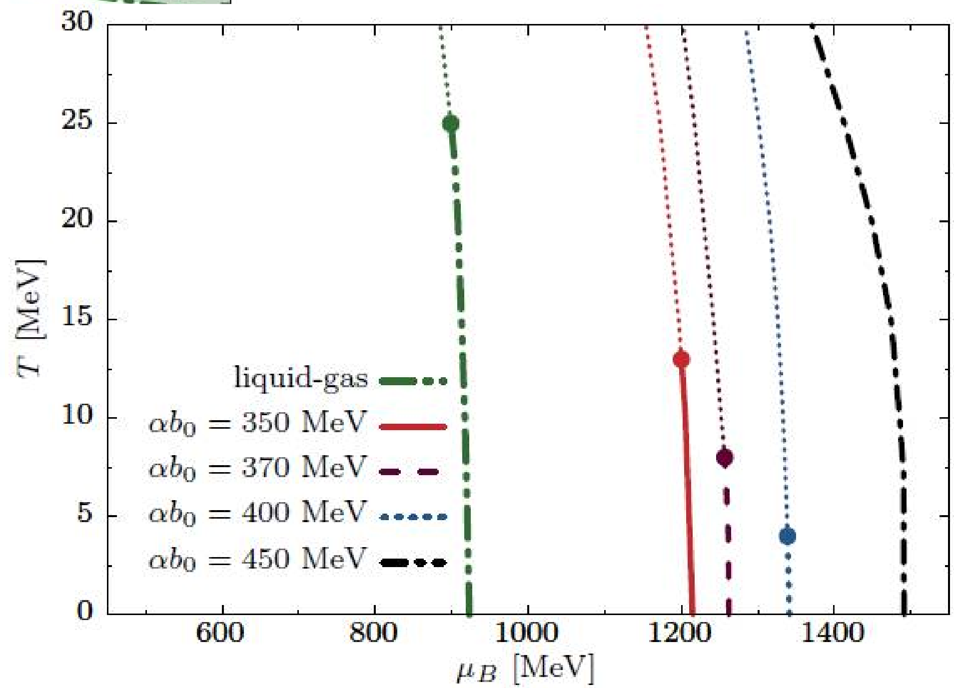


- ✓Mass
- ✓Cooling
- ✓Tidal deformability
  in the binary merger

IR cutoff

Chiral-inv. mass

CP disfavored?



2. Parity doubling of mesons

[Bando et al. (1985)]

