Universal Dynamics and Non-Thermal Fixed Points in Quantum Gases

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Synthetic Quantum Systems

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isoQuant SFB1225

AQψS Analog Quantum Simulators for Many-Body Dynamics

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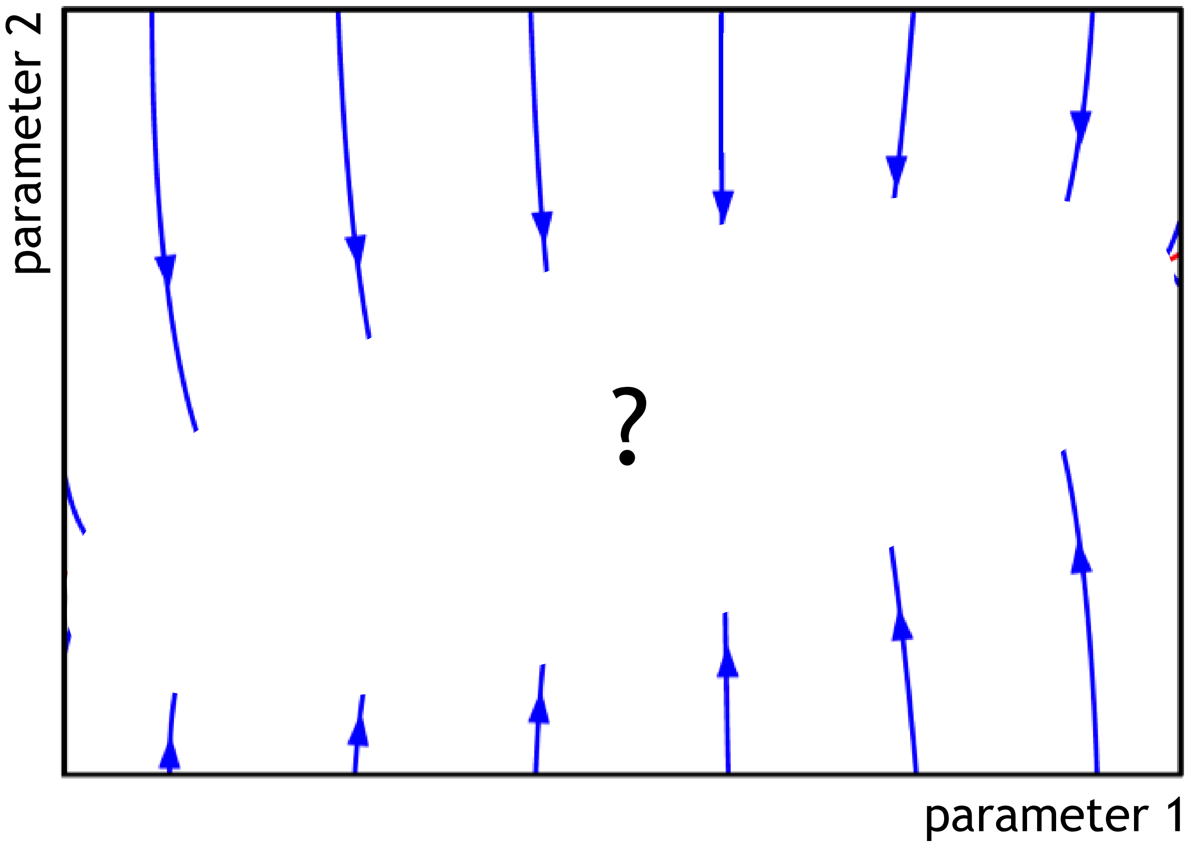
Center for Quantum Dynamics

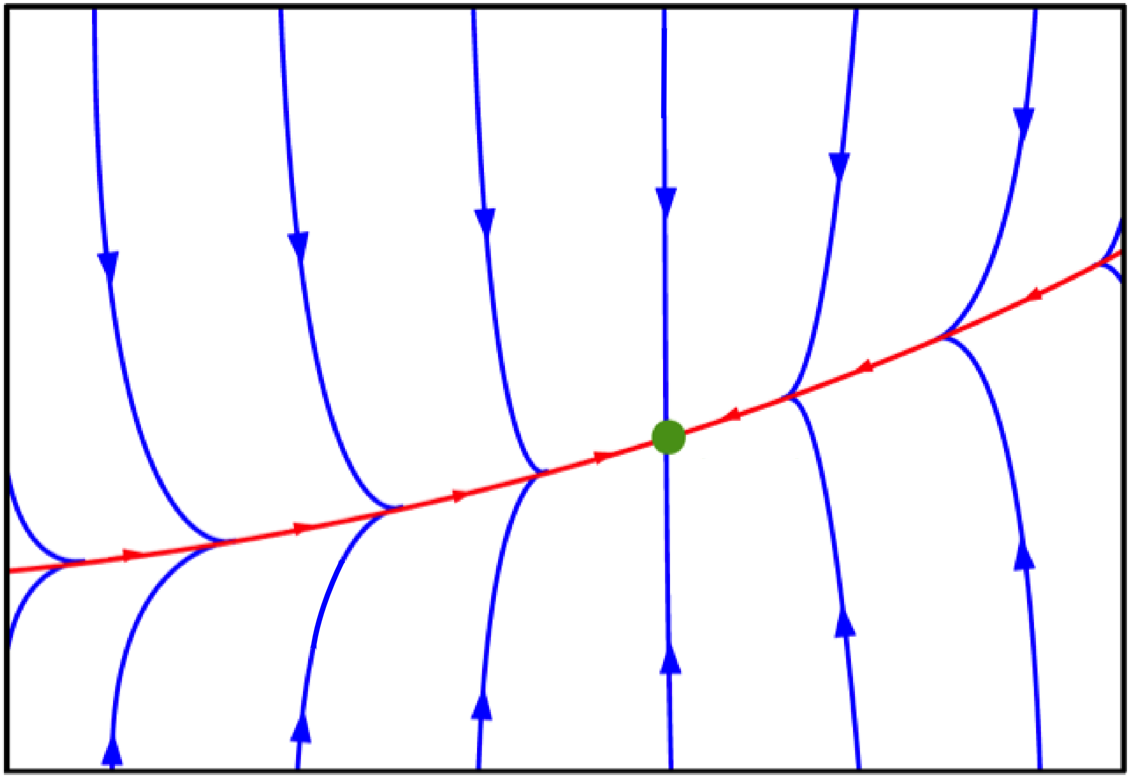
Ignazio Danti (1581-83)

Dynamics after a general quench?



parameter space
(schematically)

Partially universal quantum dynamics

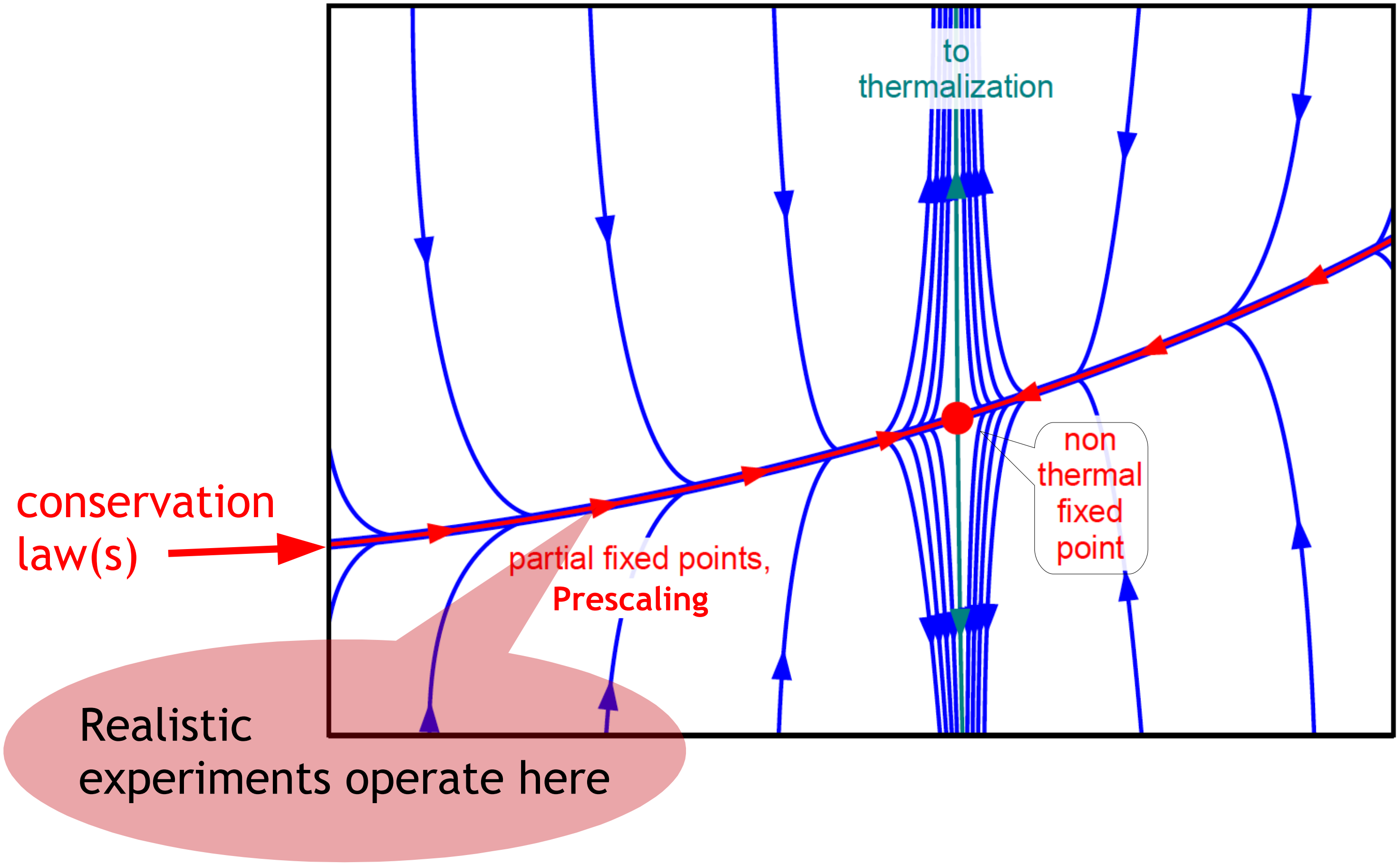


[Sketch after C. Wetterich]

(partial) loss of information about initial conditions

e.g. in terms of density matrix: loss of off-diagonal phase relations (between non-degenerate levels)

Prescaling and Non-Thermal Fixed Point



Non-thermal Fixed Points:

Berges, Rothkopf, Schmidt (08), Hoffmeister, Berges (09), Sexty, Schlichting, Piñeiro Orioli, Boguslavski, ..., Berges (10-)
Scheppach, Berges, TG (10), Nowak, Sexty, Schole, Schmidt, Erne, Karl, Schmied, ... TG (11-)
Chantesana, Piñeiro Orioli, TG (1801.09490, PRA (19)), Mikheev, Schmied, TG (1807.07514 (PRL 2019), 1807.10228)

See also the **review**: T. Langen, TG, J. Schmiedmayer, JSTAT 064009 (2016); arXiv:1603.09385
Recent **overview article**: Schmied, Mikheev, TG, arXiv:1810.08143

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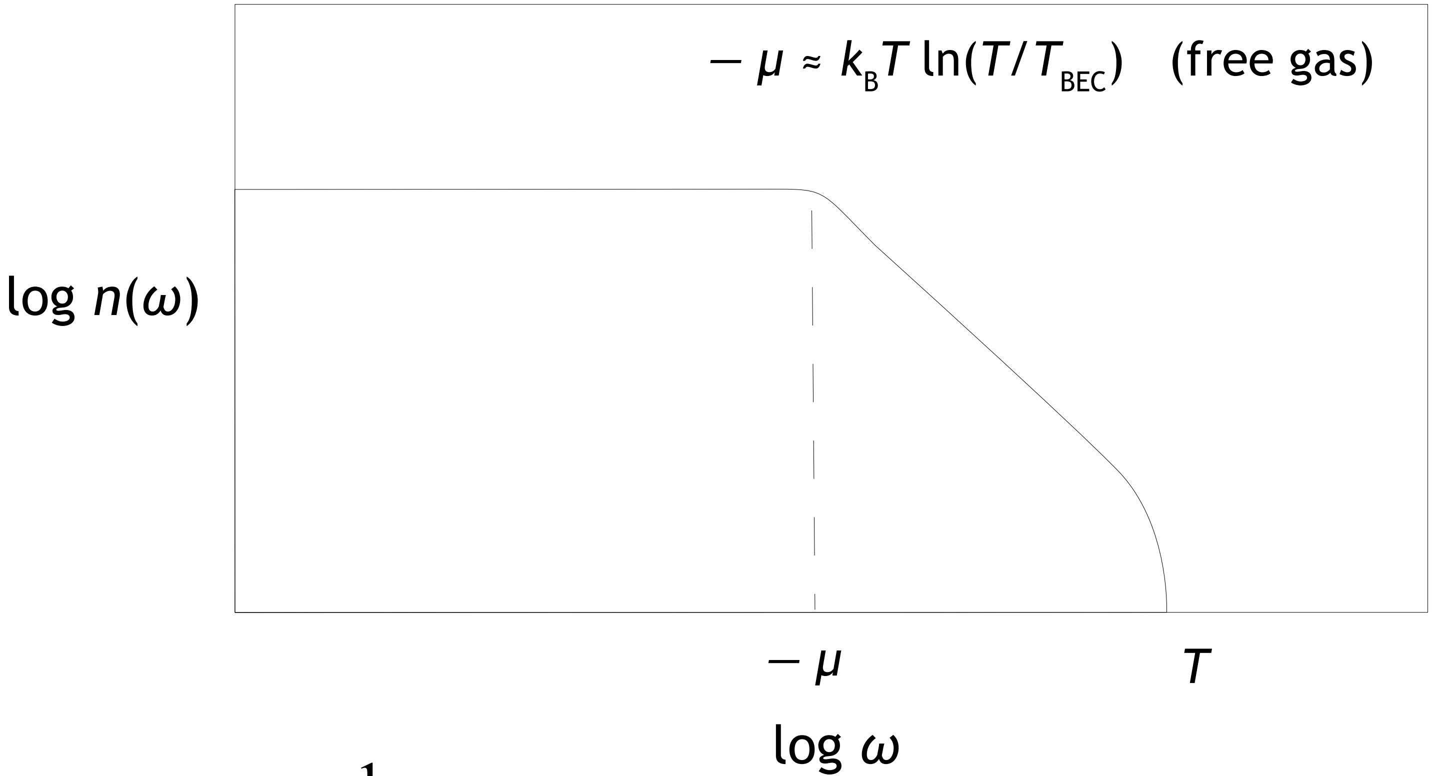
Let us consider the example of a

