Global conservation laws and mass distributions on the Cooper-Frye hypersurface

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In collaboration with
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From QCD matters to hadrons
Before talking about the **Cooper-Frye particlization** itself,

Let me briefly talk about the **hybrid approach**, since it is where the particlization comes in.
Motivation

• Our goal is to understand QCD matter
  1. how it looks like (e.g. equation of state) and
  2. how it behaves (e.g. transport coefficients)

• High-temperature QCD matter is created in heavy ion collisions (HIC)

• Q: How to validate our understanding of QCD matter?
  A: We need to have dynamical modelling
  which is as close to real experiments as possible.

• Different aspects of QGP and hadronic matter influence each other. e.g.,
  ➤ non-zero $\zeta/s$ alters the estimate of $\eta/s$.
  ➤ hadronic re-scatterings change baryonic distribution.

S. Ryu et al. PRL (2015)
Model Overview

Pre-Thermalization dynamics

Fireball evolution
(Viscous hydrodynamics)

Particlization

Hadronic re-scattering
(Microscopic transport)

e.g. S. Ryu et al. PRC 2018
Model Overview

Pre-Thermalization dynamics

**IP-Glasma** is used in this work.

B. Schenke, P. Tribedy and R. Venugopalan (2012)
Model : IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)
Classical YM dynamics with color sources in nuclei

color charge distribution
\[ \langle \rho^a(x'_T) \rho^a(x''_T) \rangle = g^2 \mu_A^2 \delta^{ab} \delta^2(x'_T - x''_T) \]

\[ \uparrow \text{gluon field from each nucleus} \]

\[ A_{(1,2)}^i(x_T) \]
\[ = \frac{i}{g} U_{(1,2)}(x_T) \partial_i U_{(1,2)}^\dagger(x_T) \]

\[ U_{(1,2)}(x_T) = \mathcal{P} \exp \left[ -ig \int dx^\pm \frac{\rho_{(1,2)}(x_T, x^\pm)}{\sqrt{\frac{2}{T} - m^2}} \right] \]

\[ \uparrow \text{initial gluon field after collision} \]

\[ A^i(\tau = +0) = A_{(1)}^i + A_{(2)}^i \]
\[ A^h(\tau = +0) = \frac{ig}{2} [A_{(1)}^i, A_{(2)}^i] \]

\[ \text{energy density profile at } \tau = \tau_0 \]

\[ \partial_\mu F^{\mu\nu} - ig [A_\mu, F^{\mu\nu}] = 0 \]
\[ T^\mu_\nu(\tau = \tau_0) u^\nu = \epsilon u^\mu \]
Model Overview

2nd Viscous hydrodynamics

**MUSIC** is used in this work.

Model : MUSIC hydro


Energy/momentum conservation

\[ \partial_\mu T^{\mu\nu} = 0 \quad \text{where} \quad T^{\mu\nu} = \epsilon_0 u^\mu u^\nu - (P_0(\epsilon_0) + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \]

Response to anisotropy/inhomogeneity

\[ \dot{\pi}^{\langle \mu\nu \rangle} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{1}{\tau_\pi} \left( 2\eta \sigma^{\mu\nu} - \delta_\pi\pi \pi^{\mu\nu} \theta + \varphi_7 \pi^{\langle \mu \pi \nu \rangle\alpha} + \varphi \pi^{\langle \mu \pi \nu \rangle\alpha} - \varphi \pi^{\langle \mu \pi \nu \rangle\alpha} + \lambda_\pi\Pi \pi^{\langle \mu \sigma \nu \rangle\alpha} \right) \]

\[ \dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi} \left( -\zeta \theta - \delta_\Pi\Pi \Pi \theta + \lambda_\Pi\pi \pi^{\mu\nu} \sigma_{\mu\nu} \right) \]

G. Denicol, S. Jeon, and C. Gale (2014)

Equation of states : HRG + Lattice (s95p-v1)

P. Huovinen, and P. Petreczky (2010)
Model Overview

Particlization

based on Cooper-Frye formalism

F. Cooper and G. Frye (1974)
Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)

Sampling particles according to the Cooper-Frye formula
(transform hydrodynamic information into particles)

\[
\frac{dN}{d^3 p} = \left. \frac{dN}{d^3 p} \right|_{1\text{-cell}} = \left[ f_0(x, p) + \delta f_{\text{shear}}(x, p) + \delta f_{\text{bulk}}(x, p) \right] \frac{p^\mu \Delta^3 \Sigma_\mu}{E_p}
\]

Isothermal hypersurface

Hydrodynamic information (temperature, flow velocity, \ldots)
Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)
sampling particles according to the Cooper-Frye formula