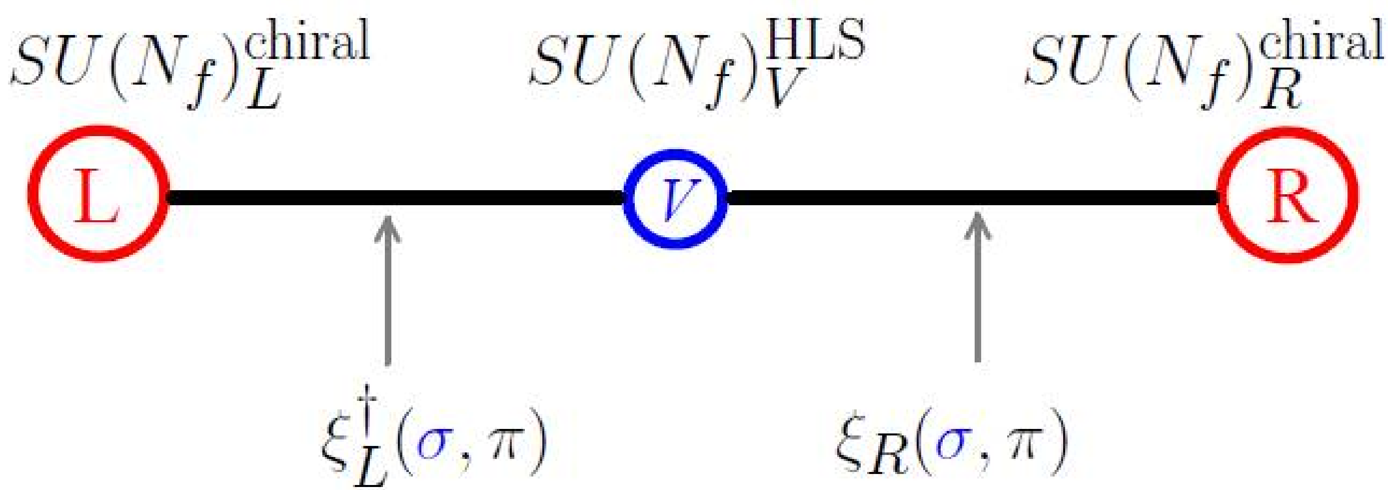
Hidden local symmetry

- Extension of non-linear chiral Lagrangian
- Vector mesons as dynamical gauge bosons

$$U = \xi^2 = e^{2i\pi/F_\pi} \quad \Rightarrow \quad U = \xi_L^\dagger \xi_R \,, \quad \xi_{L,R} = e^{i\sigma/F_\sigma} e^{\mp i\pi/F_\pi}$$

$$\boxed{U \to g_L U g_R^\dagger} \qquad \boxed{\xi_{L,R} \to h \cdot \xi_{L,R} \cdot g_{L,R}^\dagger}$$

$$h \in [SU(N_f)_V]_{\mathrm{local}} \,, \quad g_{L,R} \in [SU(N_f)_{L,R}]_{\mathrm{global}}$$



[Bando et al. (1985)]

Vector meson dominance

- 3 parameters at tree: F_π, $a=(F_\rho/F_\pi)^2$, g
- Phenomenology with a=2
  - Universality of ρ coupling $g_{\rho\pi\pi} = g$
  - KSRF relation $m_\rho^2 = 2g_{\rho\pi\pi}^2 F_\pi^2$
  - ρ meson dominance of pion EM FF $g_{\gamma\pi\pi} = 0$
- GHLS at 1 loop [Harada and CS (2006)]
  - 1st and 2nd Weinberg SR, intact at 1 loop

$$F_\pi^2 + F_{A_1}^2 = F_\rho^2 \,, \quad F_{A_1}^2 M_{A_1}^2 = F_\rho^2 M_\rho^2$$