➡ **thermal Bose-Einstein distribution...**

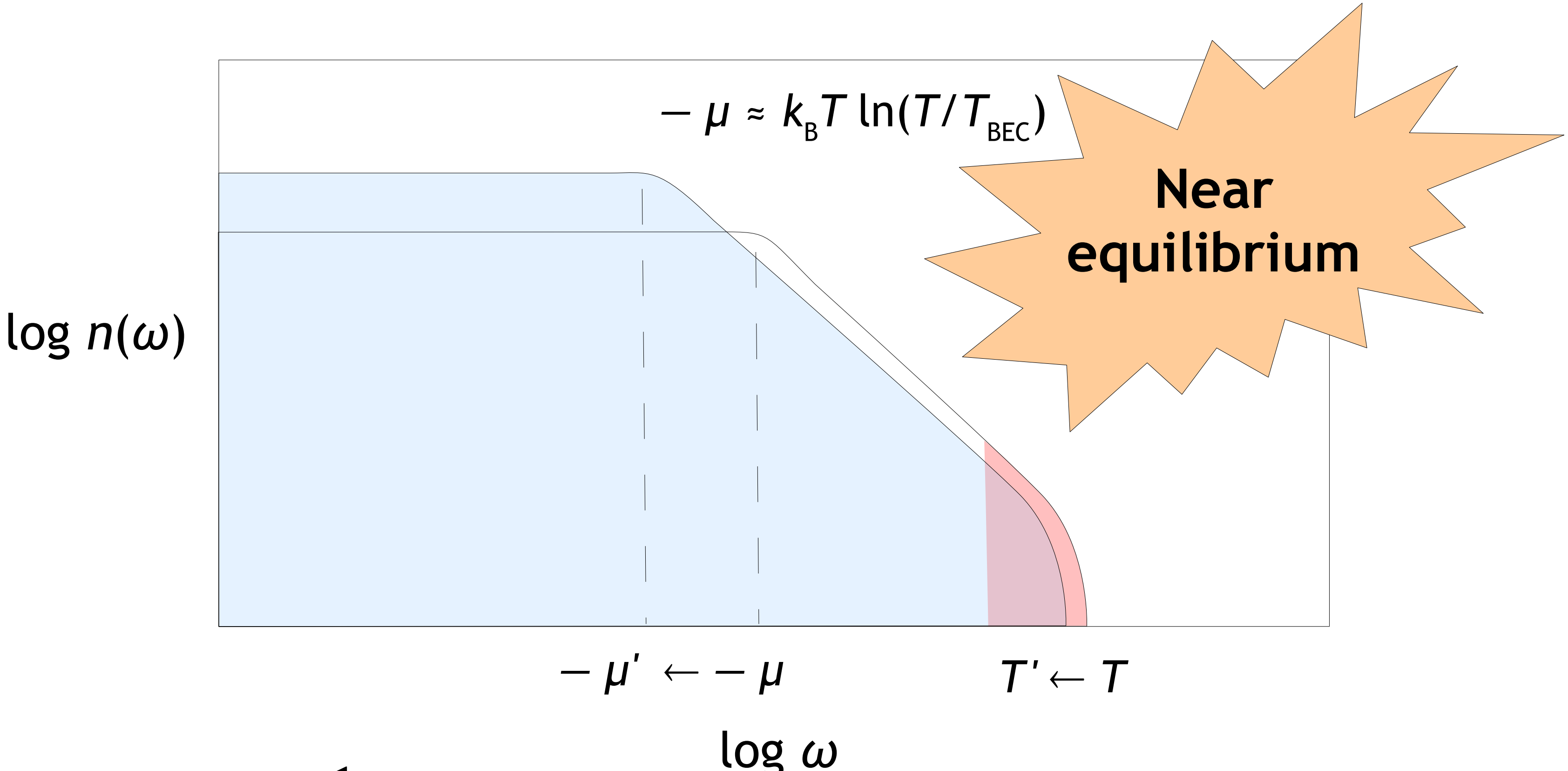


$$n(\omega) = \frac{1}{e^{(\omega-\mu)/T} - 1}$$

(**double-log!** – here sketched for $T_{BEC} < T \ll 2T_{BEC}$)

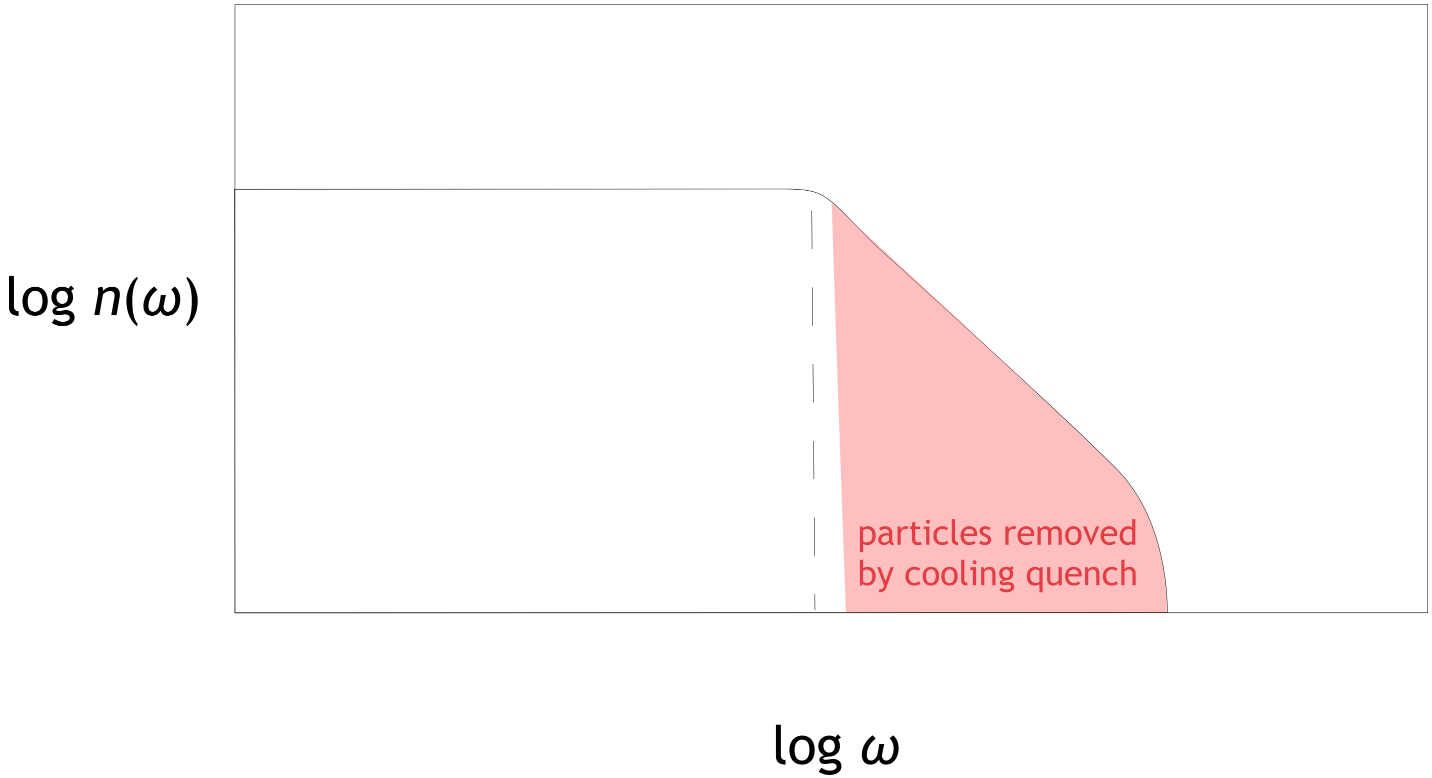
… and perform a (weak) cooling quench

...which leads to a (partially) self-similar **rescaling** of n_k



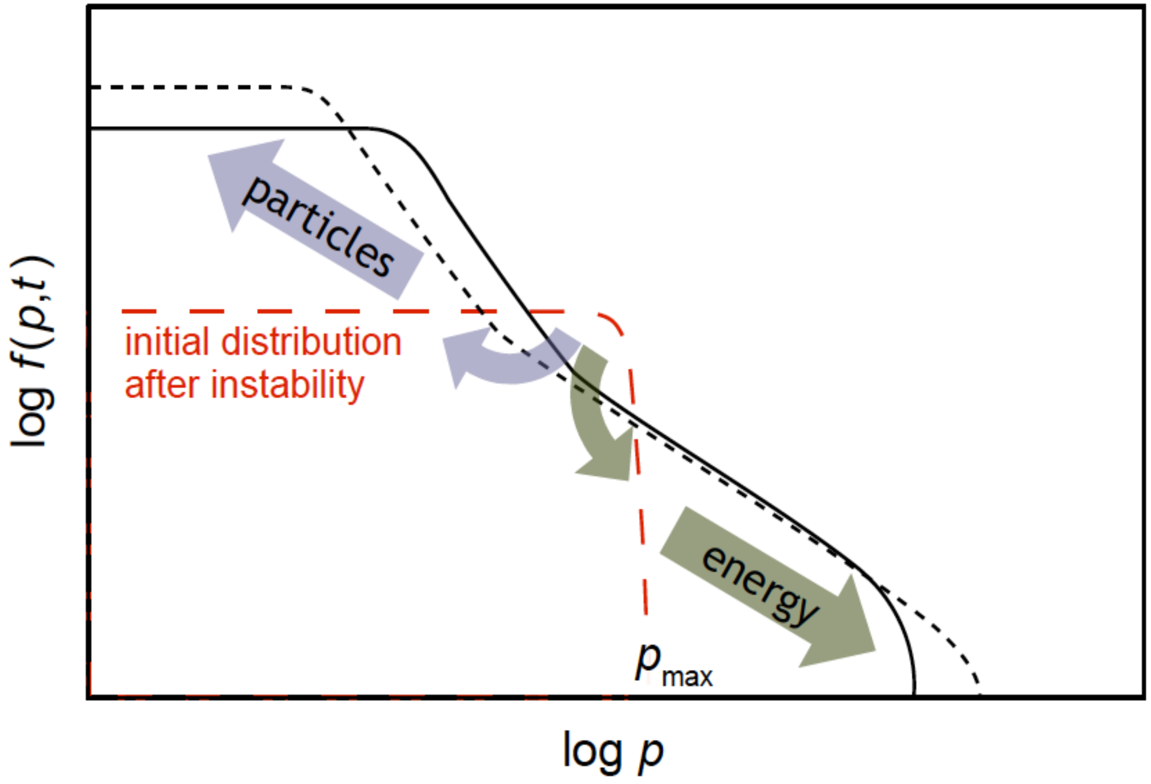
$$n(\omega) = \frac{1}{e^{(\omega-\mu)/T} - 1}$$

Strong cooling quench?