1. sample number of particles based on Poisson distribution

\[ \bar{N}_{1\text{-cell}} = \begin{cases} 
[n_0(x) + \delta n_{\text{bulk}}(x)] u^\mu \Delta \Sigma_\mu & \text{if } u^\mu \Delta \Sigma_\mu \geq 0 \\
0 & \text{otherwise} 
\end{cases} \]

\[ n_0(x) = d \int \frac{d^3k}{(2\pi)^3} f_0(k) \]

\[ \delta n_{\text{bulk}}(x) = d \int \frac{d^3k}{(2\pi)^3} \delta f_{\text{bulk}}(k) \]

2. sample momentum of each particles from the Cooper-Frye formula.
Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)

Sampling particles according to the Cooper-Frye formula

\[
\frac{dN}{d^3p} \bigg|_{\text{1-cell}} = [f_0(x, p) + \delta f_{\text{shear}}(x, p) + \delta f_{\text{bulk}}(x, p)] \frac{p^\mu \Delta^3 \Sigma_\mu}{E_p}
\]

\[
f_0(x, p) = \frac{1}{\exp[(p \cdot u)/T] \mp 1}
\]

\[
\delta f_{\text{shear}}(x, p) = f_0(1 \pm f_0) \frac{p^\mu p^\nu \pi^{\mu\nu}}{2T^2(\epsilon_0 + P_0)}
\]

\[
\delta f_{\text{bulk}}(x, p) = -f_0(1 \pm f_0) \frac{C_{\text{bulk}} \Pi}{T} \left[ c_s^2 (p \cdot u) - \frac{(-p^\mu p^\nu \Delta_{\mu\nu})}{3(p \cdot u)} \right]
\]

\[
\frac{1}{C_{\text{bulk}}} = \frac{1}{3T} \sum_n m_n^2 \int \frac{d^3k}{(2\pi)^3 E_k} f_{n,0}(1 \pm f_{n,0}) \left( c_s^2 E_k - \frac{|k|^2}{3E_k} \right)
\]

P. Bozek (2010)

Model: Cooper-Frye sampling

F. Cooper and G. Frye (1974)

Sampling particles according to the Cooper-Frye formula

\[
\frac{dN}{d^3p} \bigg|_{\text{1-cell}} = [f_0(x, p) + \delta f_{\text{shear}}(x, p) + \delta f_{\text{bulk}}(x, p)] \frac{p^\mu \Delta^3 \Sigma_\mu}{E_p}
\]

\[
f_0(x, p) = \frac{1}{\exp[(p \cdot u)/T] \mp 1}
\]

\[
\delta f_{\text{shear}}(x, p) = f_0(1 \pm f_0) \frac{p^\mu p^\nu \pi^{\mu\nu}}{2T^2(\epsilon_0 + P_0)}
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\delta f_{\text{bulk}}(x, p) = -f_0(1 \pm f_0) \frac{C_{\text{bulk}} \Pi}{T} \left[ c_s^2 (p \cdot u) - \frac{(-p^\mu p^\nu \Delta_{\mu\nu})}{3(p \cdot u)} \right]
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\[
\frac{1}{C_{\text{bulk}}} = \frac{1}{3T} \sum_n m_n^2 \int \frac{d^3k}{(2\pi)^3 E_k} f_{n,0}(1 \pm f_{n,0}) \left( c_s^2 E_k - \frac{|k|^2}{3E_k} \right)
\]

P. Bozek (2010)
Model: Cooper-Frye sampling

global conservation laws

implemented by SPREW algorithm

Reject particle with probability

$$P_{\text{reject}} = 1 - e^{-|X_{\text{particles}} - X_{\text{surface}}|}$$

if sampled particle deviates the charge $X$ from its average further.

$$X = B, S, Q$$

Rescale 3-momenta to conserve total energy.

$$E_{\text{surface}} = \sum_i \sqrt{(1 + a^2)|p_i|^2 + m_i^2}$$

C. Schwarz et al. (2017)
Model: Cooper-Frye sampling

global conservation laws

implemented by SPREW algorithm

Reject particle with probability

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\( X = B, S, Q \)

Rescale 3-momenta to conserve total energy.

\[ E_{\text{surface}} = \sum_i \sqrt{(1 + a^2)|p_i|^2 + m_i^2} \]

C. Schwarz et al. (2017)
Model : Cooper-Frye sampling

global conservation laws

(as expected)

Single-particle distribution is not affected by the conservation.