  - Fate of VMD: valid when $M_{a1}/M_\rho \to 1$

Chiral mixing in a medium

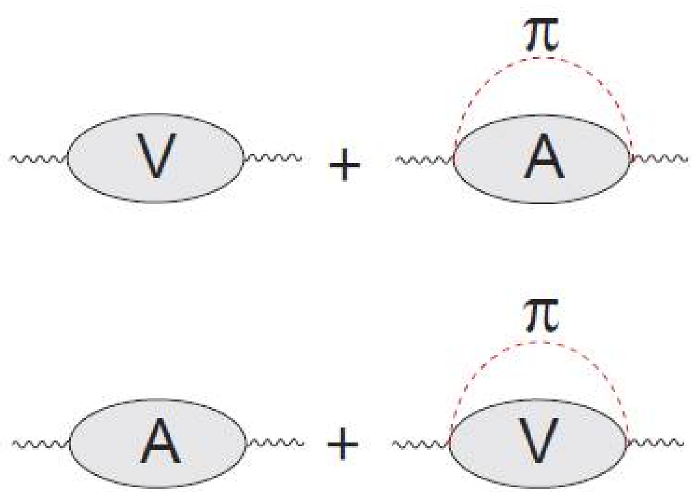
Chiral mixing in hot matter

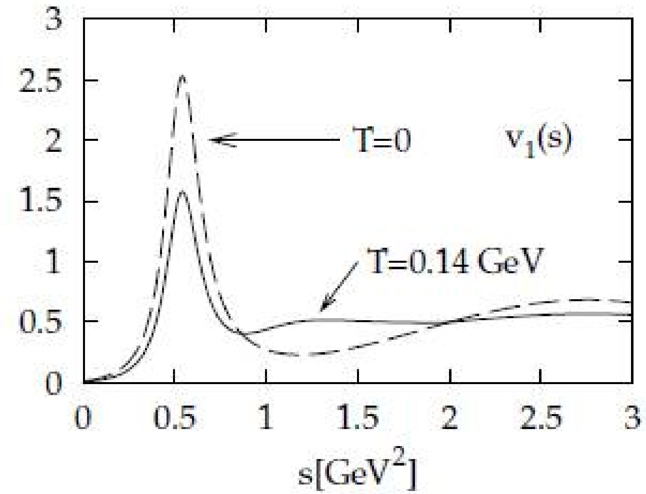
for low $T \lesssim m_\pi$: [Dey, Eletsky and Ioffe (90)]

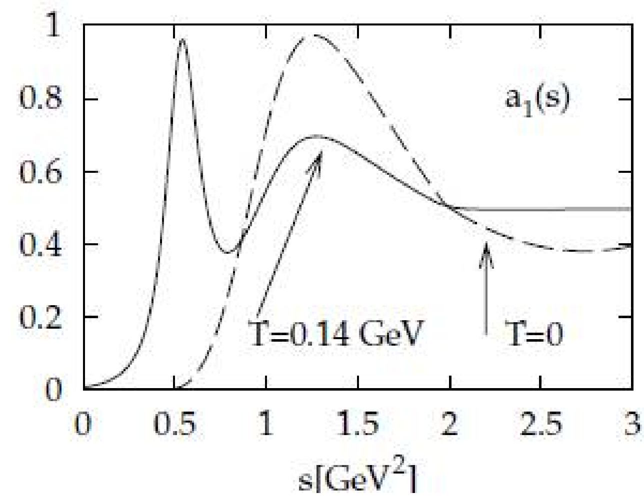
$$G_V^{\mu\nu}(T) = (1-\epsilon) G_V^{\mu\nu}(0) + \epsilon\, G_A^{\mu\nu}(0)$$

$$G_A^{\mu\nu}(T) = (1-\epsilon) G_A^{\mu\nu}(0) + \epsilon\, G_V^{\mu\nu}(0)$$

$$\epsilon = \frac{T^2}{6F_\pi^2}$$







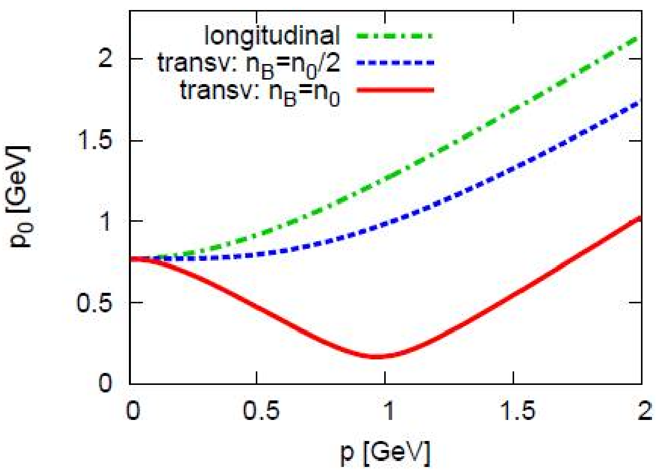
Chiral mixing in dense matter

$$\mathcal{L}_{\omega\rho a_1} = g_{\omega\rho a_1}\epsilon^{\mu\nu\lambda\sigma}\omega_\mu\left[\partial_\nu V_\lambda \cdot A_\sigma + \partial_\nu A_\lambda \cdot V_\sigma\right]$$