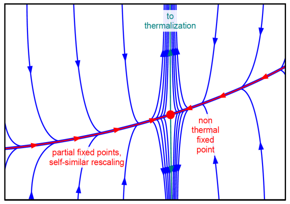
A **strong** cooling quench/instability …

...leads to **far-from-equilibrium** bidirectional transport:

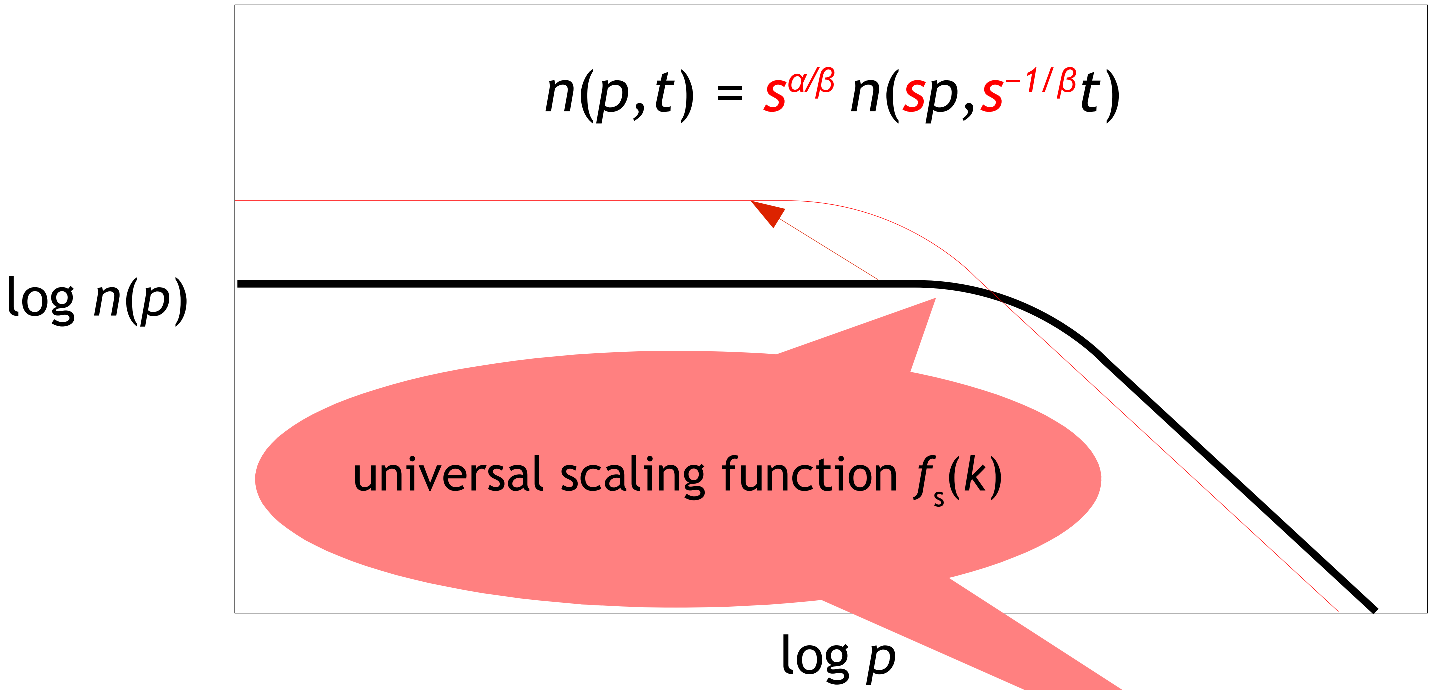


I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019)

Close to Non-Thermal Fixed Point: scaling in space & time



Time evolution ≙ Scaling transformation (*real-time* critical slowing down)

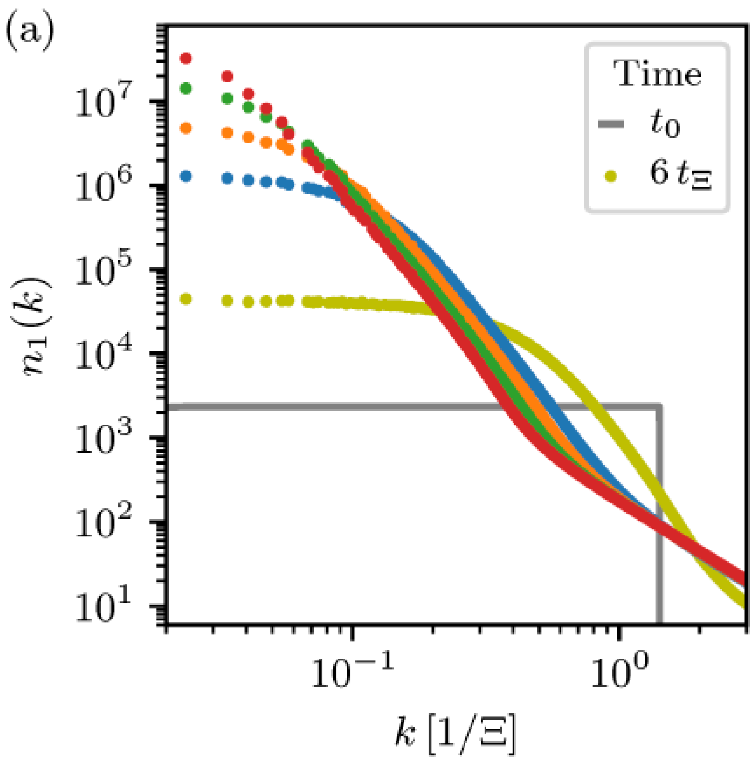


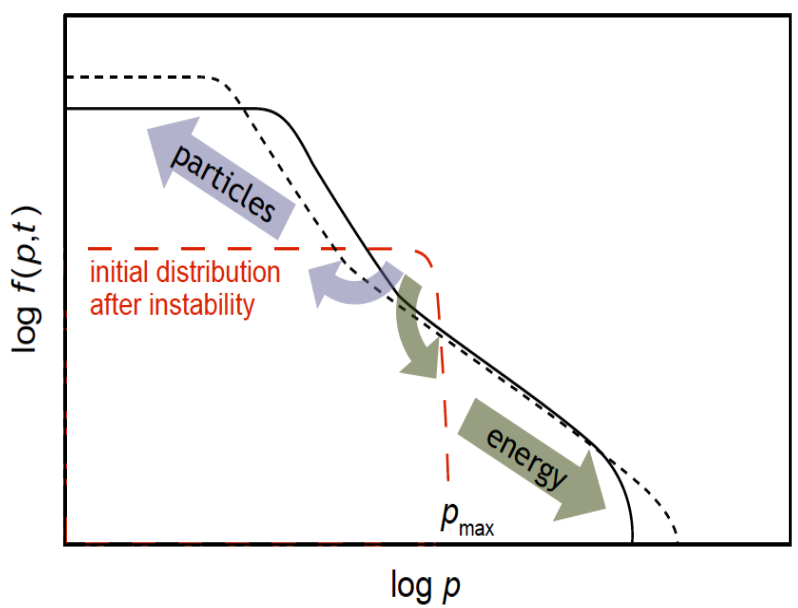
$$\text{choose } s = (t/t_0)^\beta \;\Rightarrow\; \text{scaling form: } n(p,t) = (t/t_0)^\alpha\, n(\,[t/t_0]^\beta p,\, t_0)$$
$$\equiv (t/t_0)^\alpha\, f_s(\,[t/t_0]^\beta p\,)$$

A. Piñeiro Orioli, K. Boguslavski, J. Berges, PRD **92**, 025041 (15)

Universal scaling dynamics in a three-component 3D Bose gas

$$H_{U(3)} = \int d^3x \left[-\Phi_a^\dagger \frac{\nabla^2}{2m}\Phi_a + \frac{g}{2}\Phi_a^\dagger\Phi_b^\dagger\Phi_b\Phi_a \right] \qquad a,\ b\ = 1{,}2{,}3. \quad U(3) \text{ symmetric!}$$





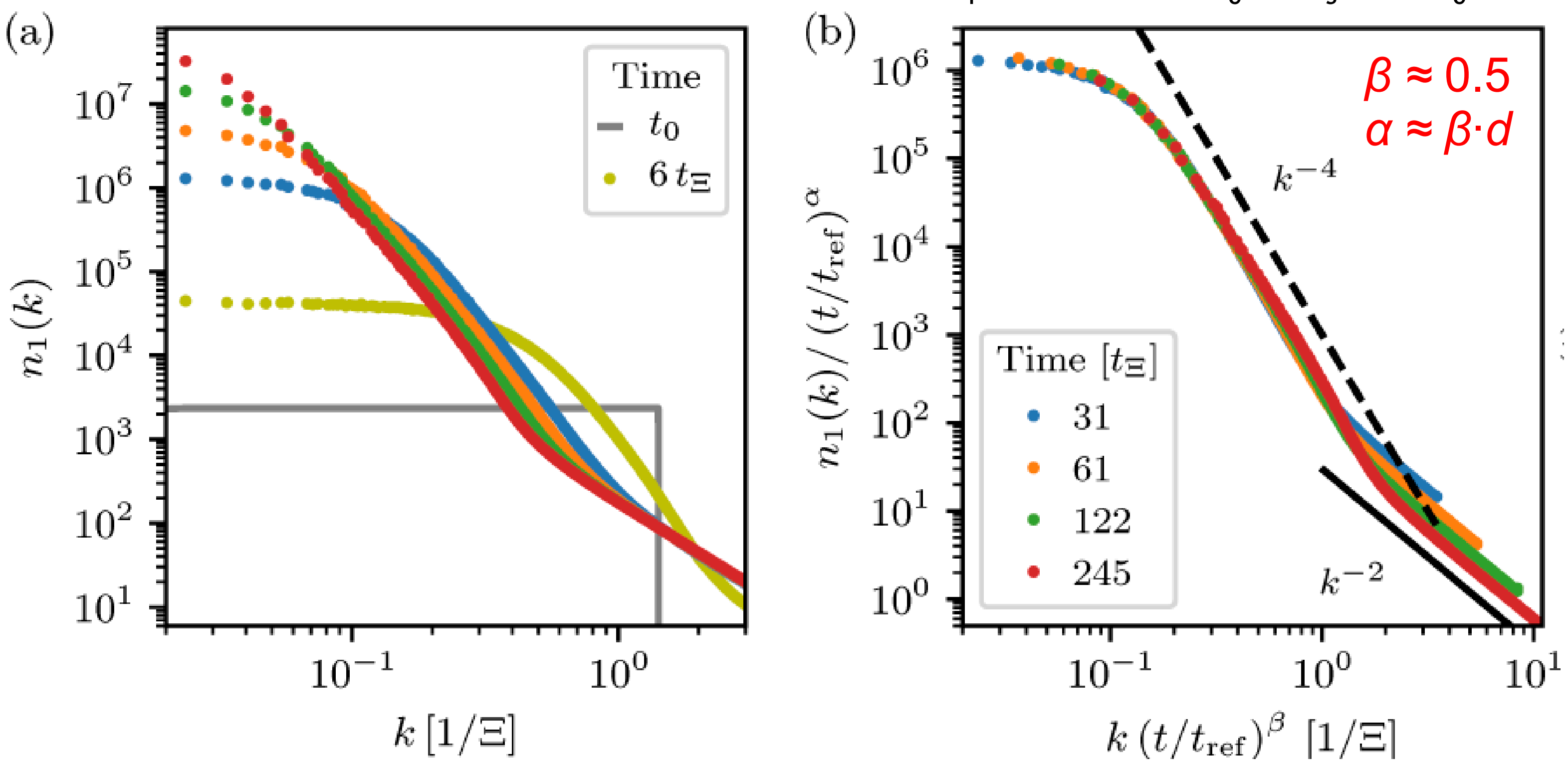
[C.M. Schmied, M. Mikheev, TG, PRL **122**, 170404 (2019); arXiv:1807.10228]

Universal scaling dynamics in a three-component 3D Bose gas

$$H_{U(3)} = \int \mathrm{d}^3x \left[-\Phi_a^\dagger \frac{\nabla^2}{2m} \Phi_a + \frac{g}{2} \Phi_a^\dagger \Phi_b^\dagger \Phi_b \Phi_a \right]$$

$a,\ b\ = 1,2,3.$ $U(3)$ symmetric!

$$n_1(p,t) = (t/t_0)^{\alpha}\ f_s(\,[t/t_0]^{\beta}\, p\,)$$



[C.M. Schmied, M. Mikheev, TG, PRL **122**, 170404 (2019); arXiv:1807.10228]

Video: Approach of a strongly anomalous non-thermal fixed point in a
1-component 2D gas

https://www.kip.uni-heidelberg.de/gasenzer/projects/lowenergyeffectiveu3a

Scaling analysis of Boltzmann transport

Radial transport equation as fixed-point equation:

$$\partial_t n_Q(\mathbf{p},t) = I[n_Q](\mathbf{p},t)$$

$$I[n_Q](\mathbf{p},t) = \int_{\mathbf{kqr}} |T_{\mathbf{pkqr}}|^2 \delta(\mathbf{p}+\mathbf{k}-\mathbf{q}-\mathbf{r})\,\delta(\omega_\mathbf{p}+\omega_\mathbf{k}-\omega_\mathbf{q}-\omega_\mathbf{r}) \times [(n_\mathbf{p}+n_\mathbf{k})n_\mathbf{q}n_\mathbf{r} - n_\mathbf{p}n_\mathbf{k}(n_\mathbf{q}+n_\mathbf{r})]$$

$$n_\mathbf{p} \equiv n_Q(\mathbf{p},t) = (t/t_0)^{\alpha}\, f_s(\,[t/t_0]^{\beta} p\,)$$

$$t^{\alpha-1} \sim t^{-\beta(3d+2m-d-z)+3\alpha}$$

$$\alpha = \beta[d+m-z/2]-1/2$$

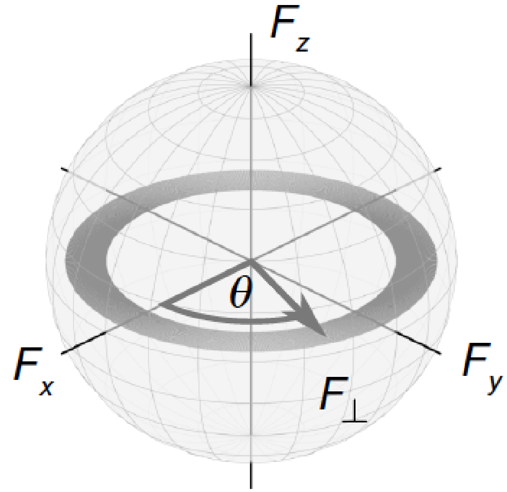
fixes, together with relation from conservation laws, both, α & β.

