Hirschegg 2019

From QCD matters to hadrons

Global conservation laws and mass distributions on the Cooper-Frye hypersurface

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In collaboration with

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

FIAS Frankfurt Institute for Advanced Studies

CRC-TR 211 Strong-interaction matter under extreme conditions

DFG

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

GSI

DAAD

Before talking about
the **Cooper-Frye particlization** itself,

Let me briefly talk about the **hybrid approach**,
since it is where the particlization comes in.

Motivation

- Our goal is to understand QCD matter
  1. how it looks like (e.g. equation of state) and
  2. how it behaves (e.g. transport coefficients)

- High-temperature QCD matter is created in heavy ion collisions (HIC)

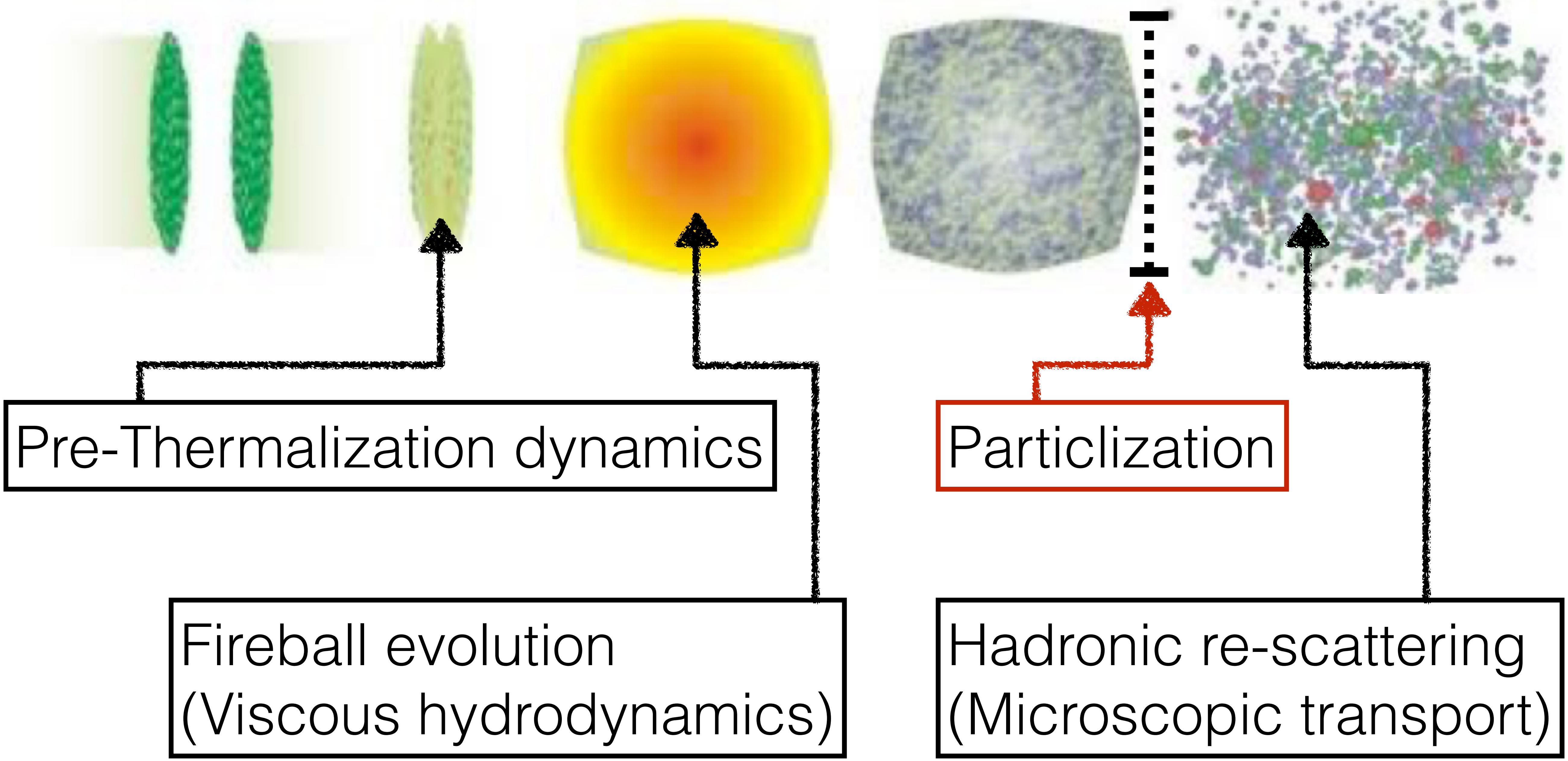
- Q: How to validate our understanding of QCD matter?
  A: We need to have **dynamical modelling**
  which is as close to real experiments as possible.

- Different aspects of QGP and hadronic matter influence each other. e.g.,
  ➤ non-zero ζ/s alters the estimate of η/s.
  ➤ hadronic re-scatterings change baryonic distribution.

S. Ryu *et al*. PRL (2015)

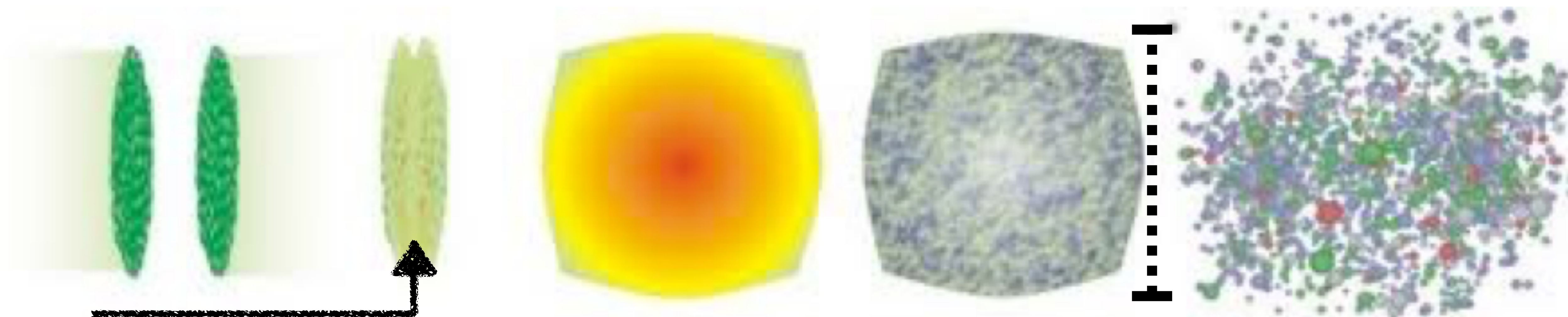
Model Overview

figure by S. Bass



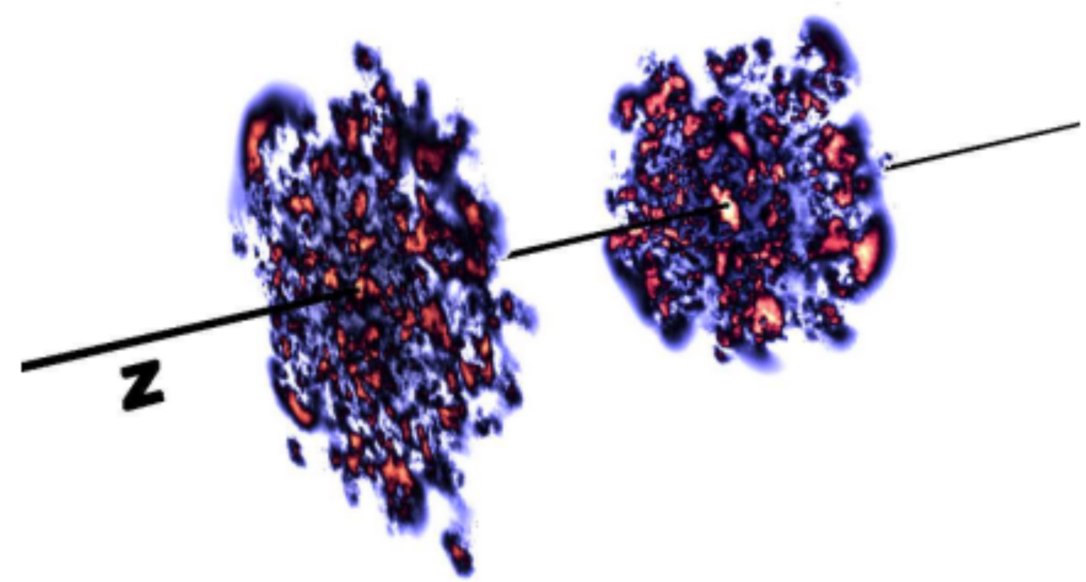
e.g. S. Ryu *et al*. PRC 2018

Model Overview



Pre-Thermalization dynamics

IP-Glasma is used
in this work.



B. Schenke, P. Tribedy and R. Venugopalan (2012)

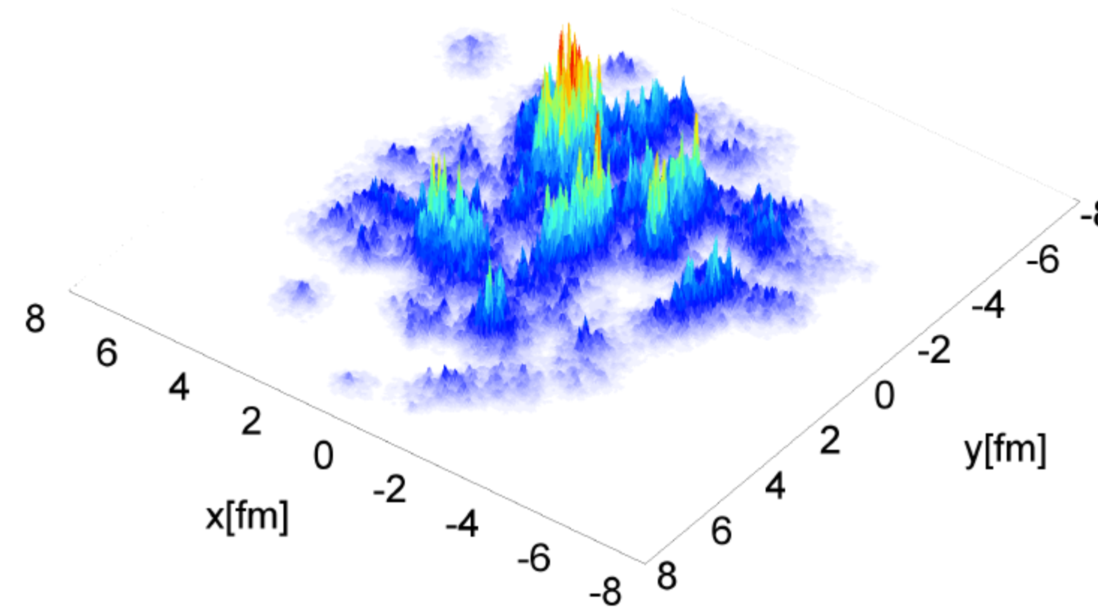
Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei

color charge distribution

$$\langle \rho^a(\mathbf{x}'_T)\,\rho^a(\mathbf{x}''_T)\rangle$$
$$= g^2\mu_A^2\delta^{ab}\delta^2(\mathbf{x}'_T-\mathbf{x}''_T)$$