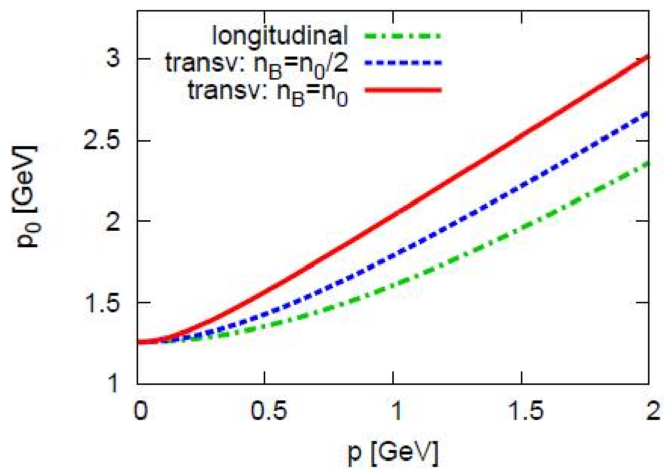
$$\langle\omega_0\rangle = g_{\omega NN}\cdot n_B/m_\omega^2 \qquad C = g_{\omega\rho a_1}\cdot g_{\omega NN}\cdot\frac{n_B}{m_\omega^2}$$

$$p_0^2 - |\vec{p}|^2 = \frac{1}{2}\left[m_\rho^2 + m_{a_1}^2 \pm \sqrt{(m_{a_1}^2 - m_\rho^2)^2 + 16C^2|\vec{p}|^2}\right]$$