I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019),
M. Mikheev, C.M. Schmied, TG, arXiv:1807.10228, PRA to appear

Experiment: Quench in a quasi-1D Spin-1 gas

$$H = \int dx \left[\vec{\Phi}^\dagger \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + q f_z^2 \right) \vec{\Phi} + \frac{c_0}{2} n^2 + \frac{c_1}{2} |\vec{F}|^2 \right]$$

$$\vec{\Phi} = (\Phi_1, \Phi_0, \Phi_{-1})^T$$

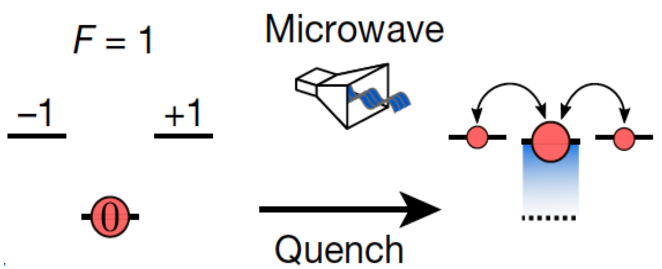


$$F_\perp = F_x + iF_y$$

Quench from polar to easy-plane (broken axisymmetric) phase

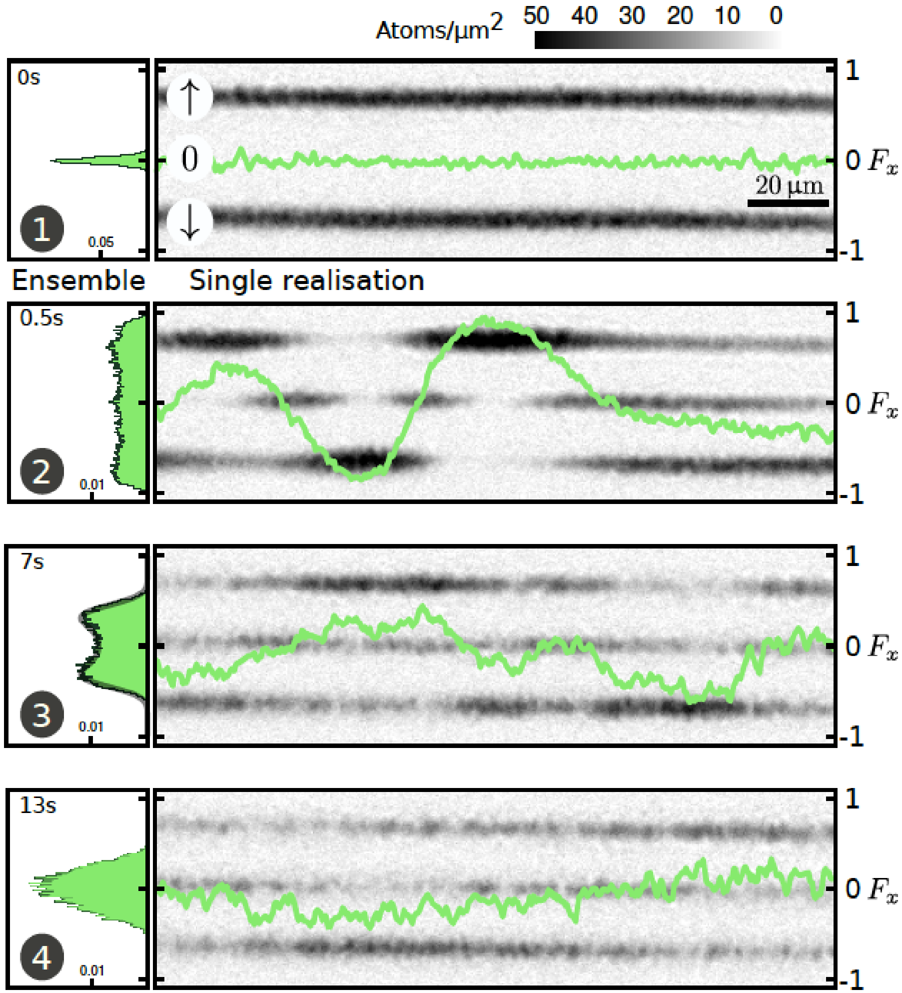
Measurement of Structure Factor:

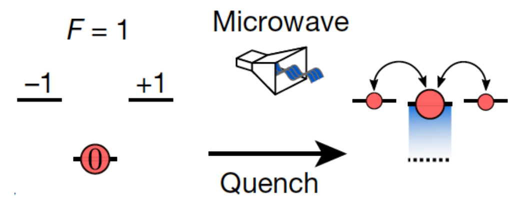
$$S(k,t) = \langle |F_\perp(k,t)|^2 \rangle$$

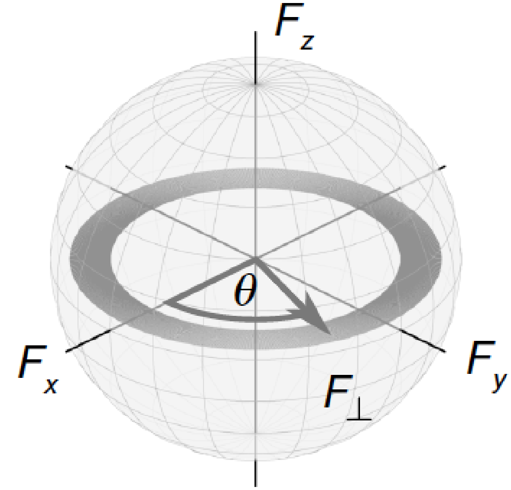


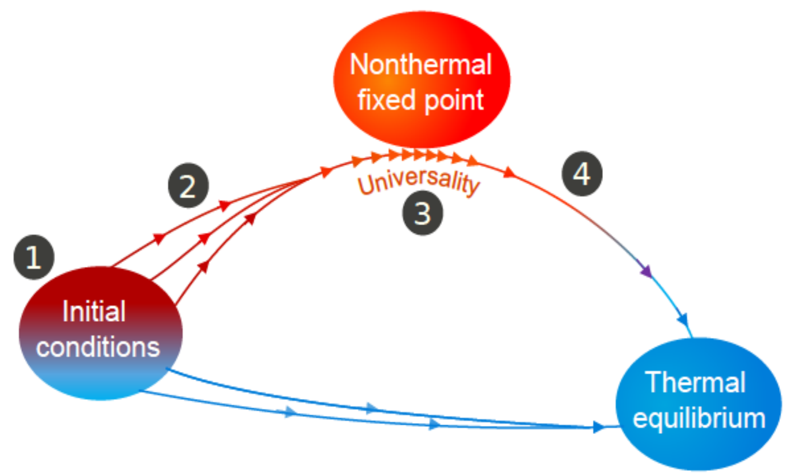
M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.-M. Schmied, J. Berges, TG, M.K. Oberthaler, Nature **563,** 217 (2018)

Quench in a quasi-1D Spin-1 gas



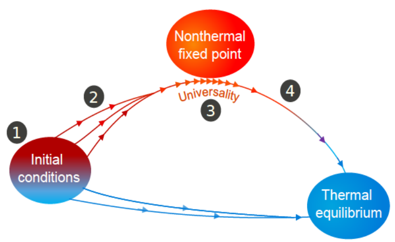


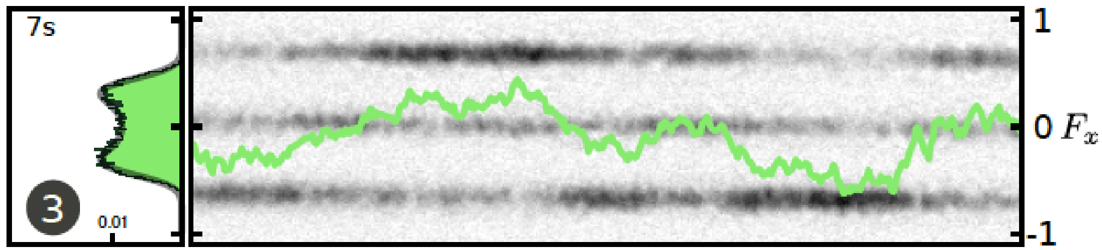


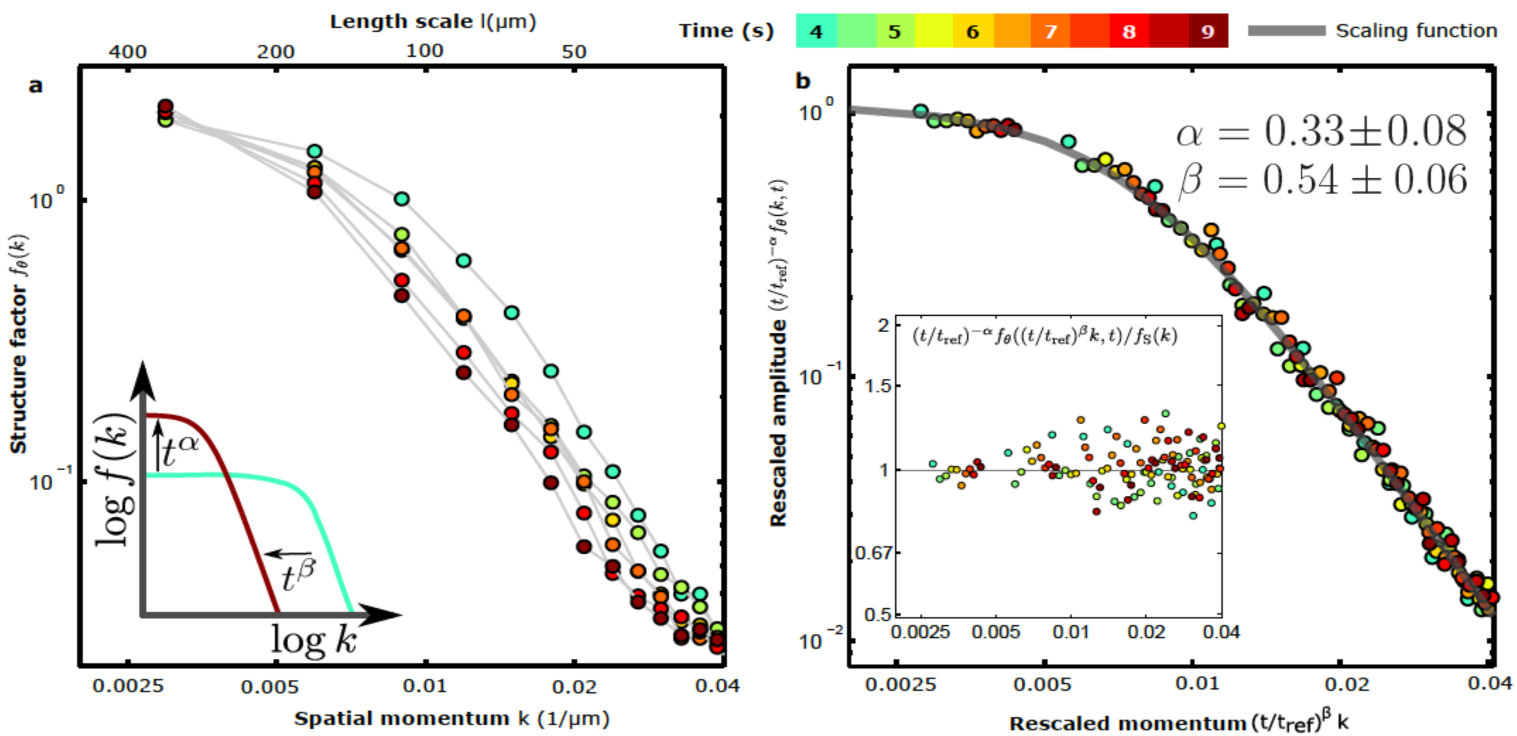


M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.-M. Schmied, J. Berges, TG, M.K. Oberthaler, Nature **563**, 217 (2018)

Universal dynamics near NTFP: scaling







M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.-M. Schmied, J. Berges, TG, M.K. Oberthaler, Nature **563**, 217 (2018)