▼ gluon field from each nucleus

$$A^i_{(1,2)}(\mathbf{x}_T)$$
$$= \frac{i}{g}U_{(1,2)}(\mathbf{x}_T)\,\partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$
$$U_{(1,2)}(\mathbf{x}_T) = \mathcal{P}\exp\left[-ig\int dx^\pm \frac{\rho_{(1,2)}(\mathbf{x}_T,x^\pm)}{\nabla_T^2-m^2}\right]$$

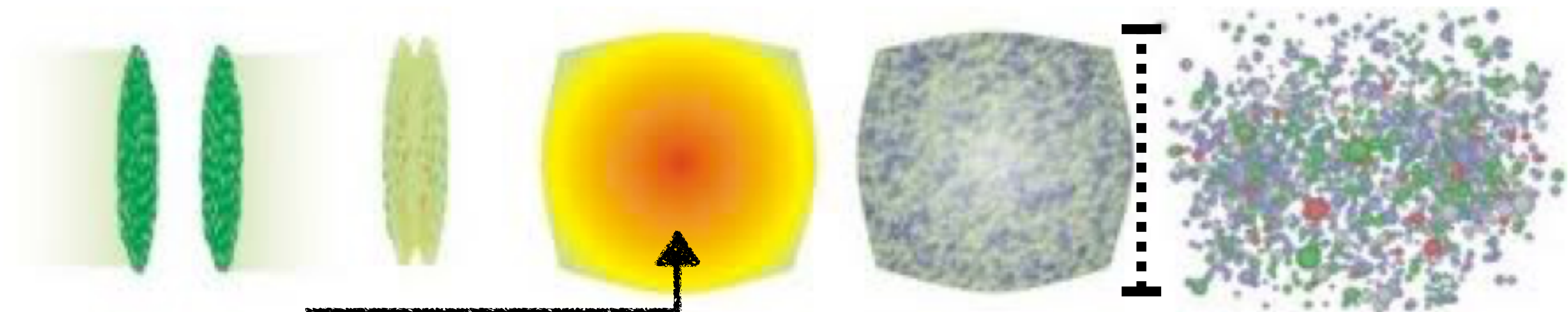
▼ initial gluon field after collision

$$A^i(\tau=+0) = A^i_{(1)} + A^i_{(2)}$$
$$A^\eta(\tau=+0) = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

▶ energy density profile at $\tau=\tau_0$

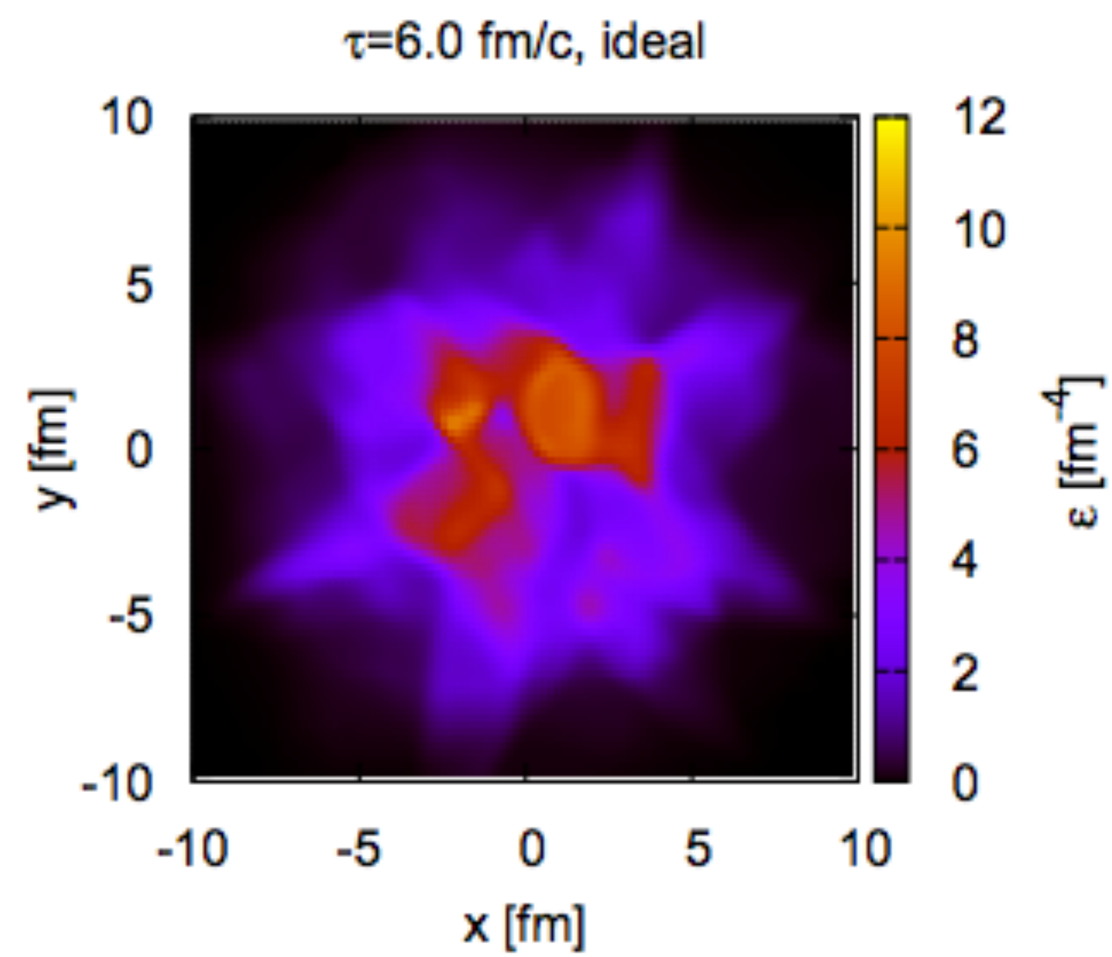
$$\partial_\mu F^{\mu\nu} - ig[A_\mu, F^{\mu\nu}] = 0$$
$$T^\mu{}_\nu(\tau=\tau_0)u^\nu = \epsilon\, u^\mu$$

Model Overview



2nd Viscous hydrodynamics

MUSIC is used
in this work.



B. Schenke, S. Jeon, and C. Gale (2010)

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

Energy/momentum conservation

$\partial_\mu T^{\mu\nu} = 0$ where $T^{\mu\nu} = \epsilon_0 u^\mu u^\nu - (P_0(\epsilon_0) + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$

Response to anisotropy/inhomogeneity

$$\begin{aligned}\dot{\pi}^{\langle\mu\nu\rangle} \quad &= \quad -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} \\ &\qquad -\tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)\end{aligned}$$

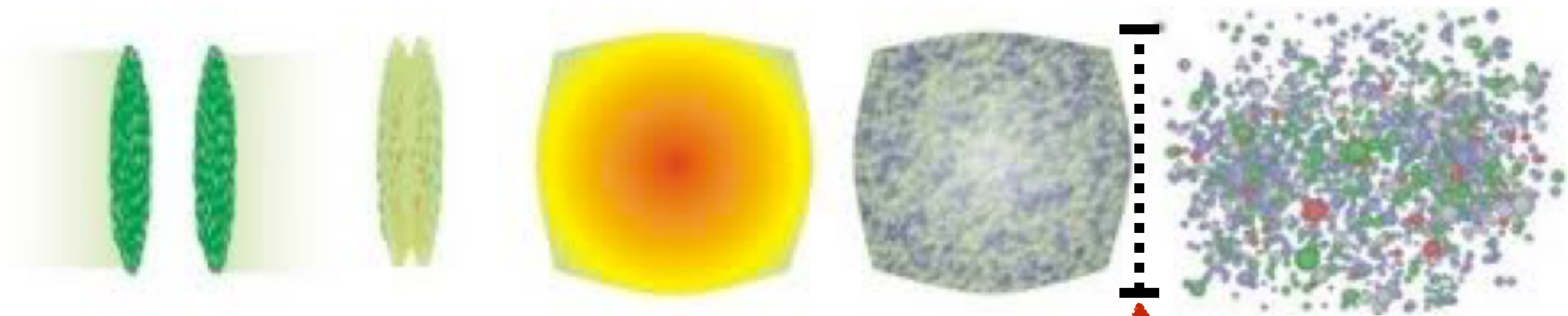
$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

G. Denicol, S. Jeon, and C. Gale (2014)

Equation of states : HRG + Lattice (s95p-v1)

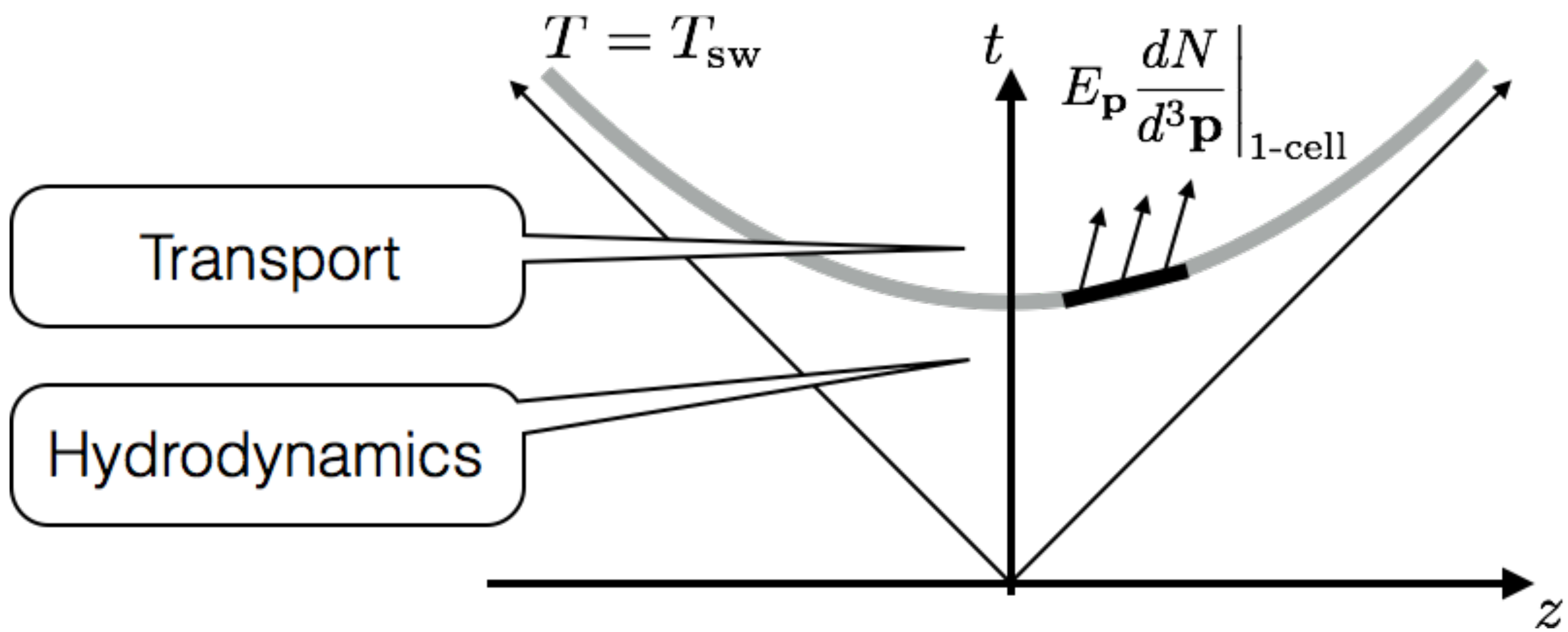
P. Huovinen, and P. Petreczky (2010)