ρ meson

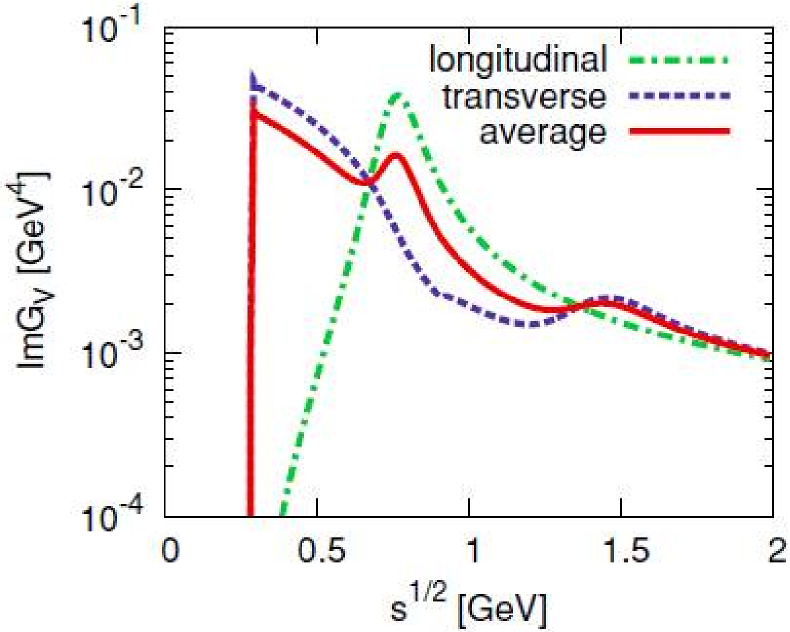


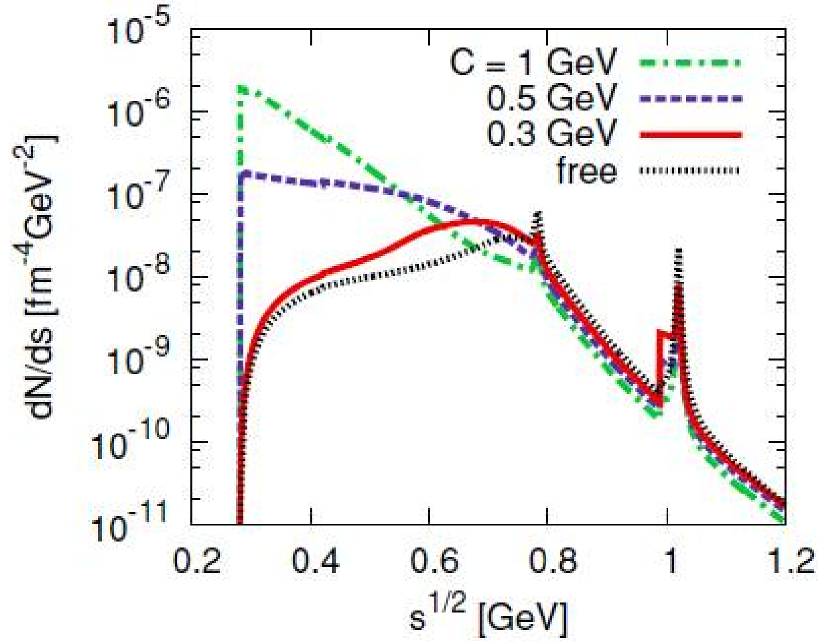
a_1 meson



Chiral mixing in dense matter

$$D_V^L = \frac{-1}{D_V}, \quad D_V^T = \frac{-D_A}{D_V D_A - 4C^2 \bar{p}^2}$$





Chiral mixing in a medium

- V-A mixing at finite T [Harada, CS and Weise ('08)]
  - Dey-Eletsky-Ioffe theorem: $\varepsilon = T^2/6F_{pi}^2 \rightarrow 1/2$ (?)
  - Higher T: $\pi\ \rho$ a1-int. reduced → V-A mixing gone
- V-A mixing at finite μ [Harada and CS ('09)]
  - $\omega\ \rho$ a1 at tree from WZW [Kaiser and Meissner ('90)]
  - V-A mixing → modified disp.relations → Not BW!
    - AdS/QCD [Domokos et al. ('07)] → strong C = 1 GeV at ρ_0
    - Chiral Walecka → weak C = 0.1 GeV at ρ_0
  - Role of high-lying states vs. large Nc

$$C_{\mathrm{hQCD}} \sim C_{\omega\rho a_1} + \sum_n C_{\omega^n \rho a_1}$$ → vector condensation at ρ_0?!

ω vs. ρ mesons

[CS et al. (2011-2013)]

HLS with nucleons

- ❑ Nucleon parity doublers [DeTar and Kunihiro ('89)]

$$m_{N_\pm} = \mp g_2 F_\pi + \sqrt{(g_1 F_\pi)^2 + m_0^2},$$

- ❑ ChPT with HLS → heavy baryon reduction

$$p^\mu = m_0 v^\mu + k^\mu \qquad \begin{pmatrix} B_+ \\ B_- \end{pmatrix} = \exp[i m_0 v \cdot x] \begin{pmatrix} N_+ \\ N_- \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_N = & \, i\bar{B} v^\mu D_\mu B - \Delta m_+ \bar{B}_+ B_+ - \Delta m_- \bar{B}_- B_- \\ & + g_V \bar{B} v^\mu \hat{\alpha}_{\parallel\mu} B + g_A \bar{B} \Big(2S^\mu \rho_3 \tanh\delta \\ & + v^\mu \rho_1 \frac{1}{\cosh\delta} \Big) \hat{\alpha}_{\perp\mu} B, \end{aligned}$$

$$\cosh\delta = \frac{m_{N_+} + m_{N_-}}{2m_0}$$

$$\Delta m_\pm = m_{N_\pm} - m_0$$

[CS et al. (2011-2013)]

ω vs. ρ mesons

- ❑ ρ as iso-triplet vs. ω as iso-singlet
- ❑ Consider HLS Lag. with $SU(2)_V \times U(1)_V$
  - Nucleon axial coupling $g_{AN_+N_+} = -g_{AN_-N_-} = g_A \tanh \delta$
  - Nucleon vector coupling

$$g_{\rho N_+N_+} = g_{\rho N_-N_-} = (g_{V\rho} - 1)g_\rho \quad g_{\omega N_+N_+} = g_{\omega N_-N_-} = (g_{V\omega} - 1)g_\omega$$

- ❑ 1-loop RGE → IR FP $\boxed{a = 1}$ & $\boxed{g_A = g_{V\rho} = 1}$ whereas $\boxed{g_{V\omega}}$ doesn't run.