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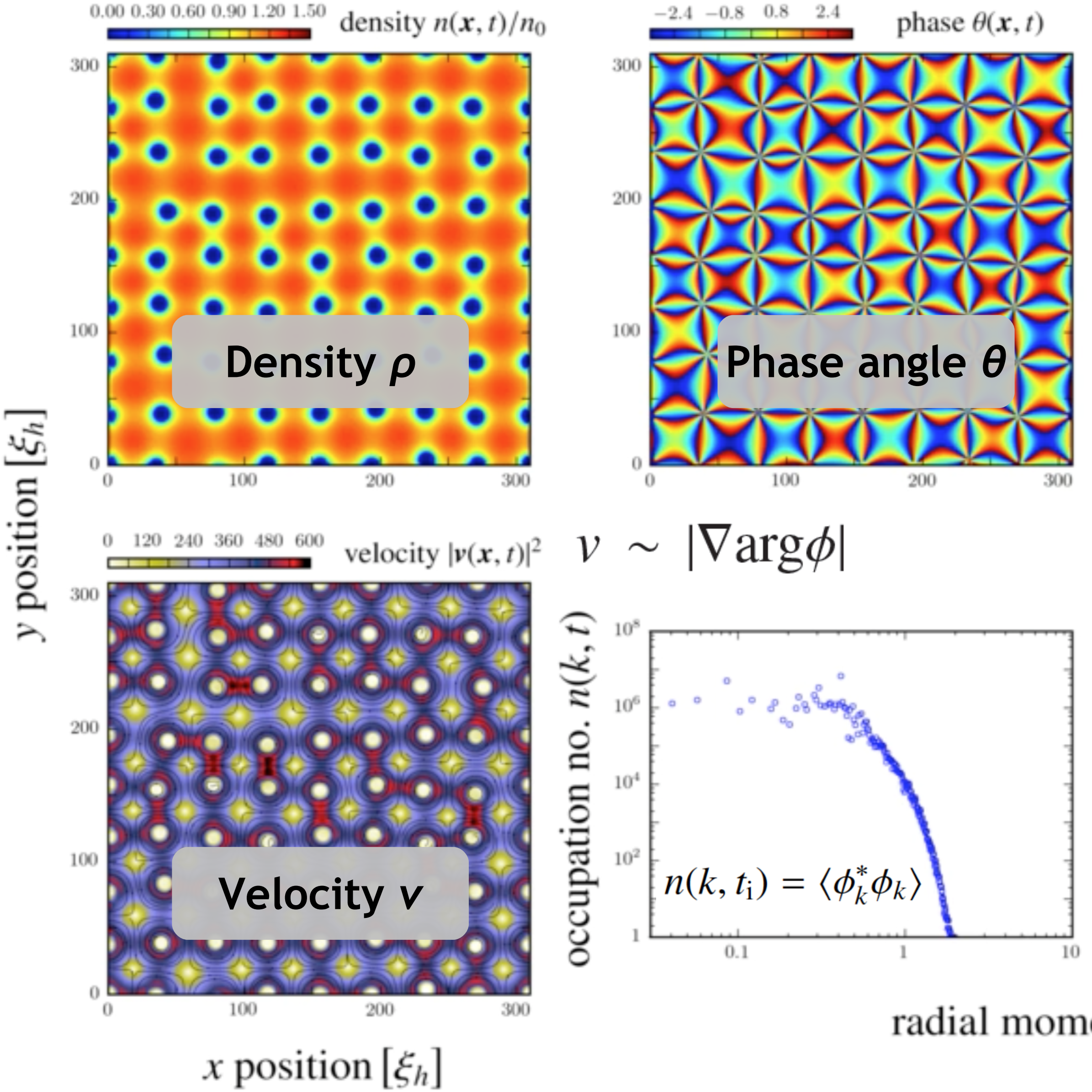
Non-linear excitations & Defects

2D complex field $\phi(\mathbf{x},t) = \rho(\mathbf{x},t)^{1/2}\exp(i\theta(\mathbf{x},t))$



$v \sim |\nabla \arg\phi|$

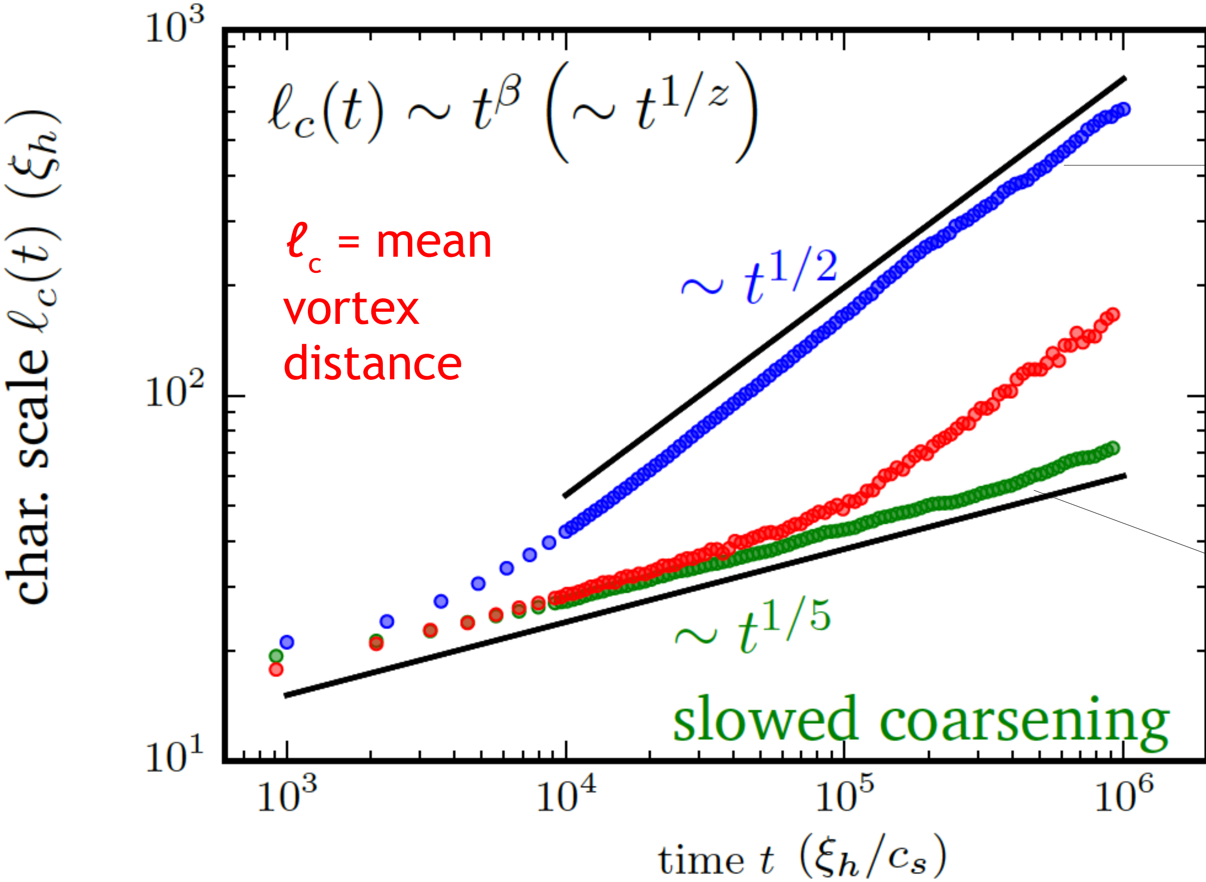
$$i\partial_t\phi - \left[-\frac{\nabla^2\phi}{2m} - \mu + g\phi^*\phi\right]\phi = 0$$

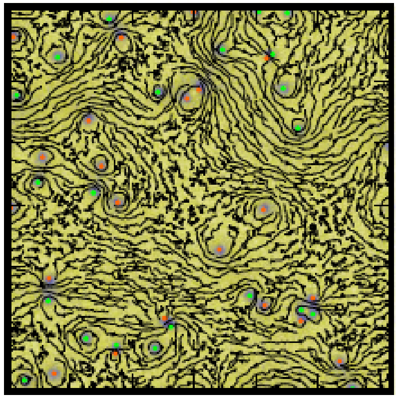
non-lin. Schrödinger Eq.

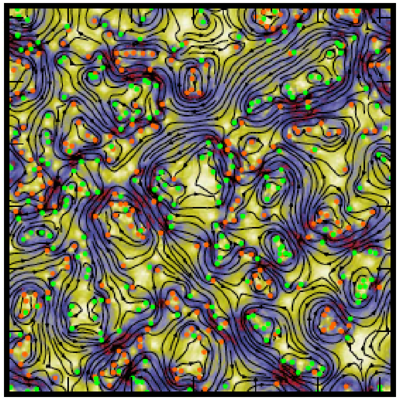
Video: Approach of a non-thermal fixed point in a 3-component 3D gas

https://www.kip.uni-heidelberg.de/gasenzer/projects/anomalousntfp

Anomalous vs near-equilibrium NTFP





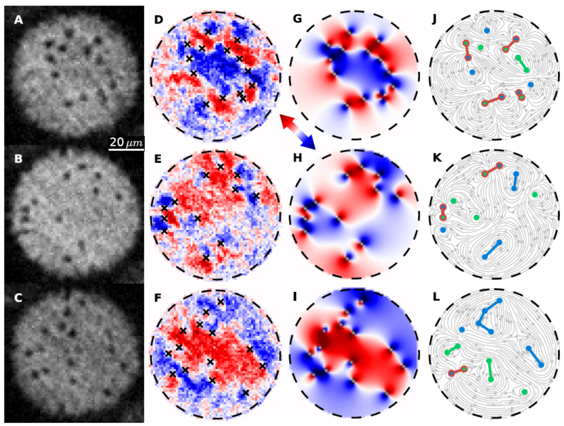


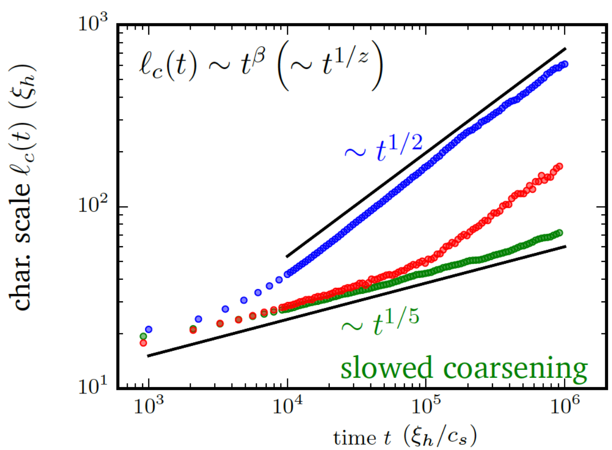
$z_d \approx 5$

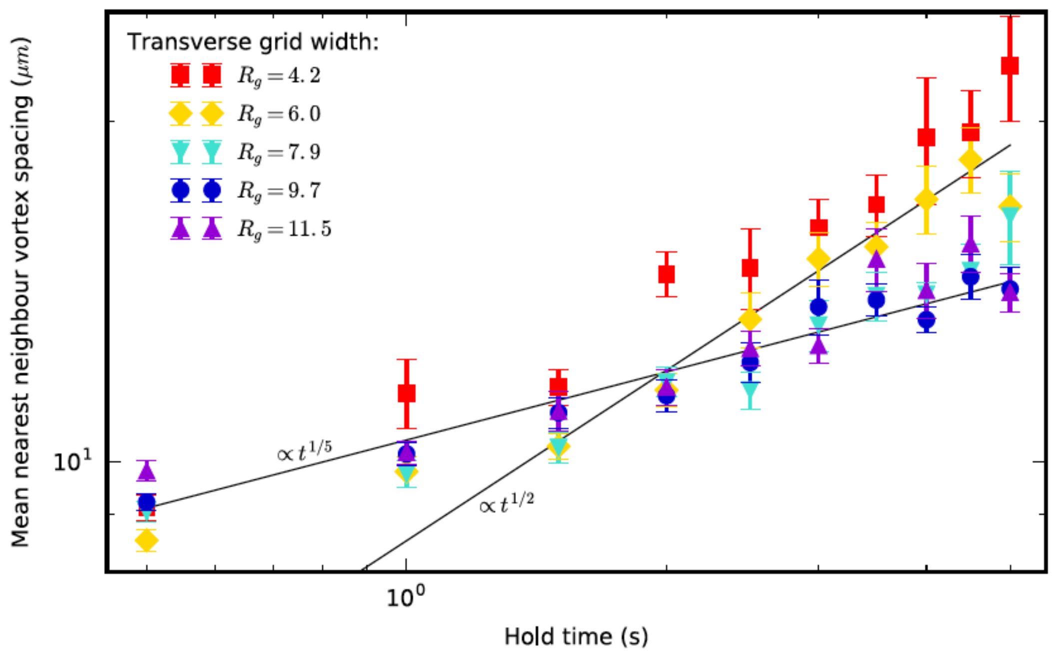
(glass-like *z*-exponent)

M. Karl, TG, NJP **19,** 093014 (2017), 1611.01163 [cond-mat.quant-gas]
See also:
J. Deng, S. Schlichting, R. Venugopalan, Q. Wang, PRA 97, 053606 (2018)
A. Groszek, M. Davis, T. Simula, 1903.05528 [cond-mat.quant-gas]

Vortex dynamics in experiment (Monash U)







S.P. Johnstone et al., 1801.06952v2 [cond-mat.quant-gas] (Helmerson group)

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Aleksandr Mikheev

Christian Marcel Schmied

Reviews, Lecture notes, Summary articles, & recent Progress:

Non-thermal fixed points: Universal dynamics far from equilibrium
C.-M. Schmied, A. N. Mikheev, TG,
in Proc. Julian Schwinger Centennial Conf. and Workshop, Singapore, 7-12 Feb 2018.
arXiv:1810.08143 [cond-mat.quant-gas]

Low-energy effective theory of non-thermal fixed points in a multicomponent Bose gas
A. N. Mikheev, C.-M. Schmied, TG,
arXiv:1807.10228 [cond-mat.quant-gas]; Phys. Rev. A, to appear

Prescaling in a far-from-equilibrium Bose gas
C.-M. Schmied, A. N. Mikheev, TG,
Phys. Rev. Lett. **122**: 170404, 2019

Kinetic theory of non-thermal fixed points in a Bose gas
I. Chantesana, A. Piñeiro Orioli, TG,
Phys. Rev. A **99**, 043620 (2019)

Prethermalization and universal dynamics in near-integrable quantum systems
T. Langen, TG, J. Schmiedmayer,
JSTAT **064009**, 2016; arXiv:1603.09385 [cond-mat.quant-gas]