Model Overview



Particlization

based on
Cooper-Frye
formalism

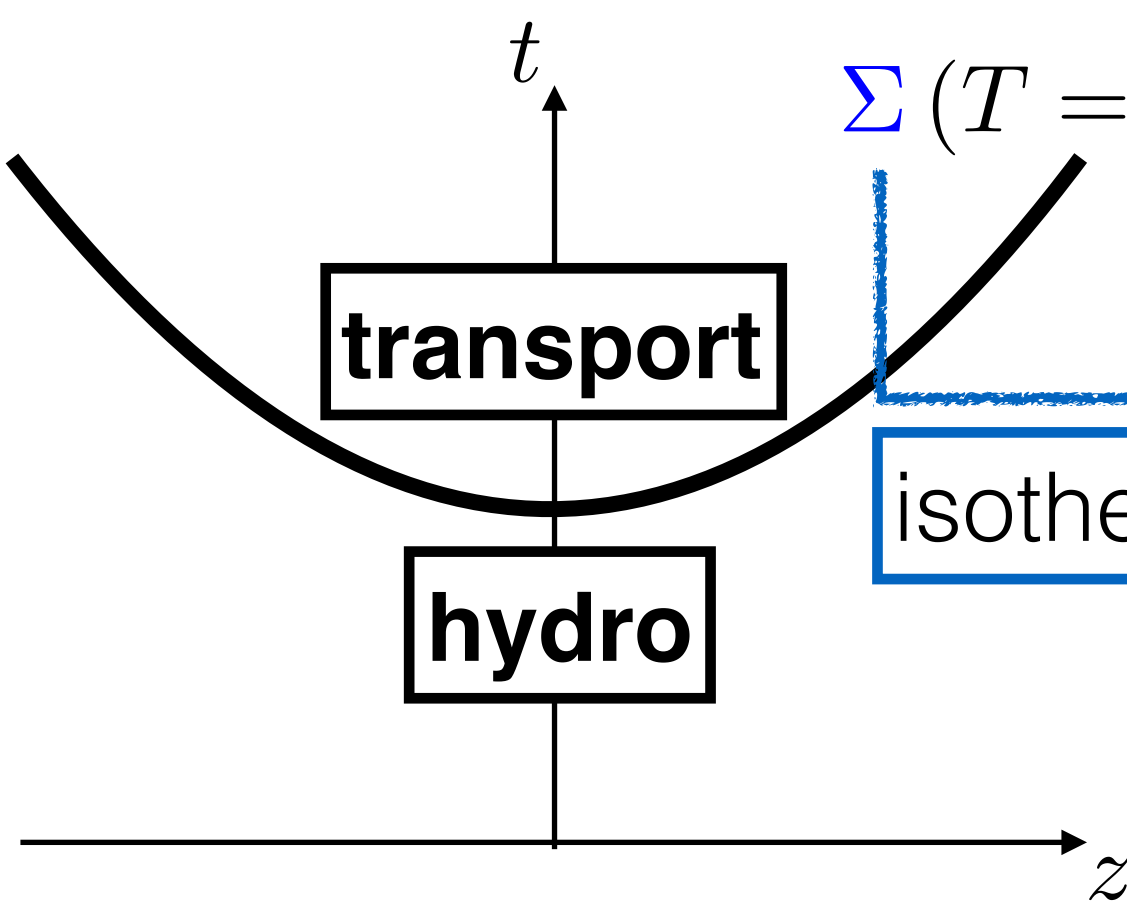


F. Cooper and G. Frye (1974)

Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula
(transform hydrodynamic information into particles)



$\Sigma \, (T = T_{\mathrm{sw}})$

$$\frac{dN}{d^3\mathbf{p}} = \int_{\Sigma} \frac{p^{\mu} d^3\Sigma_{\mu}}{E_{\mathbf{p}}} f(x, \mathbf{p})$$

isothermal hypersurface

hydrodynamic information
(temperature, flow velocity, …)

$$\left.\frac{dN}{d^3\mathbf{p}}\right|_{\text{1-cell}} = \left[f_0(x,\mathbf{p}) + \delta f_{\text{shear}}(x,\mathbf{p}) + \delta f_{\text{bulk}}(x,\mathbf{p})\right] \frac{p^{\mu}\Delta^3\Sigma_{\mu}}{E_{\mathbf{p}}}$$

Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

1. sample number of particles based on Poisson distribution

$$\bar{N}|_{\text{1-cell}} = \begin{cases} \left[n_0(x) + \delta n_{\text{bulk}}(x)\right] u^\mu \Delta\Sigma_\mu & \text{if} \quad u^\mu \Delta\Sigma_\mu \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$n_0(x) = d \int \frac{d^3\mathbf{k}}{(2\pi)^3} \, f_0(\mathbf{k})$$

$$\delta n_{\text{bulk}}(x) = d \int \frac{d^3\mathbf{k}}{(2\pi)^3} \, \delta f_{\text{bulk}}(\mathbf{k})$$

2. sample momentum of each particles from the Cooper-Frye formula.

Model : Cooper-Frye sampling

F. Cooper and G. Frye (1974)

sampling particles according to the Cooper-Frye formula

$$\left.\frac{dN}{d^3\mathbf{p}}\right|_{\text{1-cell}} = \left[f_0(x,\mathbf{p}) + \delta f_{\text{shear}}(x,\mathbf{p}) + \delta f_{\text{bulk}}(x,\mathbf{p})\right] \frac{p^\mu \Delta^3\Sigma_\mu}{E_\mathbf{p}}$$

$$f_0(x,\mathbf{p}) = \frac{1}{\exp\left[(p\cdot u)/T\right] \mp 1}$$

$$\delta f_{\text{shear}}(x,\mathbf{p}) = f_0(1 \pm f_0)\frac{p^\mu p^\nu \pi^{\mu\nu}}{2T^2(\epsilon_0 + P_0)}$$

P. Bozek (2010)

$$\delta f_{\text{bulk}}(x,\mathbf{p}) = -f_0(1 \pm f_0)\frac{C_{\text{bulk}}\Pi}{T}\left[c_s^2(p\cdot u) - \frac{(-p^\mu p^\nu \Delta_{\mu\nu})}{3(p\cdot u)}\right]$$

$$\frac{1}{C_{\text{bulk}}} = \frac{1}{3T}\sum_n m_n^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_\mathbf{k}} f_{n,0}(1 \pm f_{n,0})\left(c_s^2 E_\mathbf{k} - \frac{|\mathbf{k}|^2}{3E_\mathbf{k}}\right)$$

Model : Cooper-Frye sampling

global conservation laws

implemented by SPREW algorithm

Reject particle with probability

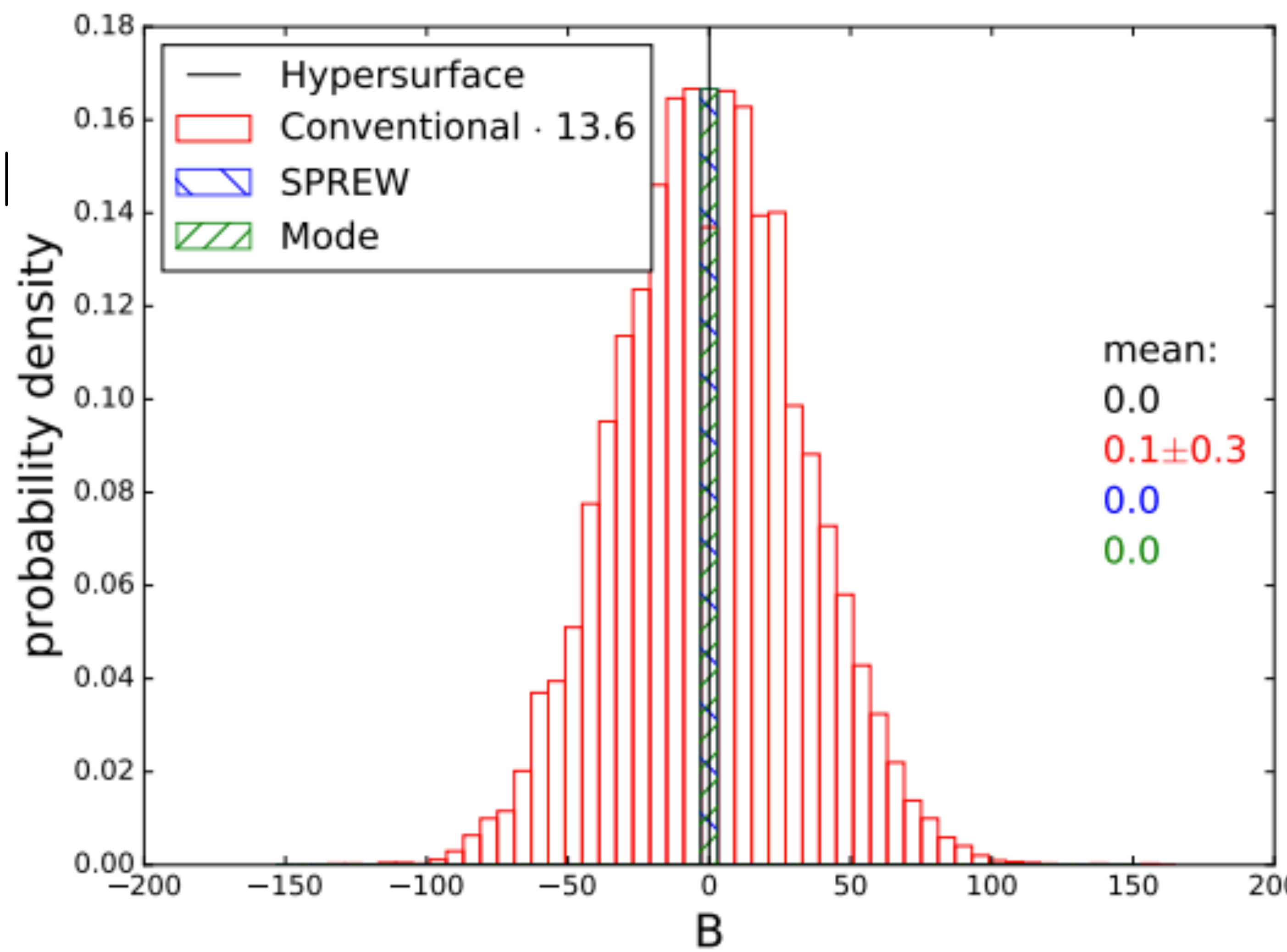
$$P_{\text{reject}} = 1 - e^{-|X_{\text{particles}} - X_{\text{surface}}|}$$

if sampled particle
deviates the charge X
from its average further.

$$X = B,\ S,\ Q$$

Rescale 3-momenta
to conserve total energy.