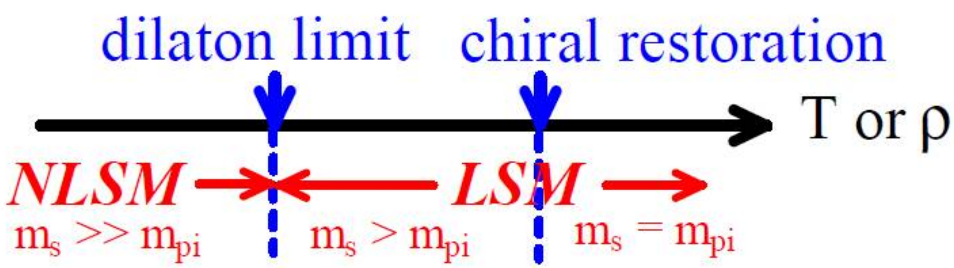
→ ρ decouples but ω doesn't.

[Beane and van Kolck (1994)]

Onset of a light scalar

1. Scale invariance in non-linear Lag.
2. From non-linear to linear basis
3. No singularity in the theory

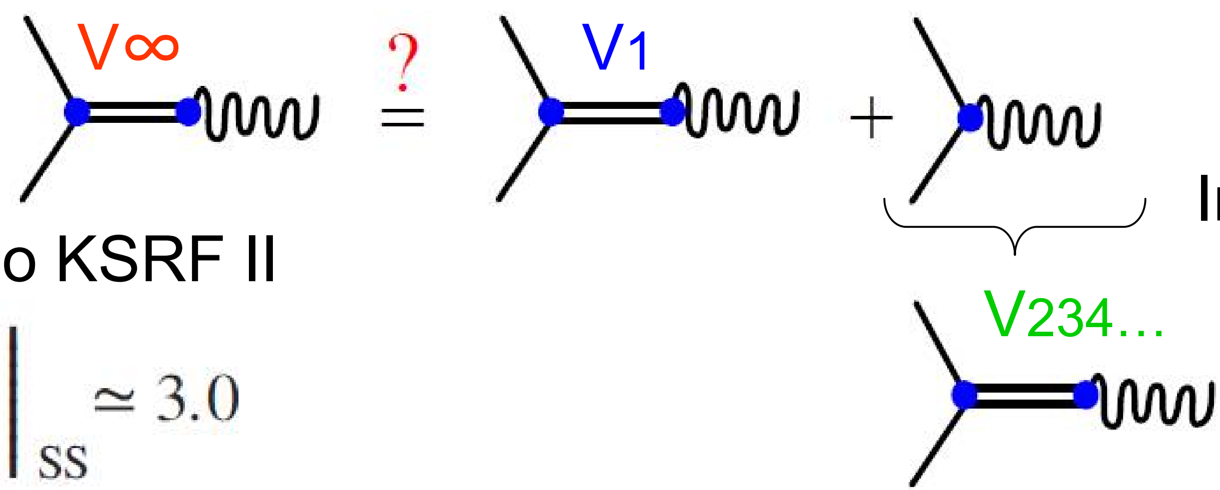
- Putting vector mesons [CS et al. (2011)]
  - Meson sector: $\boxed{a}$ unconstrained
  - Nucleon sector: $\boxed{g_A = g_{V\rho} = 1}$
  - $\boxed{g_{V\omega}}$ unconstrained
- Running/walking → medium dependence



Consequences?

- ❑ Neutron star EoS
  - ✓2 solar-mass, radius 10-12 km [Dong et al., ('13)]
  - ✓Tidal deformability, its density dep. [Ma et al., ('18)]
- ❑ Nucleon EM form factors
  - Decreasing ρ NN → VMD violated? Contact int.?
  - Holography → VMD [Sakai, Sugimoto; Hong et al.]