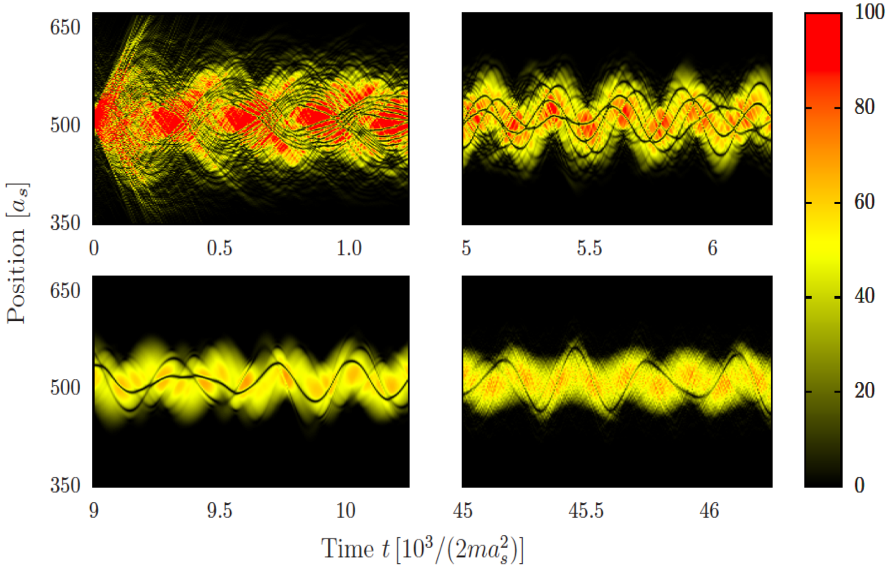
Non-thermal fixed points: universality, topology, & turbulence in Bose gases
B. Nowak, S. Erne, M. Karl, J. Schole, D. Sexty, and TG,
in Proc. Int. School on Strongly Interacting Quantum Systems Out of Equilibrium,
Les Houches, edited by T. Giamarchi et al. (OUP, Oxford, 2016)
arXiv:1302.1448 [cond-mat.quant-gas]

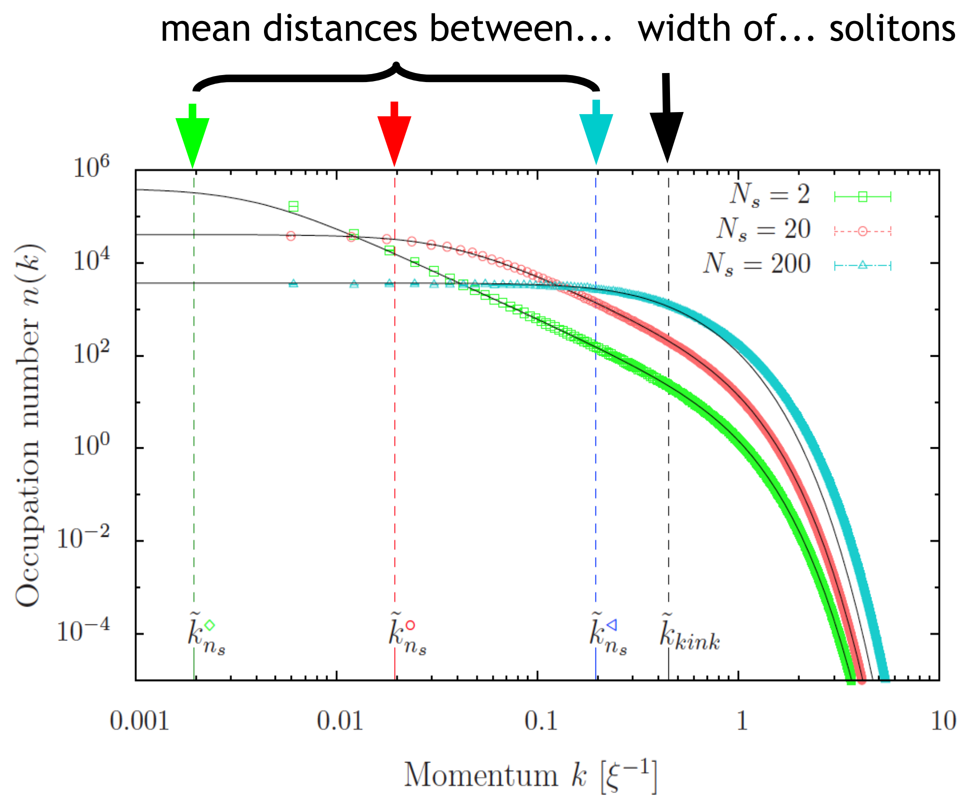
The End

Supplementary Slides

Solitons in 1D condensate

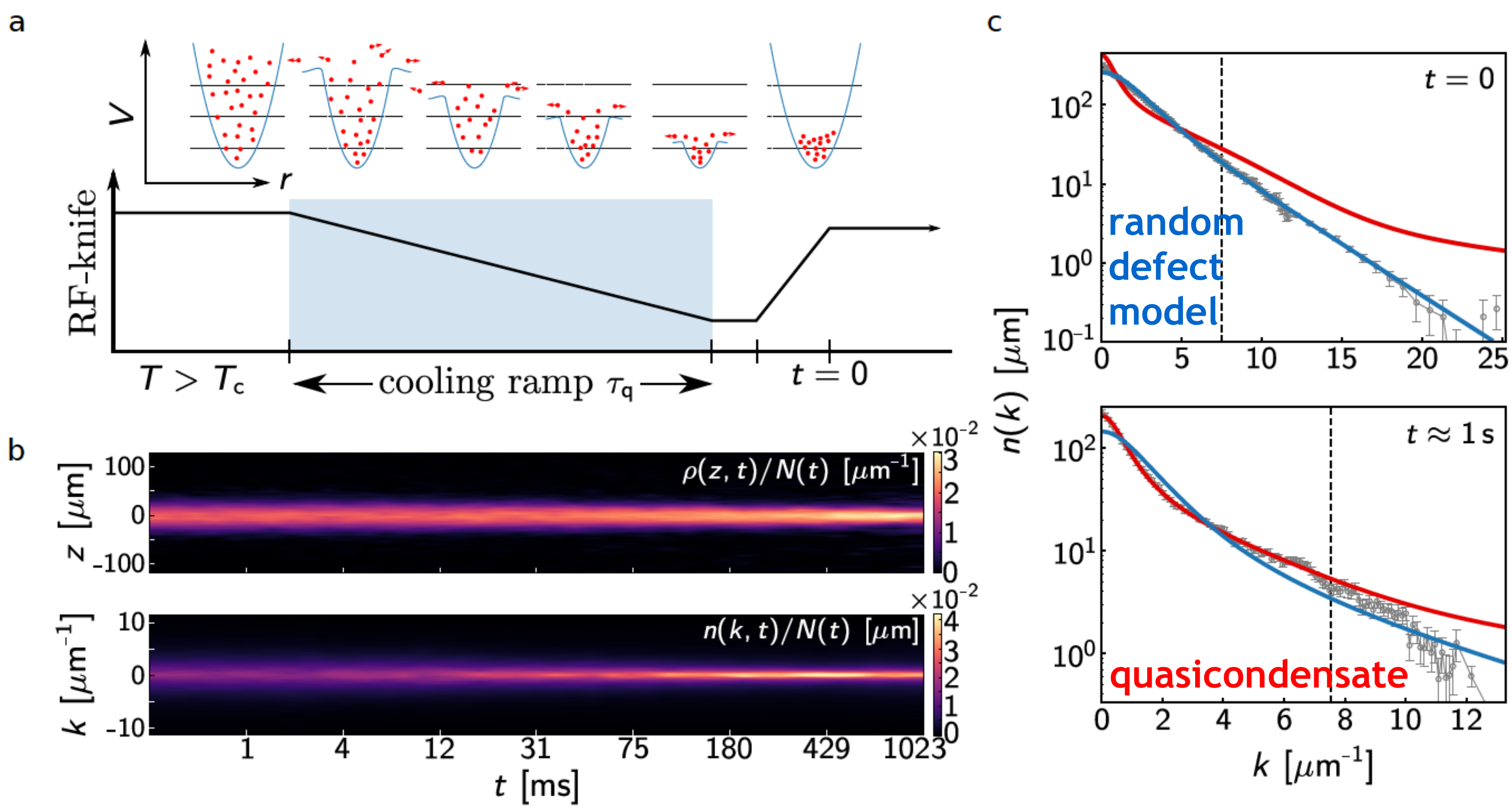
Universal distribution: $$n(k) = \left[\frac{4n_0 n_s}{(2n_s)^2 + k^2}\right] \frac{\left(\frac{\pi\xi k}{\sqrt{2}}\right)^2}{\sinh^2\left(\frac{\pi\xi k}{\sqrt{2}}\right)}$$





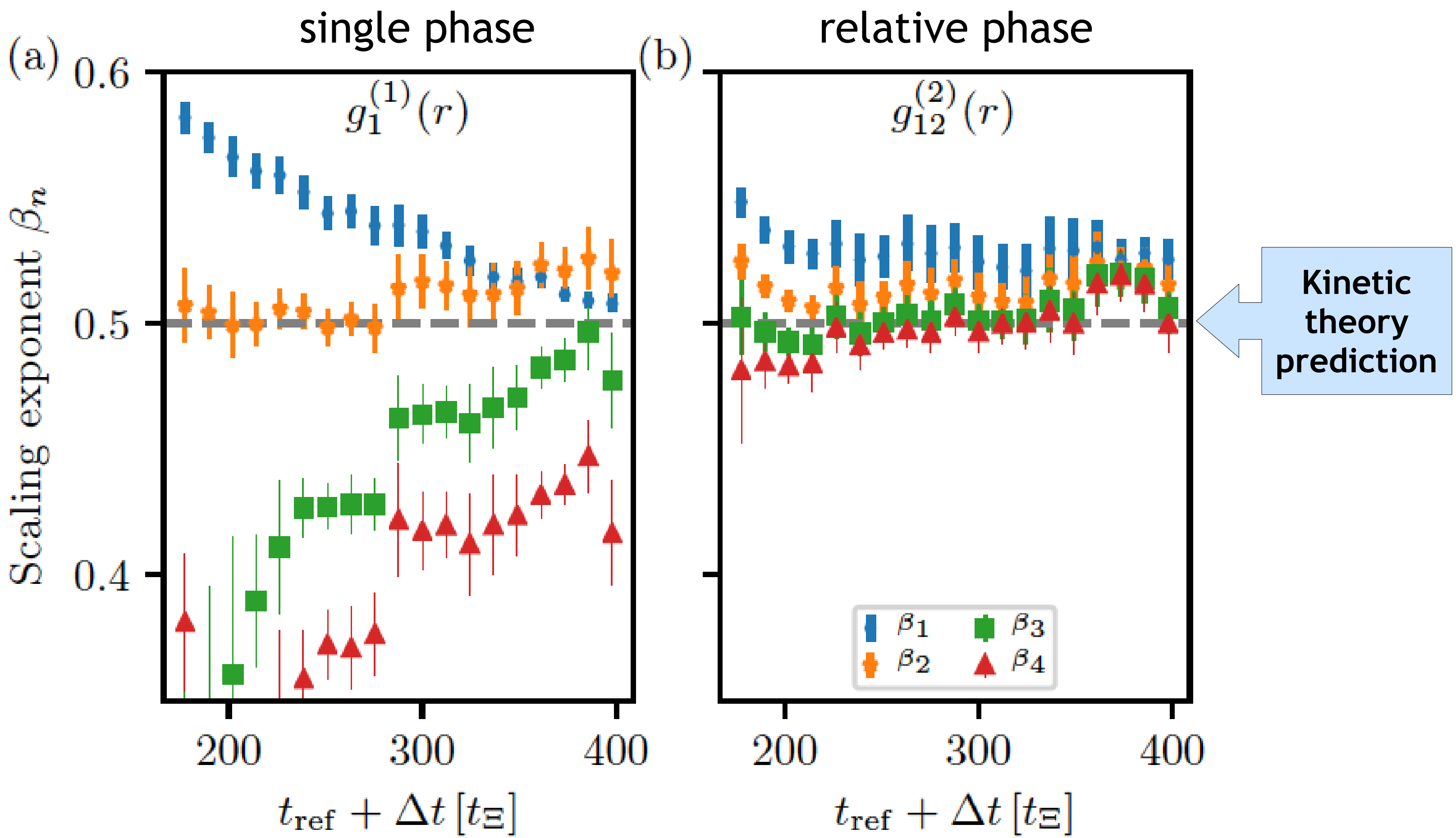
S. Erne, Diploma Thesis (2012); M. Schmidt, S. Erne, B. Nowak, D. Sexty, and TG, NJP 14 (12) 075005
S. Erne, R. Bücker, TG, J. Berges, J. Schmiedmayer, Nature **563**, 225 (2018)

NTFP in a 1D soliton gas



S. Erne, R. Bücker, TG, J. Berges, J. Schmiedmayer, Nature **563**, 225 (2018)

Scaling exponents (Prescaling)