Presented at QM2018
Model: Cooper-Frye sampling

resonance mass sampling

Retarded propagator for vector mesons

\[ \rho(M) = -\frac{2M}{\pi} \text{Im} D^R(M) \]

G. Vujanovic et al. (2014)

or

Breit-Wigner distribution with mass-dependent width

D. M. Manley et al. (1992)

Important for **dilepton production** in the rescattering phase
Model: Cooper-Frye sampling

resonance mass sampling

We cannot simply use number density at the pole mass because

\[ m_{\text{pole}} \neq \langle M \rangle_\rho = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \, M \, \rho ( M ) \]

We make the following replacement

\[ n_0 + \delta n_{\text{bulk}} \quad \text{at} \quad m_{\text{pole}} \]

\[ \downarrow \]

\[ \langle n_0 + \delta n_{\text{bulk}} \rangle_\rho = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \left[ n_0 ( M ) + \delta n_{\text{bulk}} ( M ) \right] \rho ( M ) \]

to evaluate the average multiplicity.

\( ( M_{\text{min}} \text{ from decay products and } M_{\text{max}} = m_{\text{pole}} + 10 \Gamma_{\text{pole}} ) \)

Then, we sample mass of each resonance based on

\[ \mathcal{P} ( M ) \propto [ n_0 ( M ) + \delta n_{\text{bulk}} ( M ) ] \rho ( M ) \]
Model: Cooper-Frye sampling

Resonance mass sampling

\[
\frac{1}{2\pi}(1/p_T) dN/dp_T \, (GeV^{-2})
\]

\[\eta/s = 0.06 \quad T_{sw} = 165 \, MeV\]

\[\text{Au+Au} \quad 200 \, GeV \quad 20-30\% \quad \text{shear+bulk}\]

Resonance multiplicity can be altered by finite width.
Model Overview

Hadronic re-scattering

Simulating Many Accelerated Strongly-interacting Hadrons

Microscopic transport

\[ \frac{p^\mu}{m} \partial_\mu f + F^\mu \frac{\partial f}{\partial p^\mu} = C[f] \]

with geometric collision criterion


Code is available at https://smash-transport.github.io
Simulating Many Accelerated Strongly-interacting Hadrons

Hadronic re-scattering

**Hadronic** and **EM** processes
- resonance excitation and decays
- $2 \rightarrow 2$ inelastic collisions (e.g. $NN \rightarrow NN^*$)
- dilepton and photon production (perturbative)
- string excitation processes
- baryon-antibaryon annihilations
- additive quark model

Talk by **Justin Mohs**
Results

• Effects of hadronic re-scatterings
• Effects of resonance mass sampling
Identified hadron $p_T$-spectra

- Baryon multiplicity is decreased due to annihilations.
- Baryons are also accelerated by the lighter hadrons.
Elliptic flow (identified hadrons)

- Hadronic re-scatterings broaden mass ordering (consistent with the observation made in the $p_T$ spectra.)
Results

• Effects of hadronic re-scatterings
• Effects of resonance mass sampling
The proton yield is reduced by 10% as consequence of resonance mass sampling.
Elliptic flow (identified hadrons)

Au+Au 200 GeV
20-30%

The anisotropy is not affected by resonance mass sampling.
Conclusion & Outlook

- Hybrid approach with SMASH afterburner provides reasonable description of bulk dynamics in HIC.

- Effects of hadronic re-scatterings are consistent with the previous studies.

- Global conservation laws at the particlization does not change the single-particle distribution. Multi-particle distributions or correlations might be affected.

- Resonance mass sampling at the particlization alters the proton multiplicity. Baryonic chemistry can be changed due to finite widths of resonances.
Backup Slides
Model: IP-Glasma I.C.

B. Schenke, P. Tribedy and R. Venugopalan (2012)

Classical YM dynamics with color sources in nuclei

well describes $v_n$ distribution

Model: MUSIC hydro


Hydrodynamic equations of motion

Conservation equation \( \partial_{\mu} T^{\mu\nu} = 0 \)

Decomposition

\[
T^{\mu\nu} = \epsilon_{0} u^{\mu} u^{\nu} - (P_{0}(\epsilon_{0}) + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}
\]

\( \Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu} u^{\nu} \)

Local 3-gradient

\( \nabla^{\mu} = \Delta^{\mu\nu} \partial_{\nu} \)

\( \Pi \) is the bulk pressure, \( \epsilon_{0} \) is the energy density, \( P_{0}(\epsilon_{0}) \) is the EoS at constant energy density, \( \Pi \) is the bulk pressure, \( \pi^{\mu\nu} \) is the shear stress.
Equation of motion for viscous corrections