Large Nc



Nc = 3

In progress

Caution! No KSRF II

$$\frac{m_\rho^2}{g_{\rho\pi\pi}^2 F_\pi^2}\Bigg|_{SS} \simeq 3.0$$

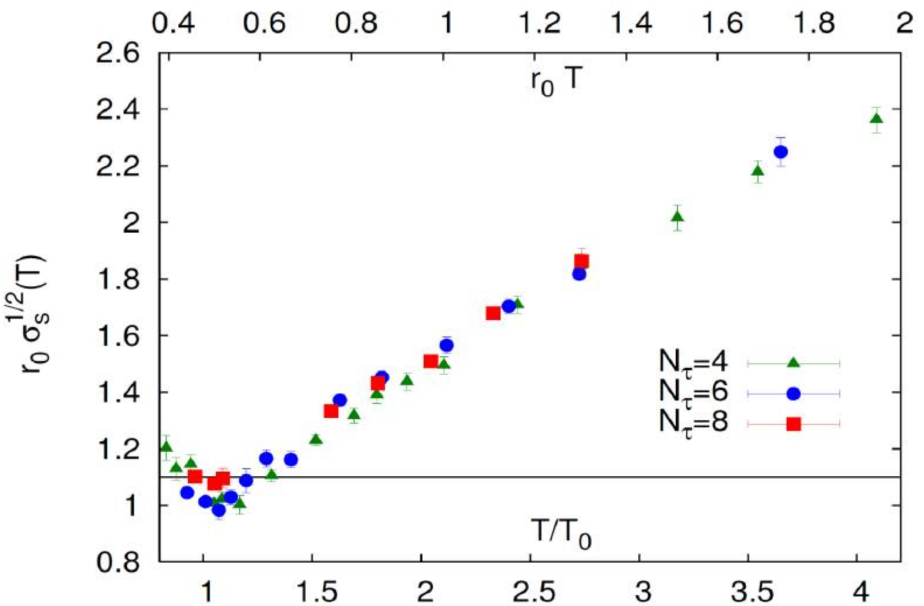
Summary

- Parity doubling of baryons
  - 2SM NS: chiral symmetric confined core
- Parity doubling of vector mesons
  - Chiral mixing: temp.-induced vs. density-induced
    (decrease) (increase)
- Role of the scalar, iso-vector/scalar mesons
  - Onset of the light scalar meson near CS rest.
  - ρ NN reduced, ω NN ≈ const.
  - Higher-lying states: explicit or integrated out

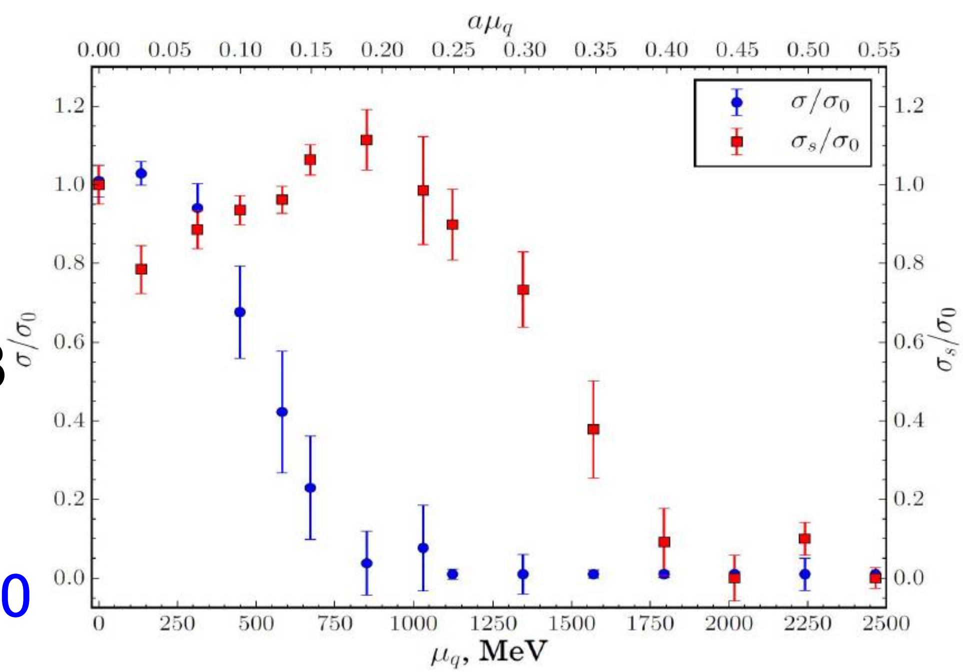
Backup

Fate of confinement: hot vs. dense

❑Non-pert. color-mag. sector → perturbative!