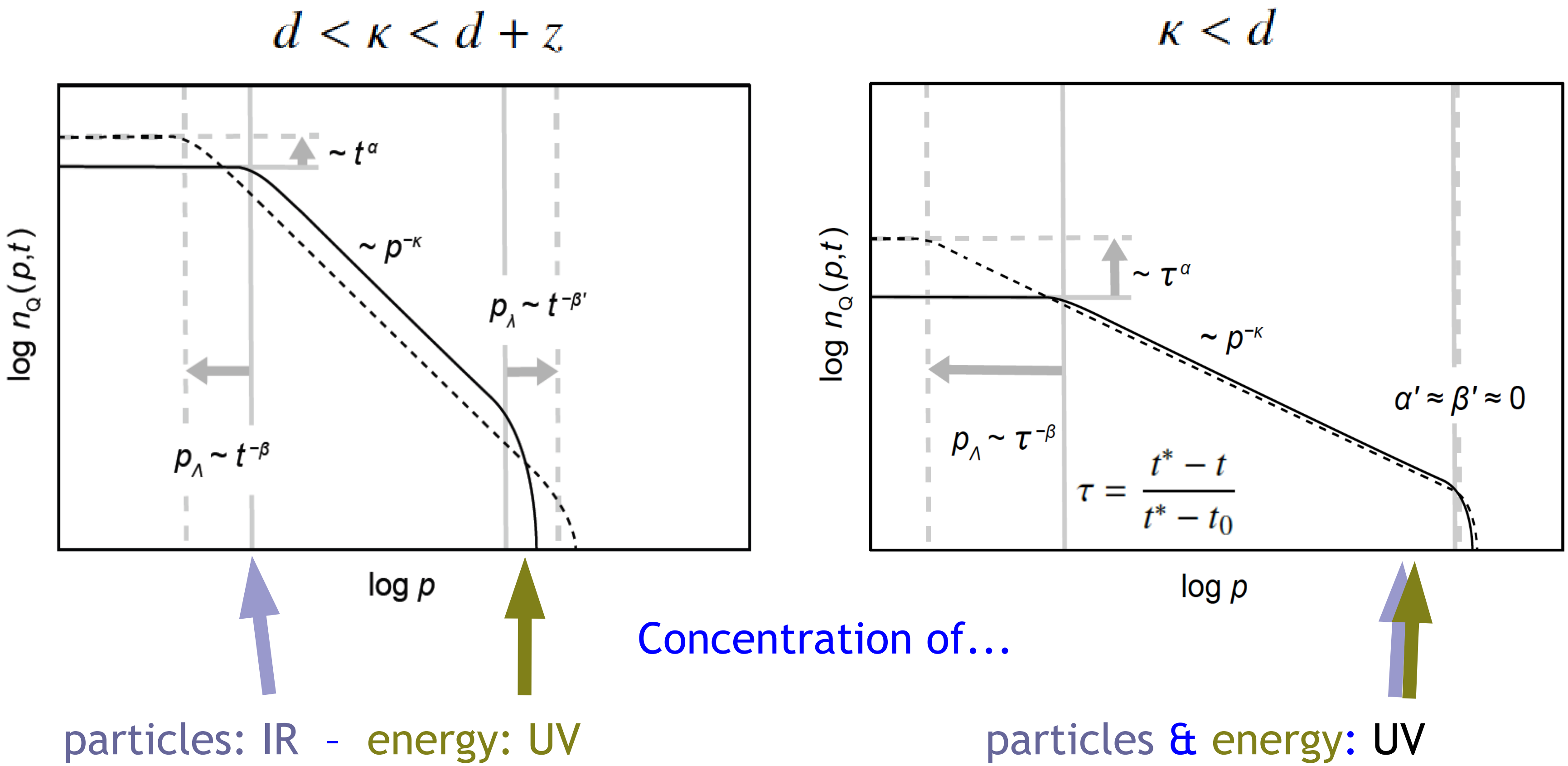


$$g_1^{(1)}(\mathbf{r}, t) = \langle \Phi_1^\dagger(\mathbf{x} + \mathbf{r}, t)\Phi_1(\mathbf{x}, t)\rangle$$

$$g_{12}^{(2)}(r, t) = \langle \Phi_1^\dagger(\mathbf{x} + \mathbf{r}, t)\Phi_2(\mathbf{x} + \mathbf{r}, t)\Phi_2^\dagger(\mathbf{x}, t)\Phi_1(\mathbf{x}, t)\rangle$$

[C.M. Schmied, M. Mikheev, TG, PRL **122**, 170404 (2019); arXiv:1807.10228]

Importance of Conservation Laws



- **NTFP vs. wave-turbulent cascades**

I. Chantesana, A. Piñeiro Orioli, TG, PRA 99, 043620 (2019) – see also B. Svistunov (1991)

NTFP:
Low-energy effective theory

(sf) Hydrodynamic formulation

Gross-Pitaevskii model (N components)   O(N) x U(1) $\simeq$ U(N)

$$H_{O(N)} = \int \mathrm{d}^d x \left[-\Phi_a^\dagger \frac{\nabla^2}{2m} \Phi_a + \frac{g}{2} \Phi_a^\dagger \Phi_b^\dagger \Phi_b \Phi_a \right]$$

Madelung 1926: $$\varphi_a(\mathbf{x}, t) = \sqrt{\rho_a(\mathbf{x}, t)} \exp\{\mathrm{i}\theta_a(\mathbf{x}, t)\}$$

Superfluid velocity: $$\mathbf{v}_a(\mathbf{x}, t) = \frac{1}{m} \nabla \theta_a(\mathbf{x}, t)$$

Superfluid hydro equations:

Euler: $$(\partial_t + \mathbf{v}_a \cdot \nabla)\, \mathbf{v}_a = -\frac{\nabla}{m} \left(g \sum_b \rho_b - \frac{1}{2m\rho_a^{1/2}} \nabla^2 \rho_a^{1/2} \right)$$

Continuity: $$\partial_t \rho_a + \nabla(\rho_a \mathbf{v}_a) = 0\,.$$

[M. Mikheev, C.M. Schmied, TG, arXiv:1807.10228, PRA to appear]

Linearized sf. hydro for broken U(N)

Linearized hydro equations in $\delta\rho_a = \rho_a - \rho_a^{(0)}$ and θ_a :

$$\partial_t \theta_a = \frac{1}{4m\rho_a^{(0)}} \nabla^2 \delta\rho_a - g \sum_b \delta\rho_b \,, \quad \partial_t \delta\rho_a = -\frac{\rho_a^{(0)}}{m} \nabla^2 \theta_a$$

⇒ Wave equation:

$$\partial_t^2 \theta_a(\mathbf{k}, t) + \frac{\mathbf{k}^2}{2m} \left(\frac{\mathbf{k}^2}{2m} \delta^{ab} + 2g\rho_b^{(0)} \right) \theta_b(\mathbf{k}, t) = 0$$

$N-1$ free Goldstones $\quad \omega_c(\mathbf{k}) \equiv \omega_{\mathrm{G}}(\mathbf{k}) = \frac{\mathbf{k}^2}{2m}, \quad c = 1, \ldots, N-1 \,,$

1 Bogoliubov mode $\quad \omega_N(\mathbf{k}) \equiv \omega_{\mathrm{B}}(\mathbf{k}) = \sqrt{\frac{\mathbf{k}^2}{2m} \left(\frac{\mathbf{k}^2}{2m} + 2g\rho^{(0)} \right)} \,,$

[M. Mikheev, C.M. Schmied, TG, arXiv:1807.10228, PRA to appear]

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Low-energy effective theory

à la Luttinger...

$$S_{\mathrm{eff,G}}[\theta] = -\int_{\mathbf{k},t} \frac{1}{2g_{\mathrm{B}}(\mathbf{k})}\theta_{\mathrm{B}}(\mathbf{k},t)\Big[\partial_t^2 + \omega_{\mathrm{B}}(\mathbf{k})^2\Big]\theta_{\mathrm{B}}(-\mathbf{k},t) - \int_{\mathbf{k},t} \frac{1}{2g_{\mathrm{G}}(\mathbf{k})}\theta_{\mathrm{G},c}(\mathbf{k},t)\Big[\partial_t^2 + \omega_{\mathrm{G}}(\mathbf{k})^2\Big]\theta_{\mathrm{G},c}(-\mathbf{k},t)$$

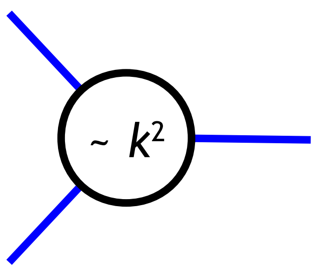
with couplings:

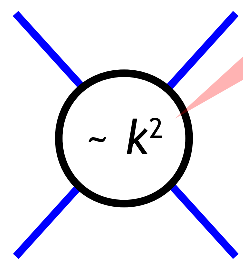
$$g_B(\mathbf{k}) = gN(1+\mathbf{k}^2/2k_\Xi^2) \qquad \text{(Bogoliubov)}$$

$$g_{\mathrm{G}}(\mathbf{k}) = \frac{g}{2}\frac{\mathbf{k}^2}{k_{\Xi,a}^2} = \frac{\mathbf{k}^2}{4m\rho_a^{(0)}} \qquad (N-1 \text{ free Goldstones})$$

IR Gaussian!

plus non-linear interactions:





[M. Mikheev, C.M. Schmied, TG, arXiv:1807.10228, PRA to appear]

Low-energy effective theory

à la Luttinger...

$$S_{\mathrm{eff,G}}[\theta] = -\int_{\mathbf{k},t} \frac{1}{2g_{\mathrm{B}}(\mathbf{k})}\theta_{\mathrm{B}}(\mathbf{k},t)\left[\partial_t^2 + \omega_{\mathrm{B}}(\mathbf{k})^2\right]\theta_{\mathrm{B}}(-\mathbf{k},t) - \int_{\mathbf{k},t} \frac{1}{2g_{\mathrm{G}}(\mathbf{k})}\theta_{\mathrm{G},c}(\mathbf{k},t)\left[\partial_t^2 + \omega_{\mathrm{G}}(\mathbf{k})^2\right]\theta_{\mathrm{G},c}(-\mathbf{k},t)$$

with couplings:

$$g_B(\mathbf{k}) = gN(1 + \mathbf{k}^2/2k_\Xi^2) \quad \text{(Bogoliubov)}$$

$$g_{\mathrm{G}}(\mathbf{k}) = \frac{g}{2}\frac{\mathbf{k}^2}{k_{\Xi,a}^2} = \frac{\mathbf{k}^2}{4m\rho_a^{(0)}} \quad (N-1 \text{ free Goldstones})$$

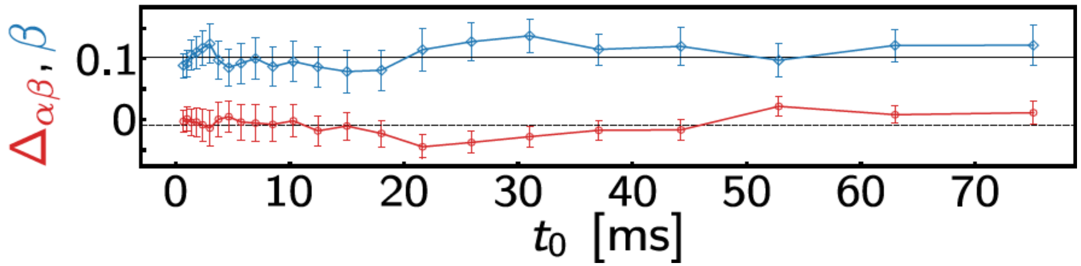
plus non-linear interactions:

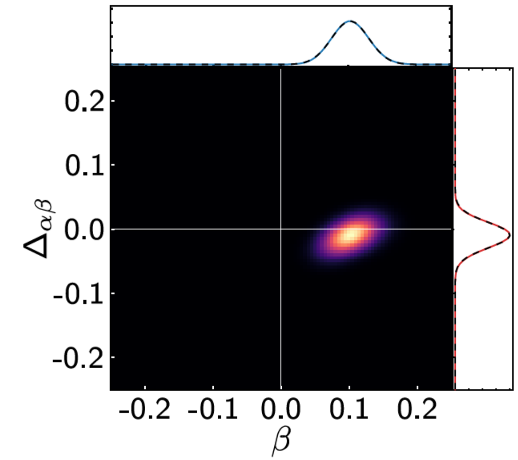
$$S^{(3)}_{\mathrm{eff,nG}}[\theta] = \int_{\mathbf{kk}',C} \frac{1}{N^{3/2}}\frac{1}{g_{1/\mathrm{N}}(\mathbf{k})}\left(\delta^{ab} - \frac{k_{\Xi,a}k_{\Xi,b}/k_\Xi^2}{1+\mathbf{k}^2/2k_\Xi^2}\right)\frac{k_\Xi}{k_{\Xi,b}} \times \frac{\mathbf{k}'(\mathbf{k}'-\mathbf{k})}{2m}\partial_t\theta_a(-\mathbf{k},t)\theta_b(\mathbf{k}',t)\theta_b(\mathbf{k}-\mathbf{k}',t)\Big\},$$

$$S^{(4)}_{\mathrm{eff,nG}}[\theta] = \int_{\mathbf{kk}'\mathbf{k}'',C} \frac{1}{2N^2}\frac{1}{g_{1/\mathrm{N}}(\mathbf{k})}\left(\frac{\delta^{ab}k_\Xi^2}{k_{\Xi,a}^2} - \frac{1}{1+\mathbf{k}^2/2k_\Xi^2}\right) \times \frac{\mathbf{k}'(\mathbf{k}'+\mathbf{k})}{2m}\frac{\mathbf{k}''(\mathbf{k}''-\mathbf{k})}{2m} \times \theta_a(\mathbf{k}',t)\theta_a(-\mathbf{k}-\mathbf{k}',t)\theta_b(\mathbf{k}'',t)\theta_b(\mathbf{k}-\mathbf{k}'',t)\Big\}.$$