$$E_{\text{surface}} = \sum_i \sqrt{(1+a^2)|\mathbf{p}_i|^2 + m_i^2}$$



C. Schwarz *et al.* (2017)

Model : Cooper-Frye sampling

global conservation laws

implemented by SPREW algorithm

Reject particle with probability

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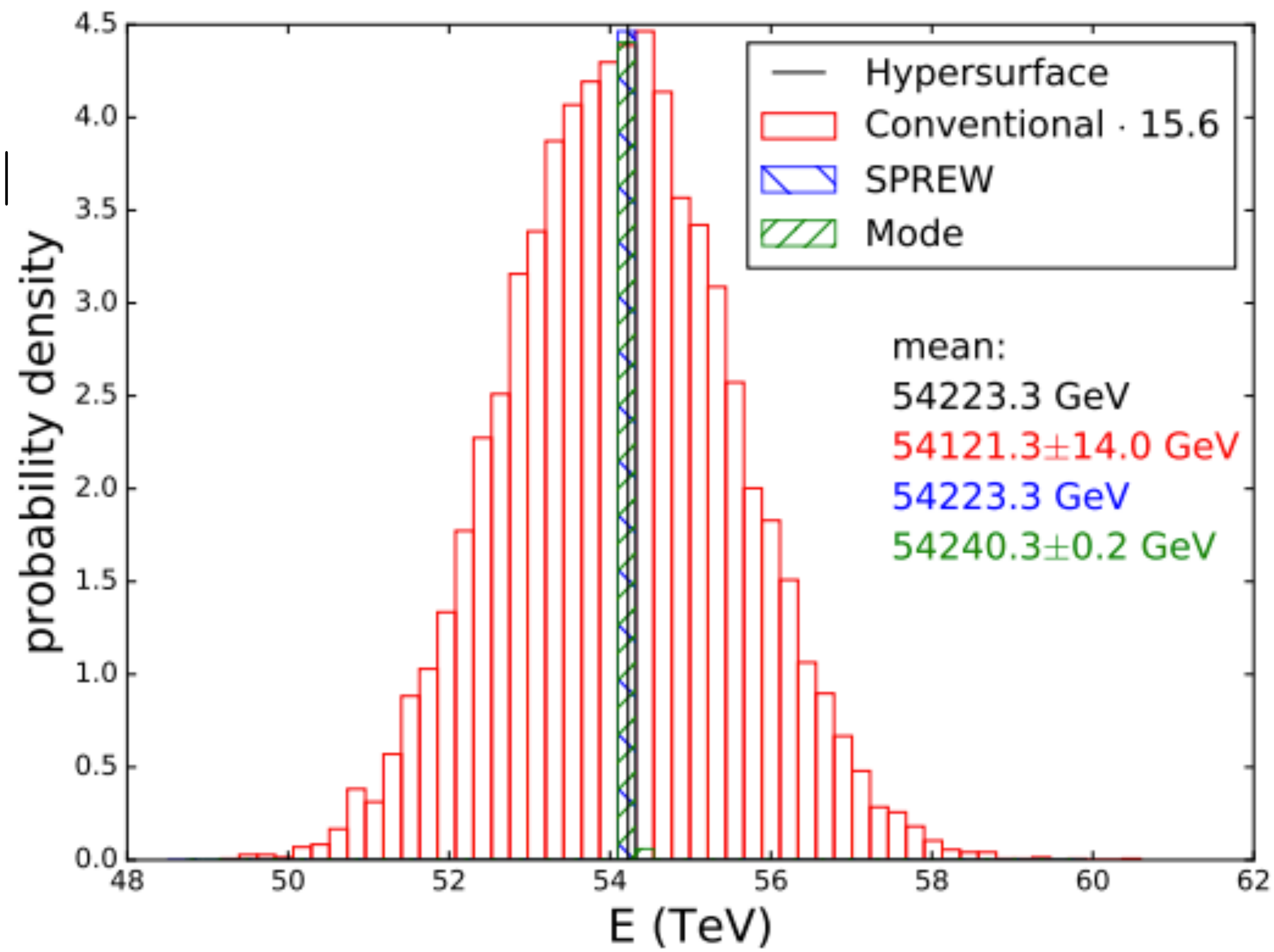
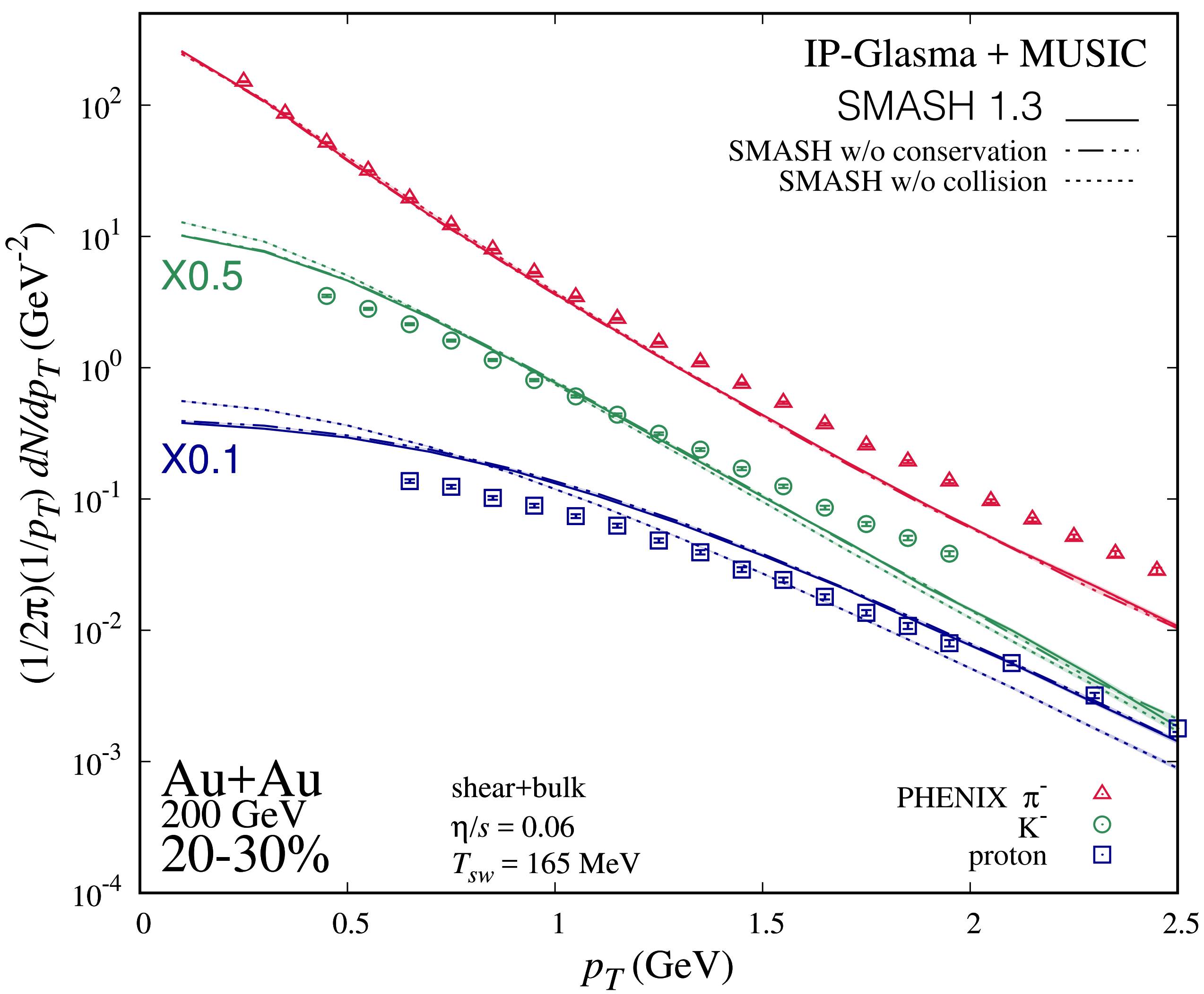


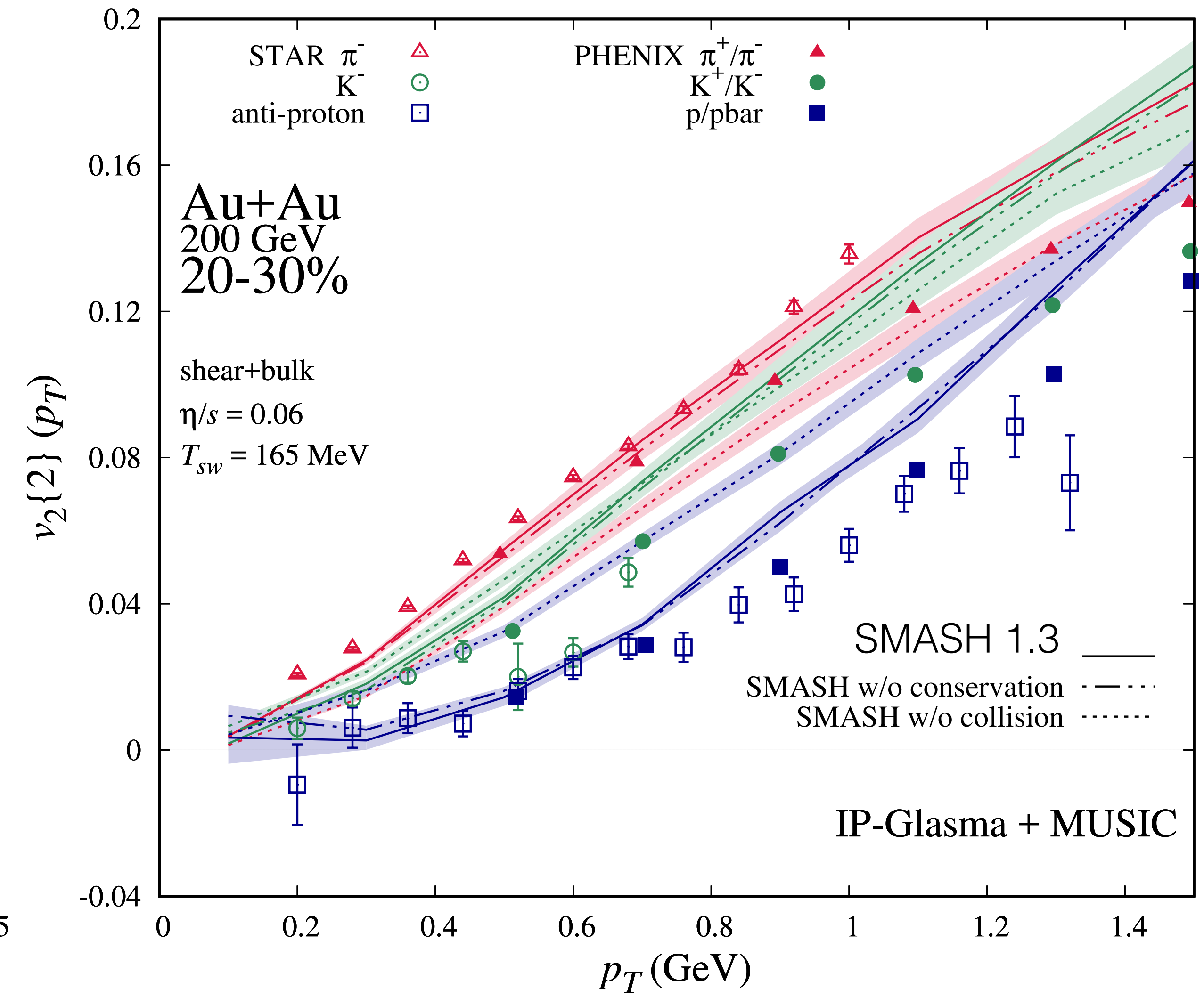
C. Schwarz *et al.* (2017)