← SU(3)c, T >0, μ = 0;
Cheng et al., PRD 2008
[mpi = 220 MeV].



→ SU(2)c, T = 0, μ > 0;
Bornyakov et al., JHEP 2018
[mpi = 740 MeV].

✓m0(μ =0) vs. m0($\mu \neq$0)

✓Color-mag.monopoles at $\mu \neq 0$

Generalized GT relations

$$g_A \equiv \begin{pmatrix} g_{AN_+N_+} & g_{AN_+N_-} \\ g_{AN_+N_-} & g_{AN_-N_-} \end{pmatrix} = \begin{pmatrix} \tanh\,\delta & -\dfrac{1}{\cosh\,\delta} \\ -\dfrac{1}{\cosh\,\delta} & -\tanh\,\delta \end{pmatrix}$$

$$g_{\pi N_+N_+} = g_{AN_+N_+}\frac{m_+}{\sigma_0}, \quad g_{\pi N_-N_-} = g_{AN_-N_-}\frac{m_-}{\sigma_0},$$

$$g_{\pi N_+N_-} = g_{AN_+N_-}\frac{m_+ - m_-}{2\sigma_0}.$$

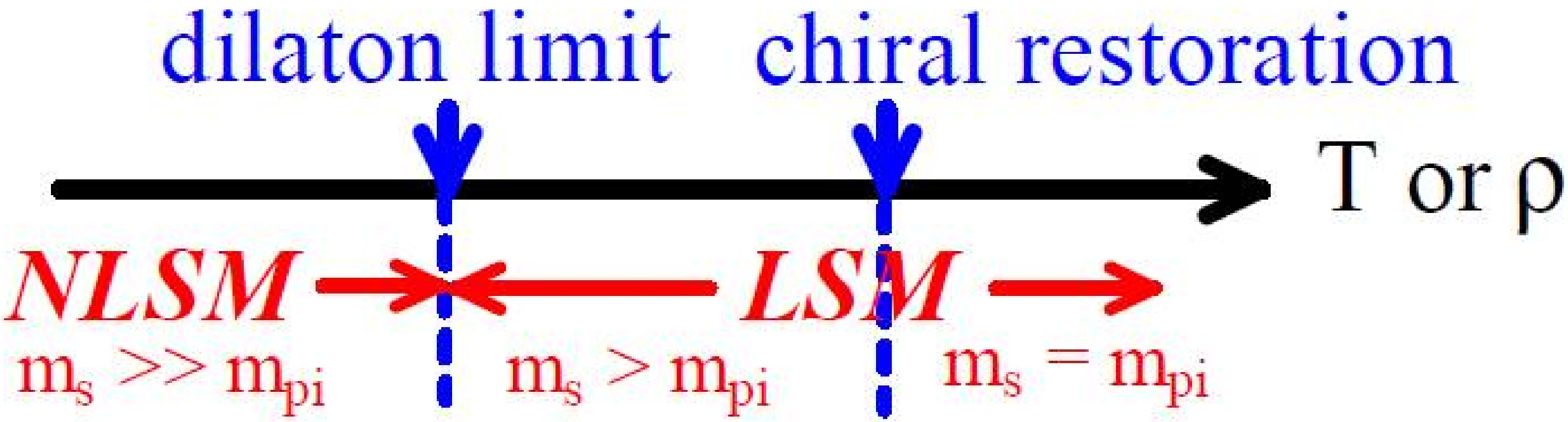
[Beane and van Kolck (1994)]

Dilaton limit

$$U = \xi^2 = e^{2i\pi/F_\pi}, \sqrt{\kappa} = F_\pi / F_{\chi s}$$

$$\Sigma = \sqrt{\kappa} U \chi = s + i\vec{\tau}\cdot\vec{\pi}$$

$$B = \frac{1}{2}[(\xi + \xi^\dagger) - \gamma_5(\xi - \xi^\dagger)]\psi$$



$$\kappa \to 1 \ \& \ g_A \to 1$$