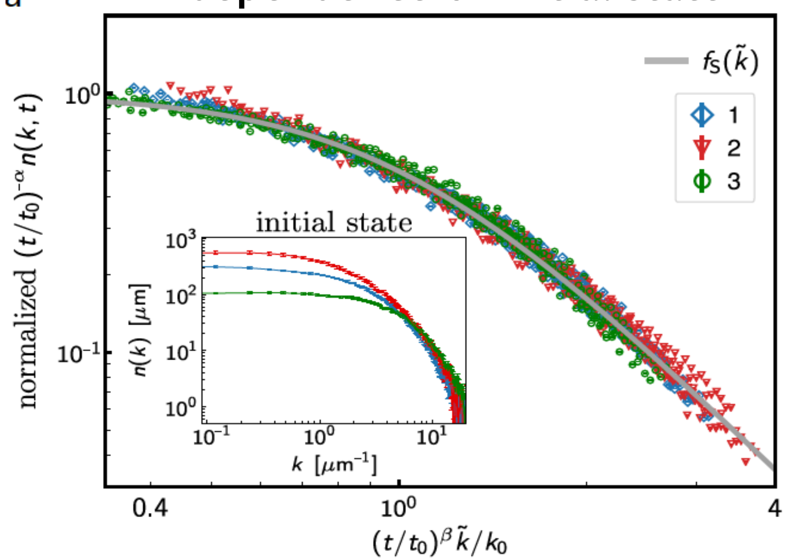
[M. Mikheev, C.M. Schmied, TG, arXiv:1807.10228, PRA to appear]

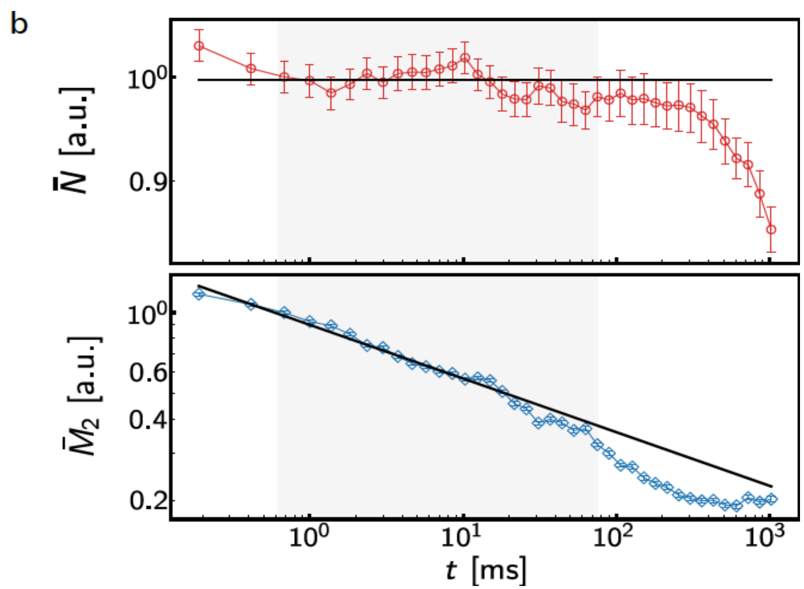
NTFP in a 1D soliton gas





Independence of initial state





S. Erne, R. Bücker, TG, J. Berges, J. Schmiedmayer, Nature **563**, 225 (2018)

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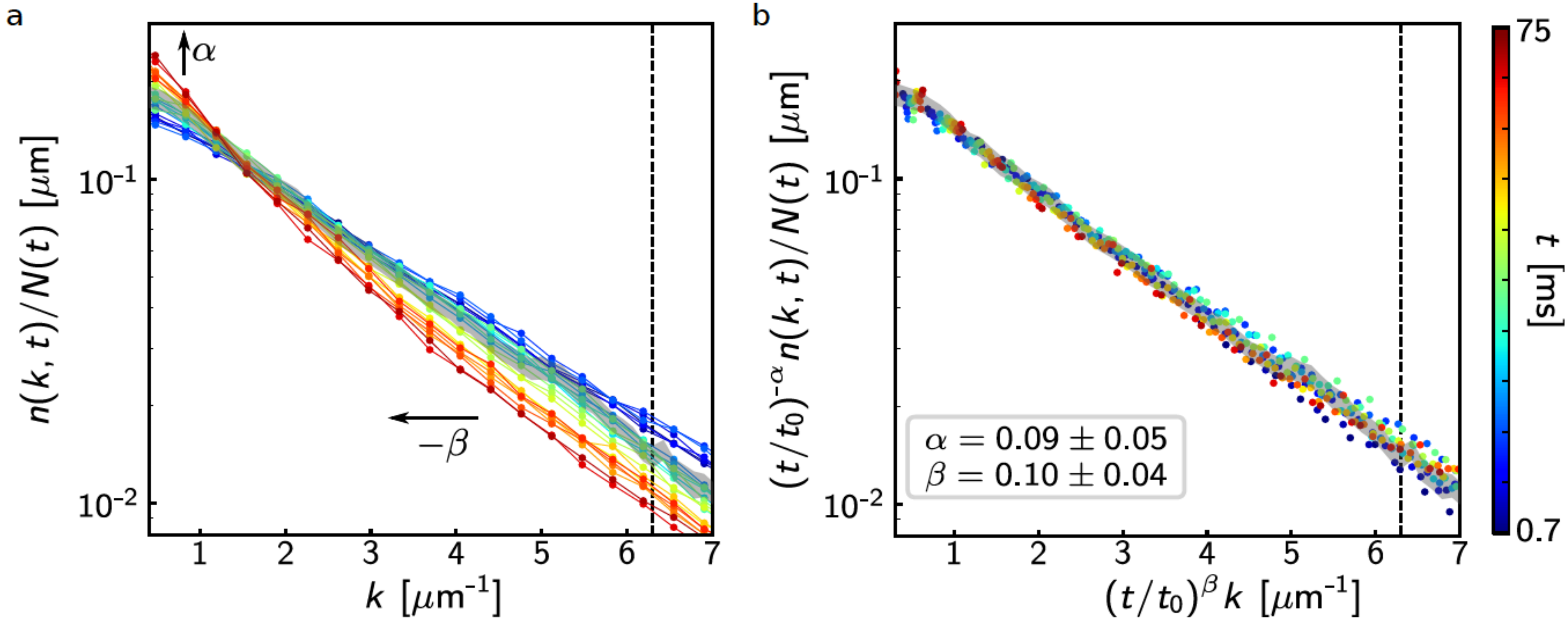
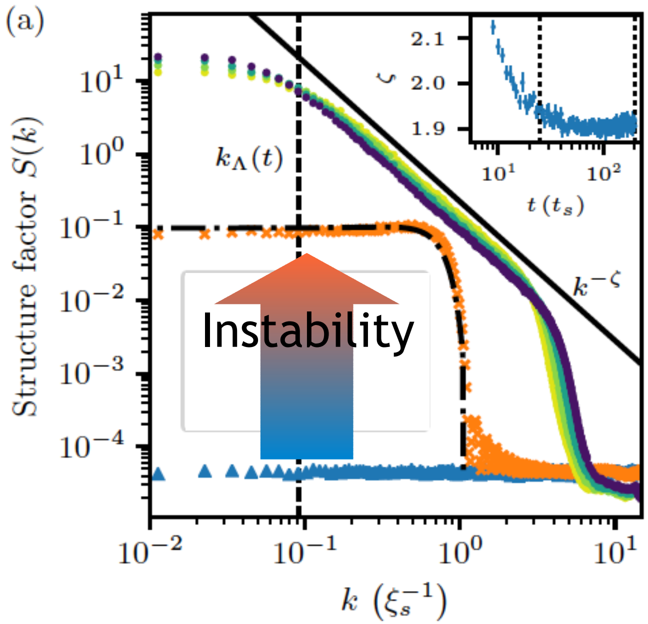


S. Erne, R. Bücker, TG, J. Berges, J. Schmiedmayer, Nature **563**, 225 (2018)

Universal scaling dynamics in a **1D** Spin-1 Bose gas

$$\vec{\Phi} = (\Phi_1, \Phi_0, \Phi_{-1})^T$$

$$H = \int dx \left[\vec{\Phi}^\dagger \left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + q f_z^2 \right) \vec{\Phi} + \frac{c_0}{2} n^2 + \frac{c_1}{2} |\vec{F}|^2 \right]$$



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[C.M. Schmied, M. Prüfer, M.K. Oberthaler, TG, PRA 99, 033611 (2019)]

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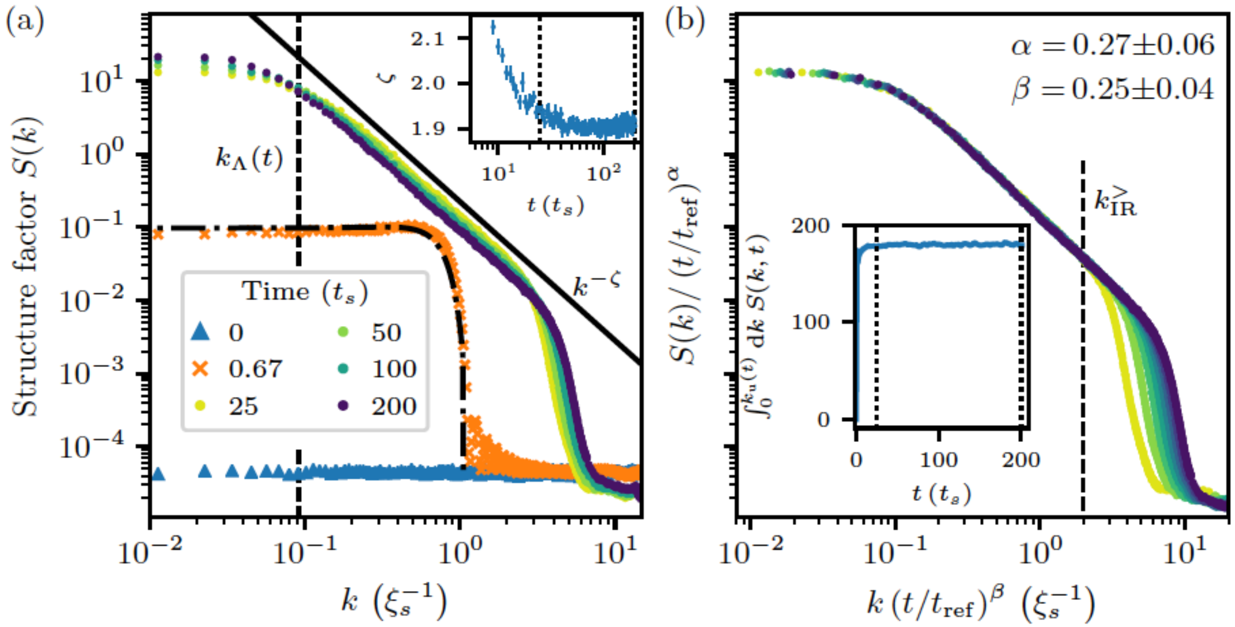

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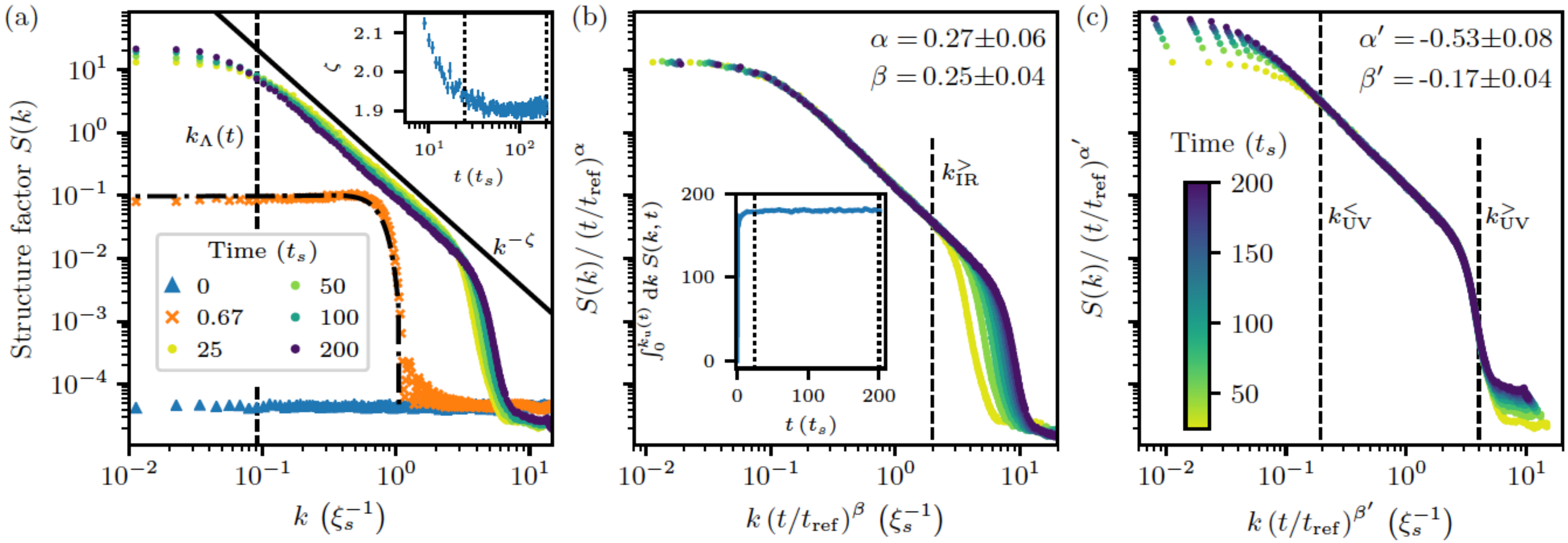

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