Model : Cooper-Frye sampling

global conservation laws

Presented at QM2018





(as expected)

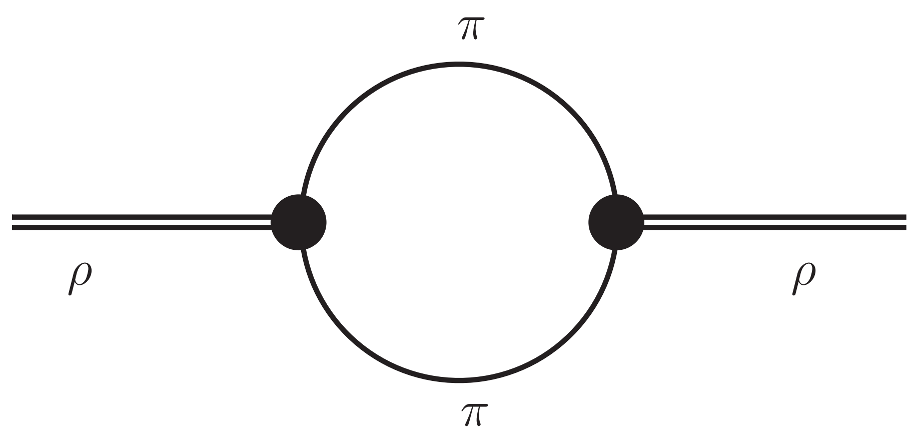
Single-particle distribution is not affected by the conservation.

Model : Cooper-Frye sampling

resonance mass sampling

Retarded propagator
for vector mesons

$$\rho(M) = -\frac{2M}{\pi}\,\mathrm{Im}\,D^R(M)$$

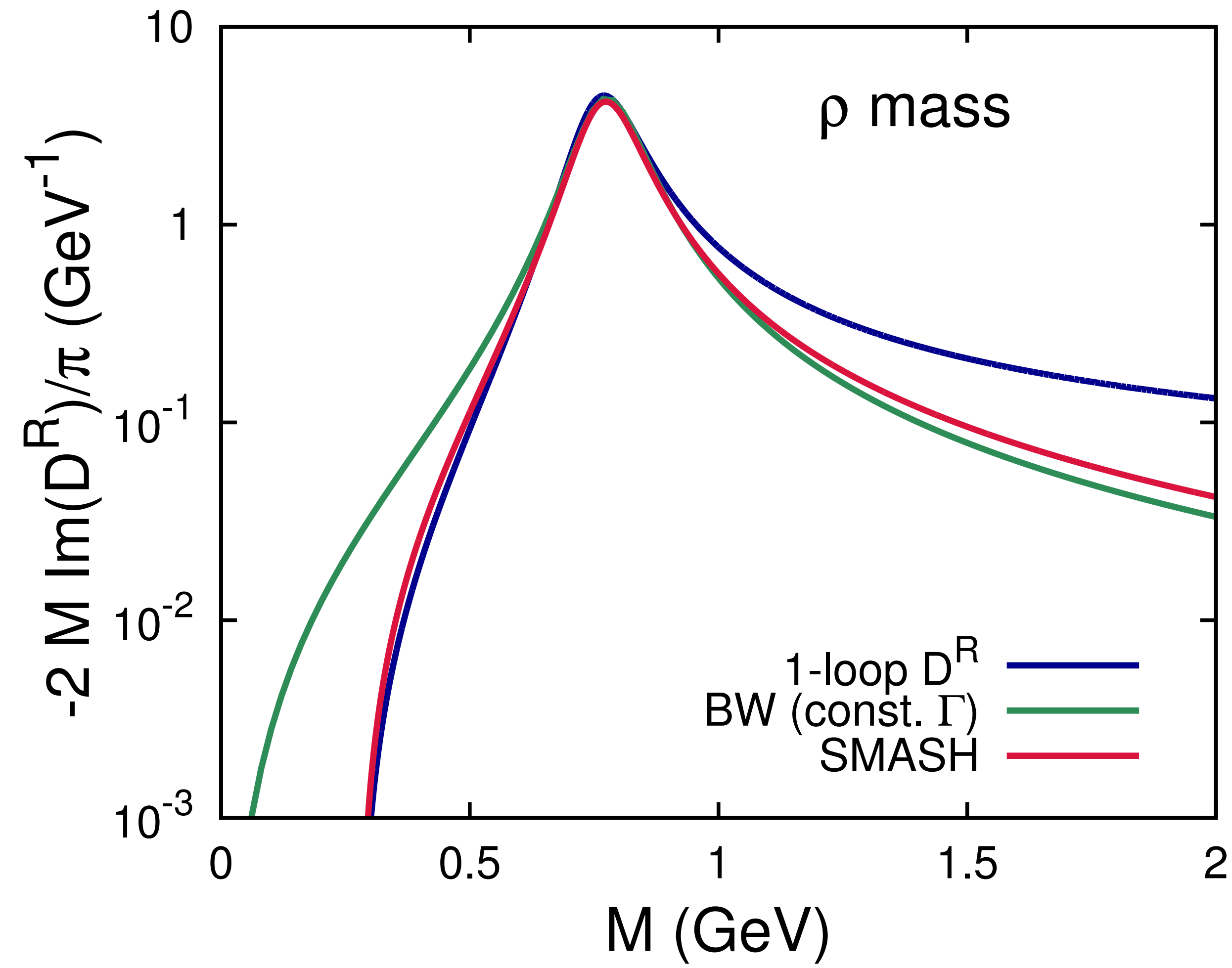


G. Vujanovic *et al*. (2014)

or

Breit-Wigner distribution with mass-dependent width

D. M. Manley *et al*. (1992)



Important for **dilepton production** in the rescattering phase

Model : Cooper-Frye sampling

resonance mass sampling

We cannot simply use number density at the pole mass

because $m_{\text{pole}} \neq \langle M \rangle_\rho = \int_{M_{\min}}^{M_{\max}} dM\, M\, \rho(M)$

We make the following replacement

$$n_0 + \delta n_{\text{bulk}} \qquad \text{at } m_{\text{pole}}$$
$$\downarrow$$
$$\langle n_0 + \delta n_{\text{bulk}} \rangle_\rho \quad = \int_{M_{\min}}^{M_{\max}} dM\, \left[n_0(M) + \delta n_{\text{bulk}}(M)\right] \rho(M)$$

to evaluate the average multiplicity.

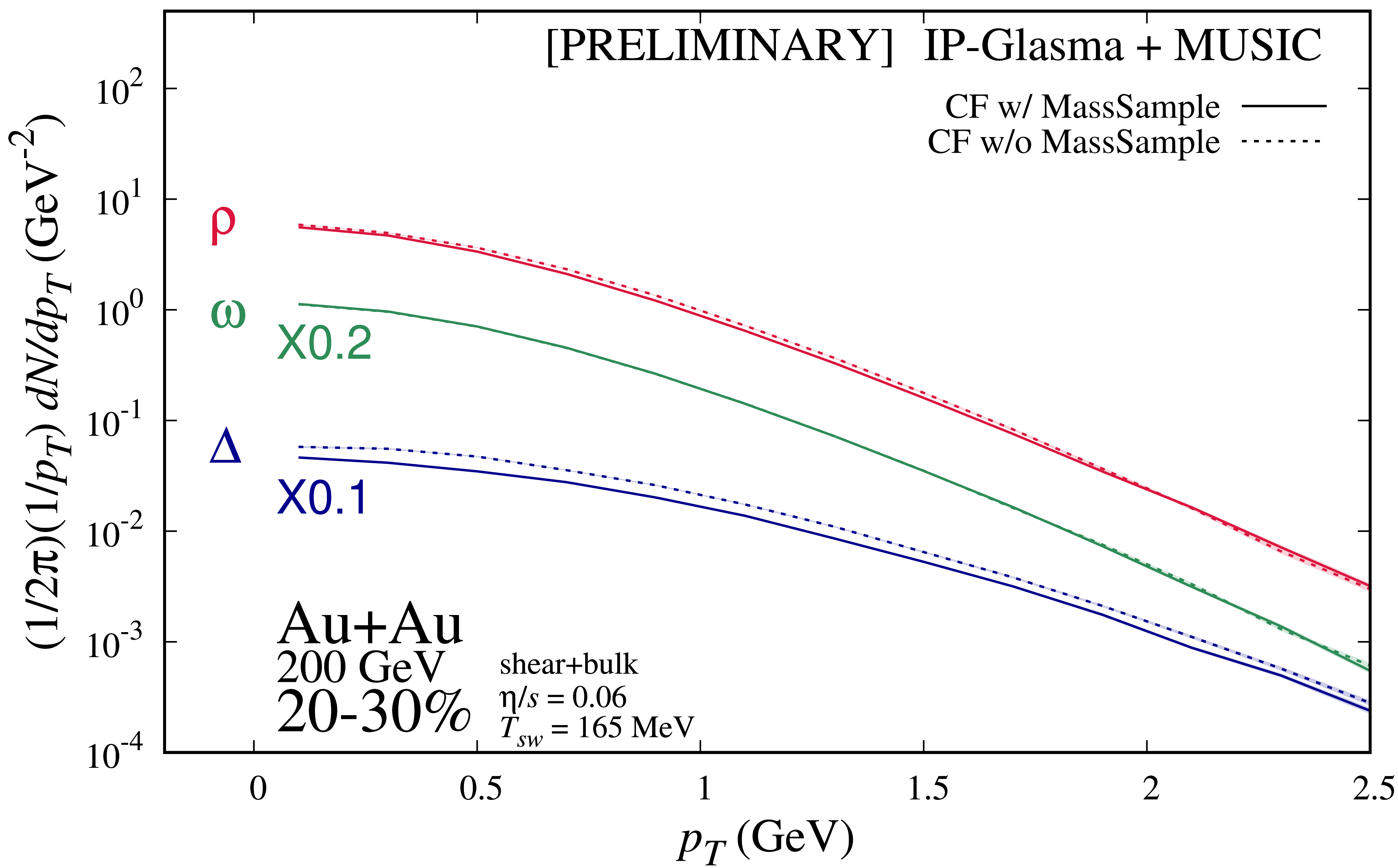
($M_{\min}$ from decay products and $M_{\max} = m_{\text{pole}} + 10\,\Gamma_{\text{pole}}$)

Then, we sample mass of each resonance based on

$$\mathcal{P}(M) \propto \left[n_0(M) + \delta n_{\text{bulk}}(M)\right] \rho(M)$$