Shear viscosity relaxation equation

\[ \dot{\pi}^{\langle \mu \nu \rangle} = -\pi^{\mu \nu}_{\tau} + \frac{1}{\tau_{\pi}} \left( 2\eta \sigma^{\mu \nu} - \delta_{\pi \pi} \pi^{\mu \nu} \theta + \varphi \pi^{\langle \mu \pi \nu \rangle \alpha} + \varphi \pi^{\langle \mu \pi \nu \rangle \alpha} + \lambda \pi_{\Pi} \Pi_{\sigma^{\mu \nu}} \right) \]

Expansion rate

\[ \theta = \nabla_{\mu} u^{\mu} \]

Shear tensor

\[ \sigma^{\mu \nu} = \frac{1}{2} \left[ \nabla^{\mu} u^{\nu} + \nabla^{\nu} u^{\mu} - \frac{3}{2} \Delta^{\mu \nu} (\nabla_{\alpha} u^{\alpha}) \right] \equiv \nabla^{\langle \mu} u^{\nu \rangle} \]
Model: MUSIC hydro


Equation of motion for viscous corrections

Shear viscosity relaxation equation

\[
\dot{\pi}^{\langle \mu \nu \rangle} = -\frac{\pi^{\mu \nu}}{\tau_\pi} + \frac{1}{\tau_\pi} \left( 2\eta \sigma^{\mu \nu} - \delta_{\pi \pi} \pi^{\mu \nu} \theta + \varphi_7 \pi^{\langle \mu \pi \rangle \alpha} \pi^{\langle \nu \pi \rangle \alpha} - \tau_{\pi \pi} \pi^{\langle \mu \sigma \nu \rangle \alpha} + \lambda_{\pi \Pi \Pi} \sigma^{\mu \nu} \right)
\]

\[\frac{\eta}{s} = \text{const}\]
Model : MUSIC hydro

equation of motion for viscous corrections

\[ \dot{\pi}^{\langle \mu \nu \rangle} = -\frac{\pi^{\mu \nu}}{\tau_\pi} + \frac{1}{\tau_\pi} \left( 2\eta \sigma^{\mu \nu} - \delta_{\pi \pi} \pi^{\mu \nu} \theta + \varphi_7 \pi^{\langle \mu \pi \nu \rangle \alpha} \right. \\
\left. - \tau_{\pi \pi} \pi^{\langle \mu \sigma \nu \rangle \alpha} + \lambda_{\pi \Pi} \Pi \sigma^{\mu \nu} \right) \]

14-moment approximation in the small mass limit

G. Denicol, S. Jeon, and C. Gale (2014)

\[ \frac{\eta}{\tau_\pi} = \frac{1}{5} (\epsilon_0 + P_0) \quad \frac{\delta_{\pi \pi}}{\tau_\pi} = \frac{4}{3} \quad \frac{\lambda_{\pi \Pi}}{\tau_\pi} = \frac{6}{5} \quad \frac{\tau_{\pi \pi}}{\tau_\pi} = \frac{10}{7} \]

second-order transport coefficients
Model : MUSIC hydro


equation of motion for viscous corrections

bulk viscosity relaxation equation

\[ \dot{\Pi} = -\frac{\Pi}{\tau_\Pi} + \frac{1}{\tau_\Pi} \left( -\zeta \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \right) \]


Model: MUSIC hydro


Equation of motion for viscous corrections

**Bulk viscosity relaxation equation**

\[
\dot{\Pi} = -\frac{\Pi}{\tau_{\Pi}} + \frac{1}{\tau_{\Pi}} \left( -\zeta \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \right)
\]

**14-moment approximation in the small mass limit**

G. Denicol, S. Jeon, and C. Gale (2014)

\[
\frac{\zeta}{\tau_{\Pi}} = 15 \left( \frac{1}{3} - c_s^2 \right)^2 (\epsilon_0 + P_0)
\]

\[
\frac{\delta_{\Pi\Pi}}{\tau_{\Pi}} = \frac{2}{3}, \quad \frac{\lambda_{\Pi\pi}}{\tau_{\Pi}} = \frac{8}{5} \left( \frac{1}{3} - c_s^2 \right)
\]

**Second-order transport coefficients**
Model: Equation of state

P. Huovinen, and P. Petreczky (2010)

Equation of state: hadron gas + lattice data

Only those included in UrQMD

Cross over phase transition around $T = 180$ MeV

\begin{align*}
\varepsilon/T^4 & \quad \text{HotQCD} \quad \text{Laine} \quad \text{EoS L} \quad \text{Krakow} \\
p/T^4 & \quad \text{p} [\text{GeV/fm}^3] \\
T [\text{MeV}] & \\
\varepsilon [\text{GeV/fm}^3] & \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2
\end{align*}