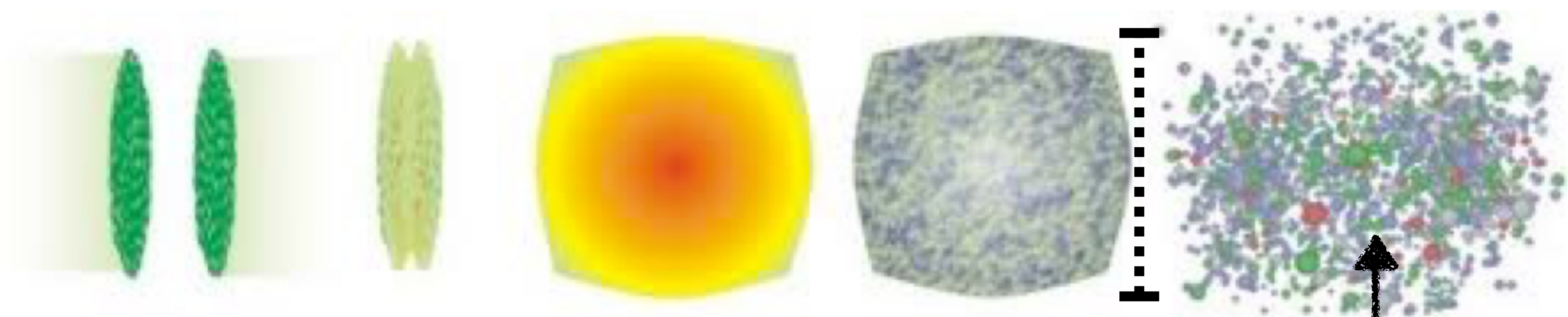
Model : Cooper-Frye sampling

resonance mass sampling



Resonance multiplicity can be altered by finite width.

Model Overview



Hadronic re-scattering

Simulating
Many
Accelerated
Strongly-interacting
Hadrons

Microscopic transport

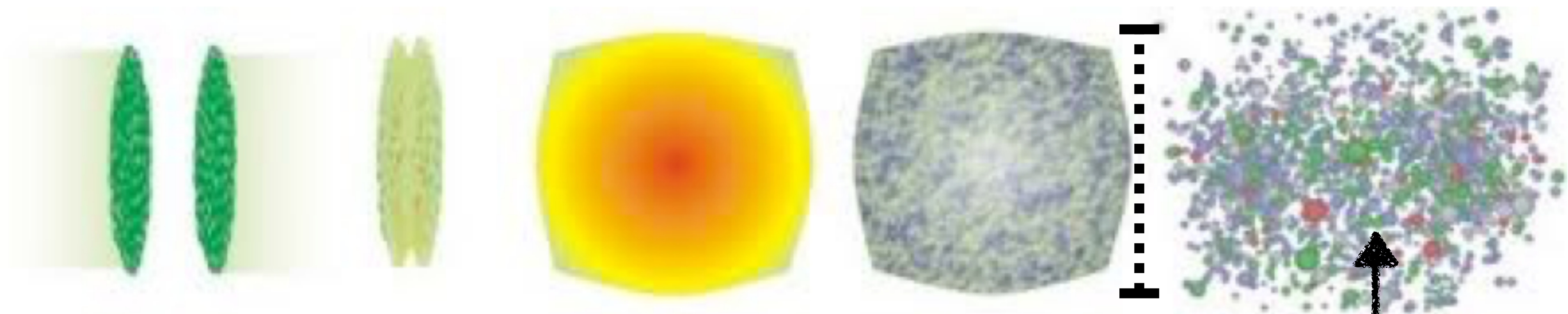
$$\frac{p^\mu}{m}\partial_\mu f + F^\mu \frac{\partial f}{\partial p^\mu} = \mathcal{C}[f]$$

with geometric collision criterion

J. Weil *et al.*, Phys. Rev. **C**94, 054905 (2016)

Code is available at `https://smash-transport.github.io`

Model : SMASH cascade



Hadronic re-scattering

Simulating
Many
Accelerated
Strongly-interacting
Hadrons

Hadronic and **EM** processes

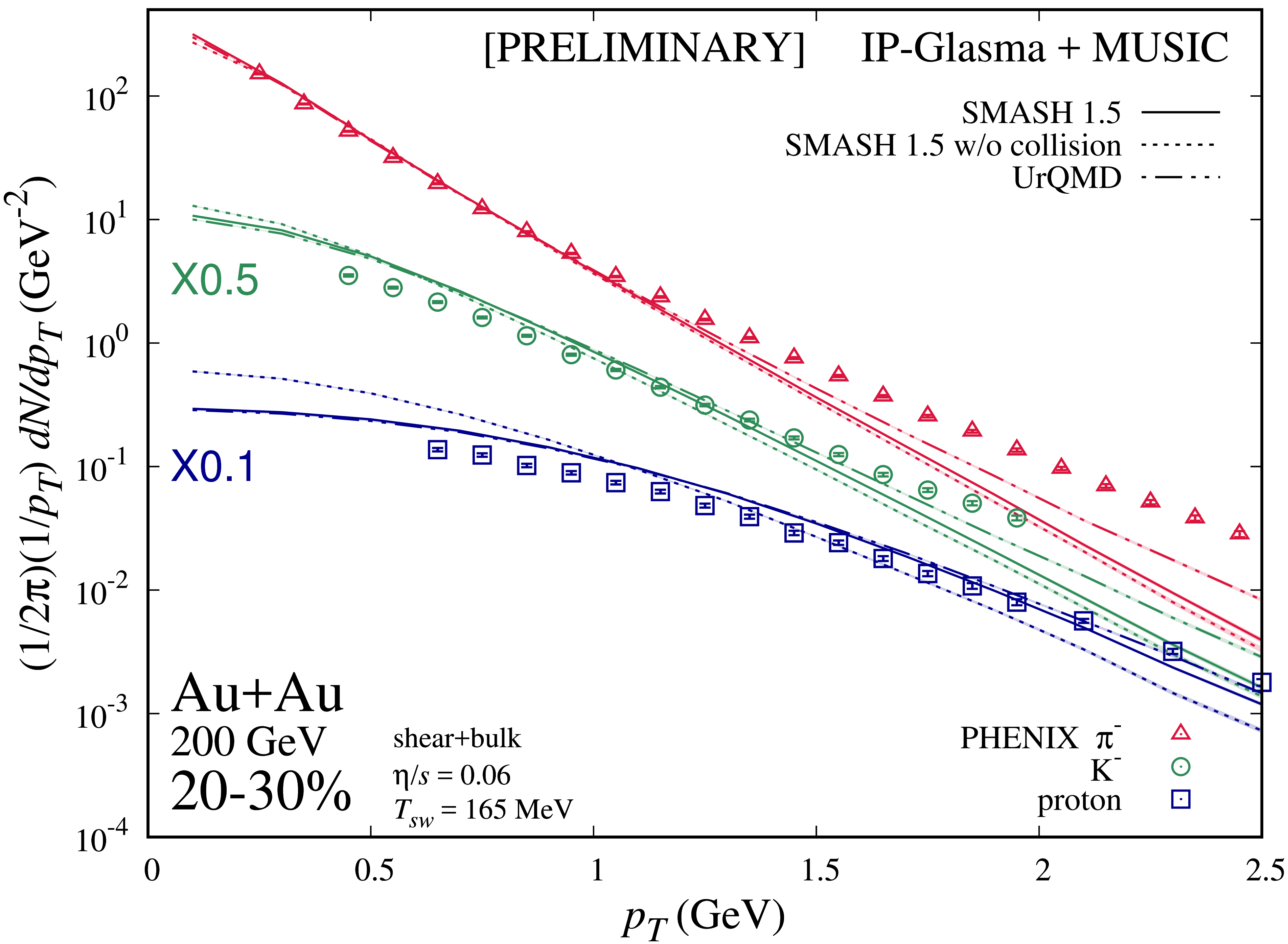
- resonance excitation and decays
- 2➔2 inelastic collisions (e.g. $NN \to NN^*$)
- dilepton and photon production (perturbative)
- string excitation processes
- baryon-antibaryon annihilations
- additive quark model

Talk by **Justin Mohs**

Results

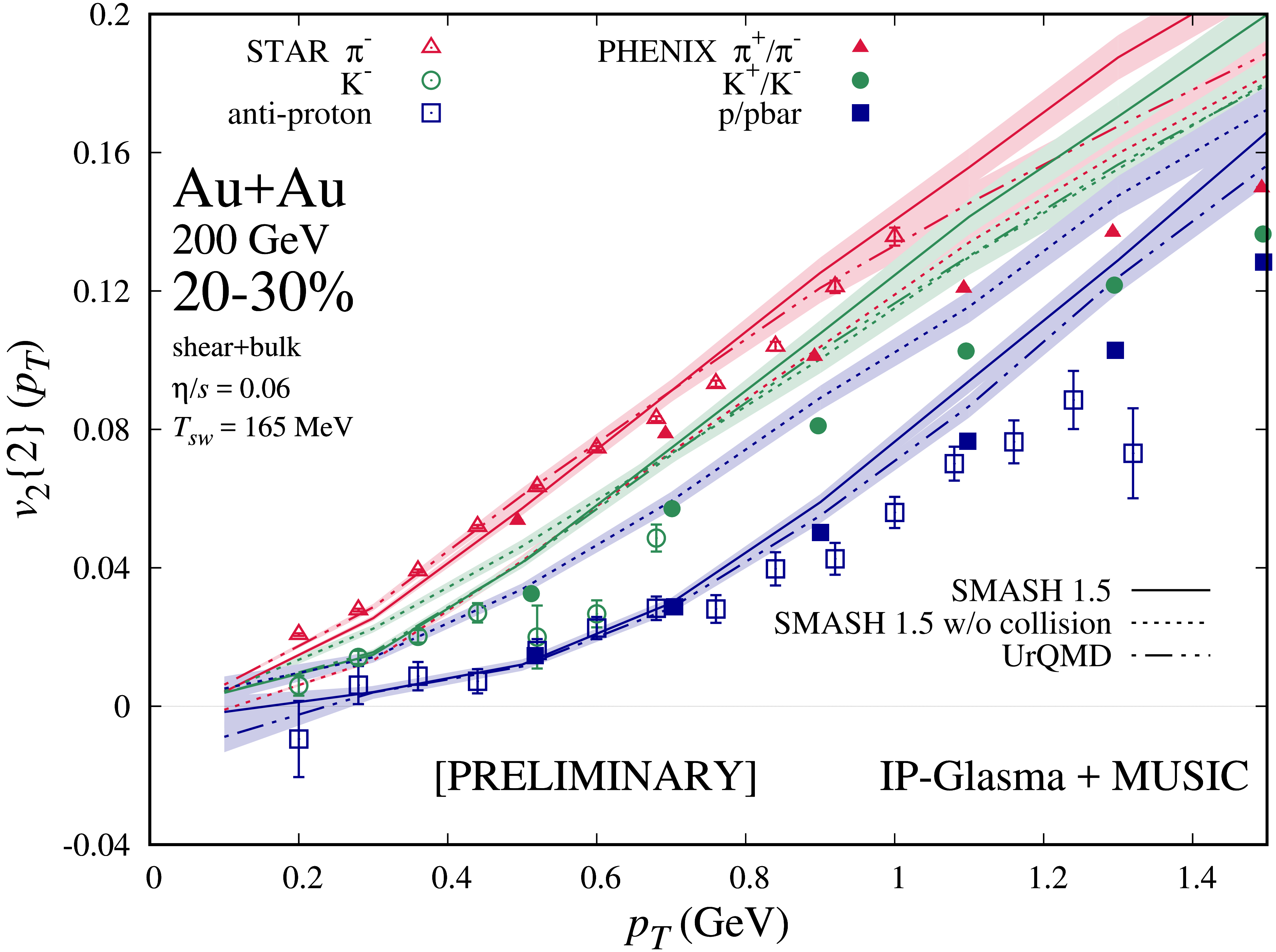
- Effects of hadronic re-scatterings
- Effects of resonance mass sampling

Identified hadron p_T-spectra



- Baryon multiplicity is decreased due to annihilations.
- Baryons are also accelerated by the lighter hadrons.

Elliptic flow (identified hadrons)

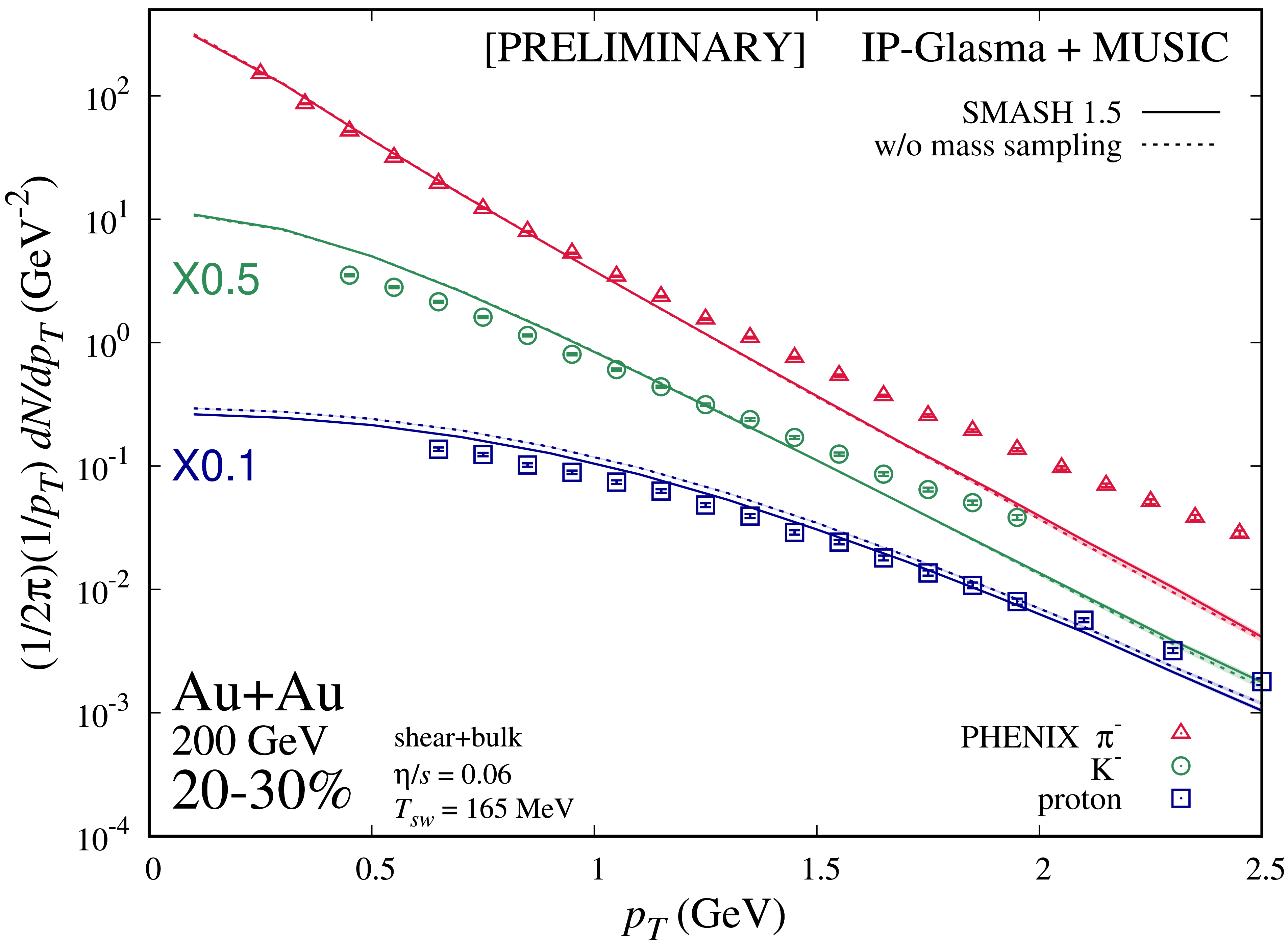


- Hadronic re-scatterings broaden mass ordering (consistent with the observation made in the p_T spectra.)

Results

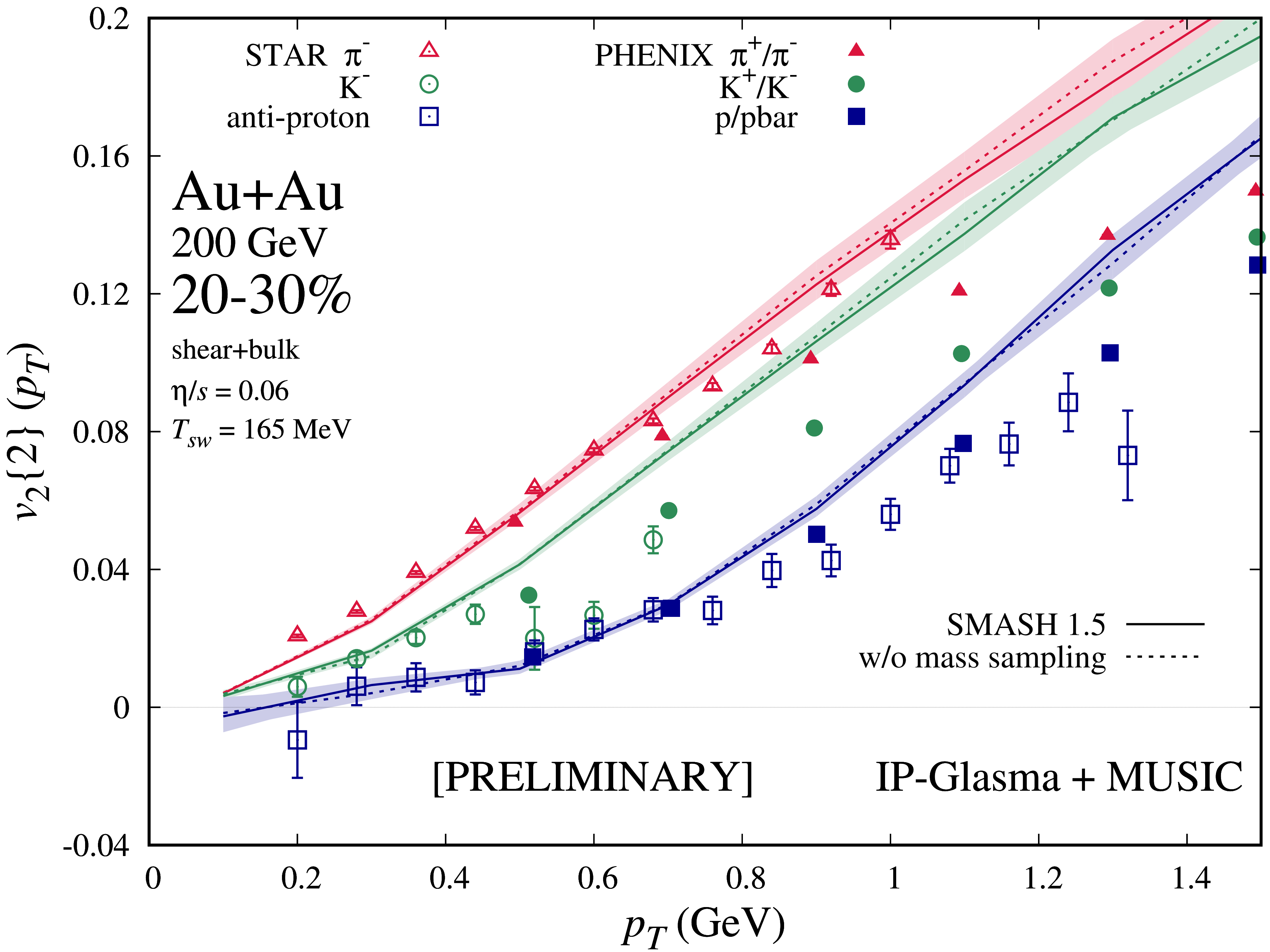
- Effects of hadronic re-scatterings
- Effects of resonance mass sampling

Identified hadron p_T-spectra



The proton yield is reduced by 10%
as consequence of resonance mass sampling.

Elliptic flow (identified hadrons)



The anisotropy is not affected by resonance mass sampling.

Conclusion & Outlook

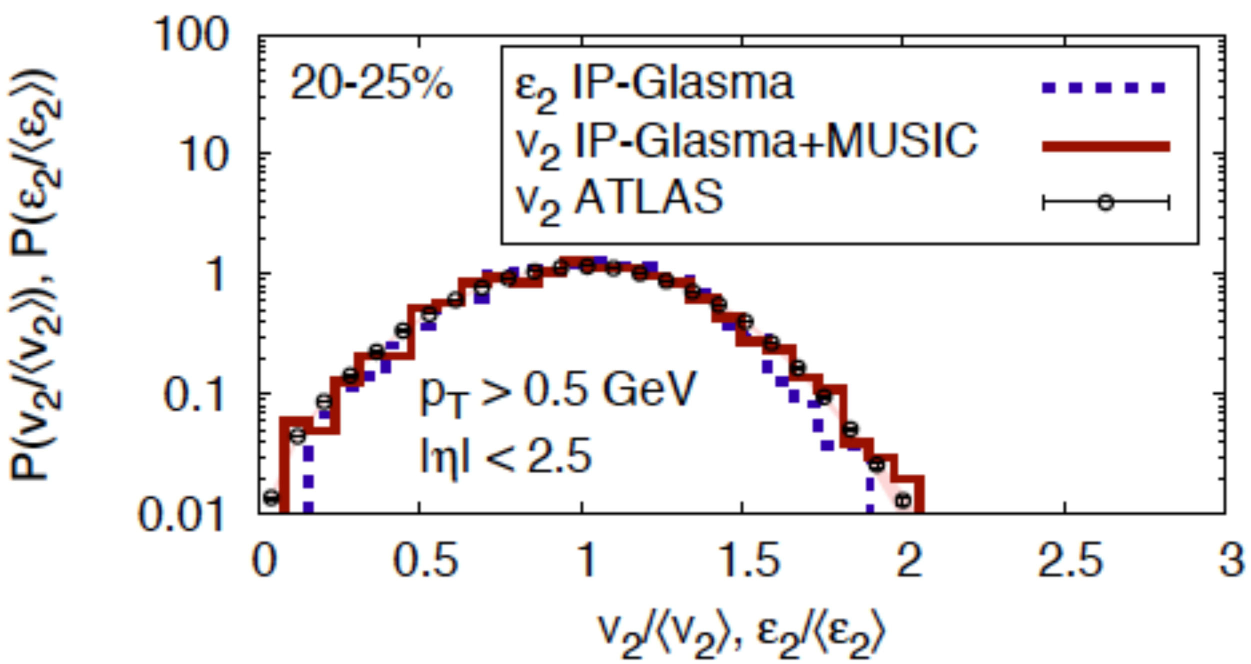
- Hybrid approach with SMASH afterburner provides reasonable description of bulk dynamics in HIC.
- Effects of hadronic re-scatterings are consistent with the previous studies.
- Global conservation laws at the particlization does not change the single-particle distribution.
  Multi-particle distributions or correlations might be affected.
- Resonance mass sampling at the particlization alters the proton multiplicity.
  Baryonic chemistry can be changed due to finite widths of resonances.

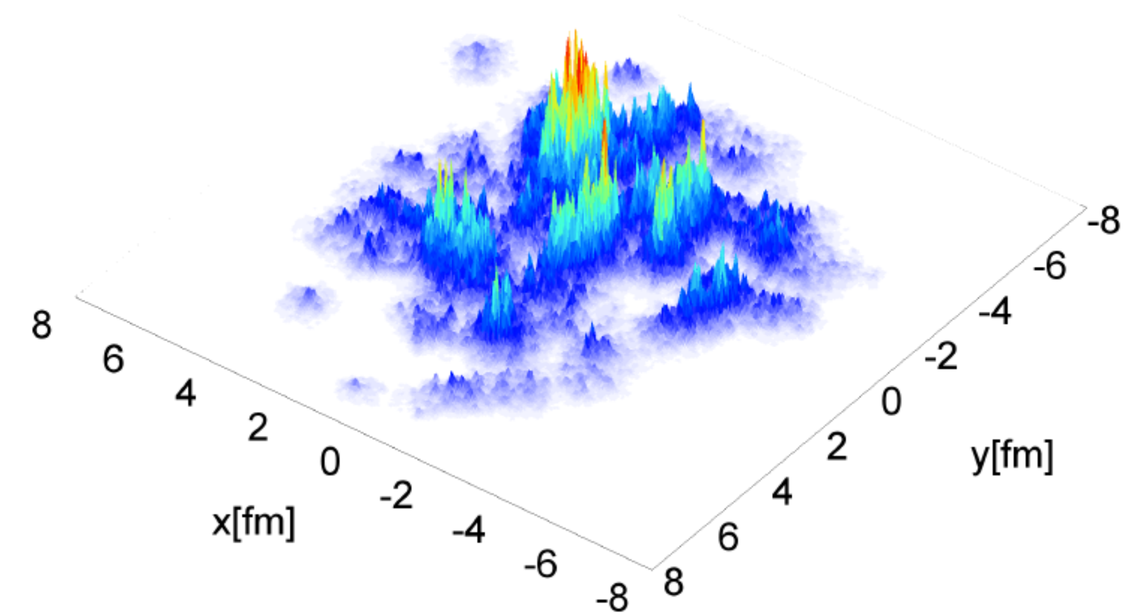
Backup Slides

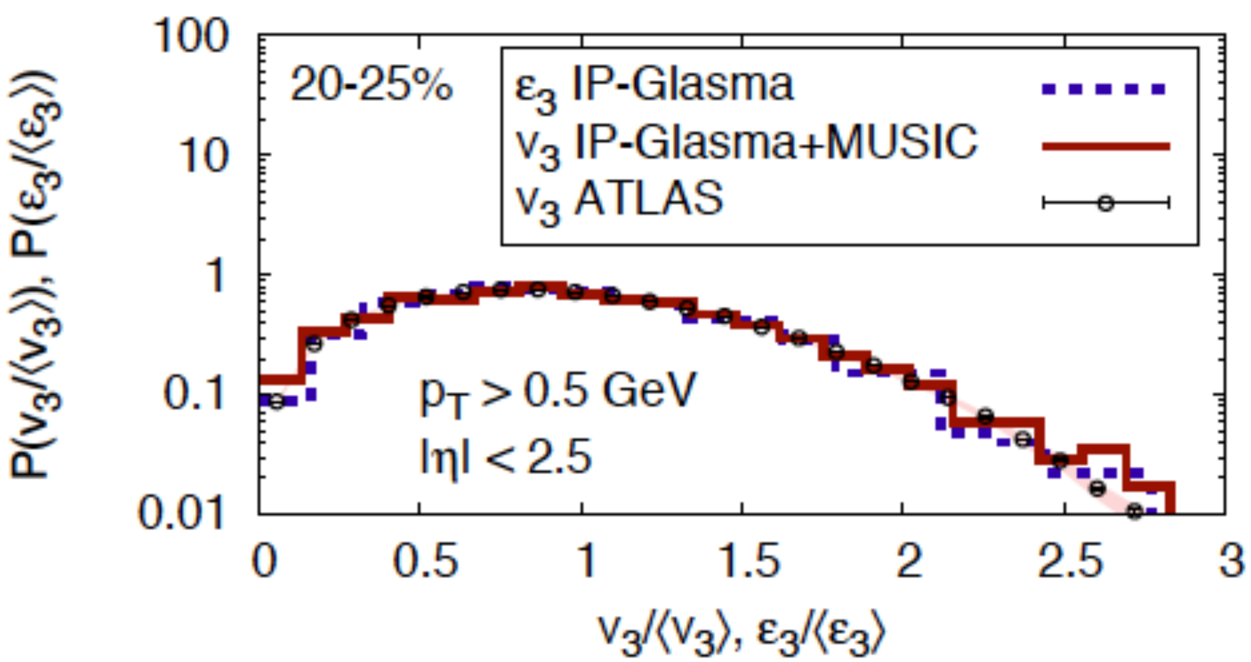
Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei







well describes v_n distribution

C. Gale, S. Jeon, B. Schenke,
P. Tribedy and R. Venugopalan (2012)

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

hydrodynamic equations of motion

Conservation equation $\partial_\mu T^{\mu\nu} = 0$

Decomposition $T^{\mu\nu} = \epsilon_0 u^\mu u^\nu - (P_0(\epsilon_0) + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu}$

EoS bulk shear

Local 3-metric $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Local 3-gradient $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

expansion rate

$$\theta = \nabla_\mu u^\mu$$

shear tensor

$$\sigma^{\mu\nu} = \frac{1}{2}\left[\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{3}{2}\Delta^{\mu\nu}\left(\nabla_\alpha u^\alpha\right)\right] \equiv \nabla^{\langle\mu}u^{\nu\rangle}$$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

shear

$$\frac{\eta}{s} = \mathrm{const}$$

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

shear viscosity relaxation equation

$$\dot{\pi}^{\langle\mu\nu\rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi}\Big(2\eta\,\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_7\pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha} - \tau_{\pi\pi}\pi_\alpha^{\langle\mu}\sigma^{\nu\rangle\alpha} + \lambda_{\pi\Pi}\Pi\,\sigma^{\mu\nu}\Big)$$

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

$$\frac{\eta}{\tau_\pi} = \frac{1}{5}\left(\epsilon_0 + P_0\right) \qquad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3} \qquad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5} \qquad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}$$

second-order transport coefficients

Model : MUSIC hydro

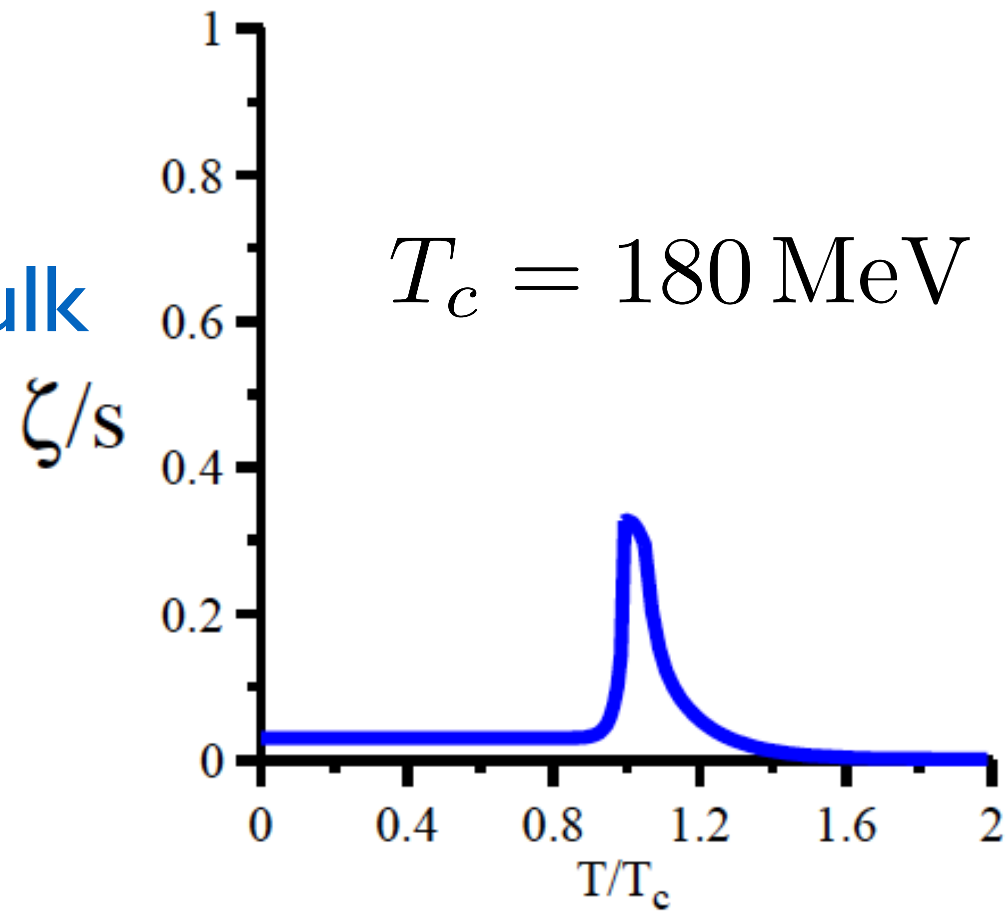
B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

bulk viscosity relaxation equation

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

bulk



F. Karsch, D. Kharzeev and K. Tuchin (2008)

J. Noronha-Hostler, J. Noronha and C. Greiner (2009)

Model : MUSIC hydro

B. Schenke, S. Jeon, and C. Gale (2010)

equation of motion for viscous corrections

bulk viscosity relaxation equation

$$\dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi}\left(-\zeta\,\theta - \delta_{\Pi\Pi}\Pi\,\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}\right)$$

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

$$\frac{\zeta}{\tau_\Pi} = 15\left(\frac{1}{3} - c_s^2\right)^2 (\epsilon_0 + P_0)$$

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3} \qquad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)$$

second-order
transport coefficients

Model : Equation of state

P. Huovinen, and P. Petreczky (2010)

Equation of state : hadron gas + lattice data

Only those included in UrQMD

Cross over phase transition around T = 180 MeV