Sensitivity and parametrisation of NJL like model for the prediction of the chiral Critical End Point

Also, can dense and hot quark matter be constrained by compact star phenomenology ?

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Motivation: Hot and Dense QCD

Study of the hot and dense phases of QCD with a quark effective model, at equilibrium, with a focus on the chiral transition.

Dense phase: Very few experimental and theoretical knowledge (non-perturbative finite density properties difficult to access in QCD or Lattice QCD) ; Different critical properties (first order) than zero density ; Compact (neutron) star phenomenology.

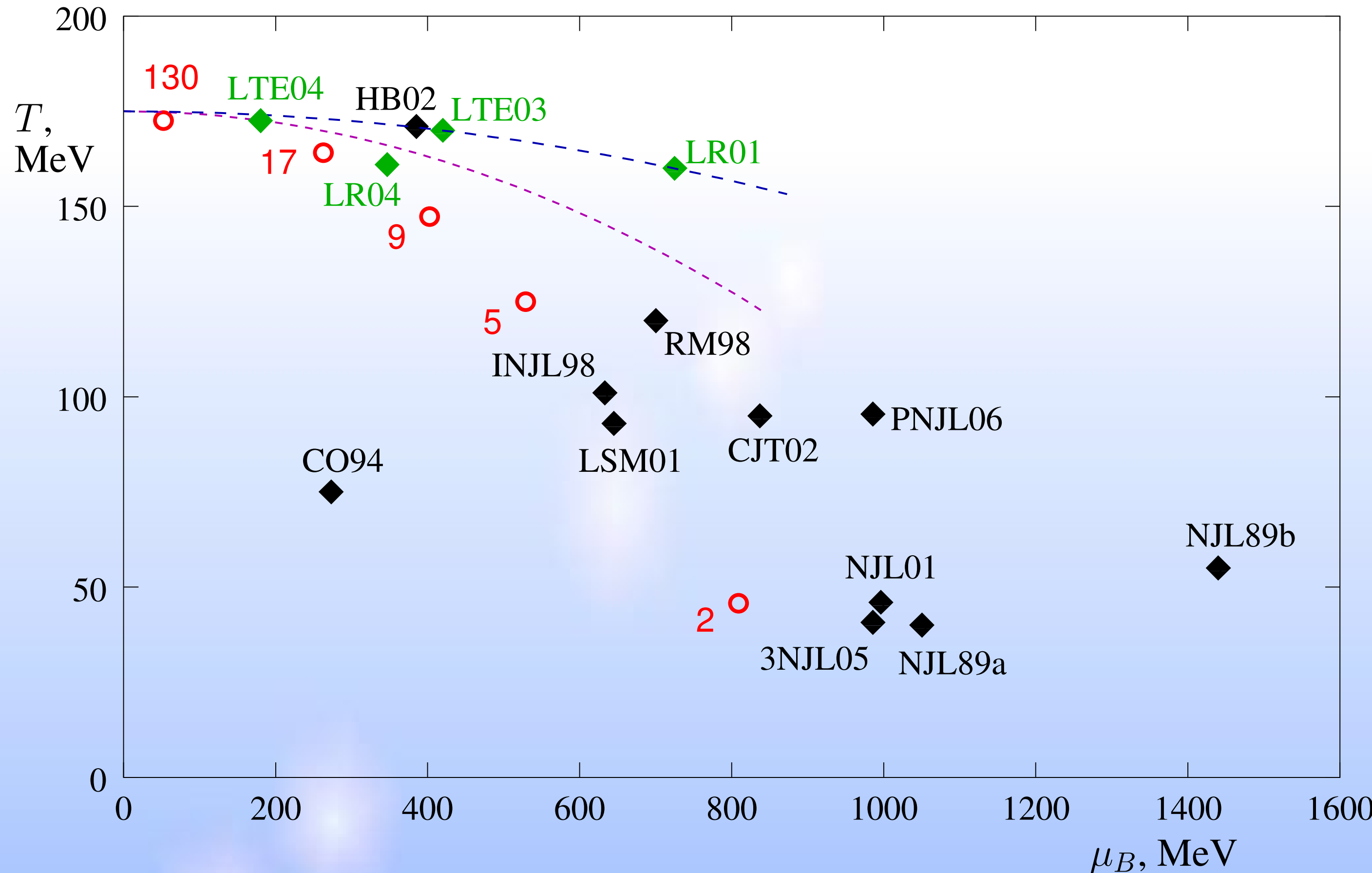
Equilibrium properties: First step before understanding out of equilibrium ; can be an input for transport code based on local thermal equilibrium ; quark matter at equilibrium may exists in core of compact stars.

Chiral physics: Chiral symmetry governs important properties of hadronic physics (e.g. in the low mass region pion $\leftrightarrow$ nucleon-nucleon interaction, rho meson ; CEP, etc.)

Effective models

- Basically, model provides an extrapolation (based on some QCD ingredients, not a polynomial expansion) from known inputs to some predictions.
- calculation of phases and critical properties ; mesonic fluctuations description ; provide microscopic predictions (cross sections, viscosity, etc) ; microscopic mechanism related to QCD (chiral symmetry breaking, statistical confinement effect, etc.) ;
- can then be used as input for transport, compact star, etc.

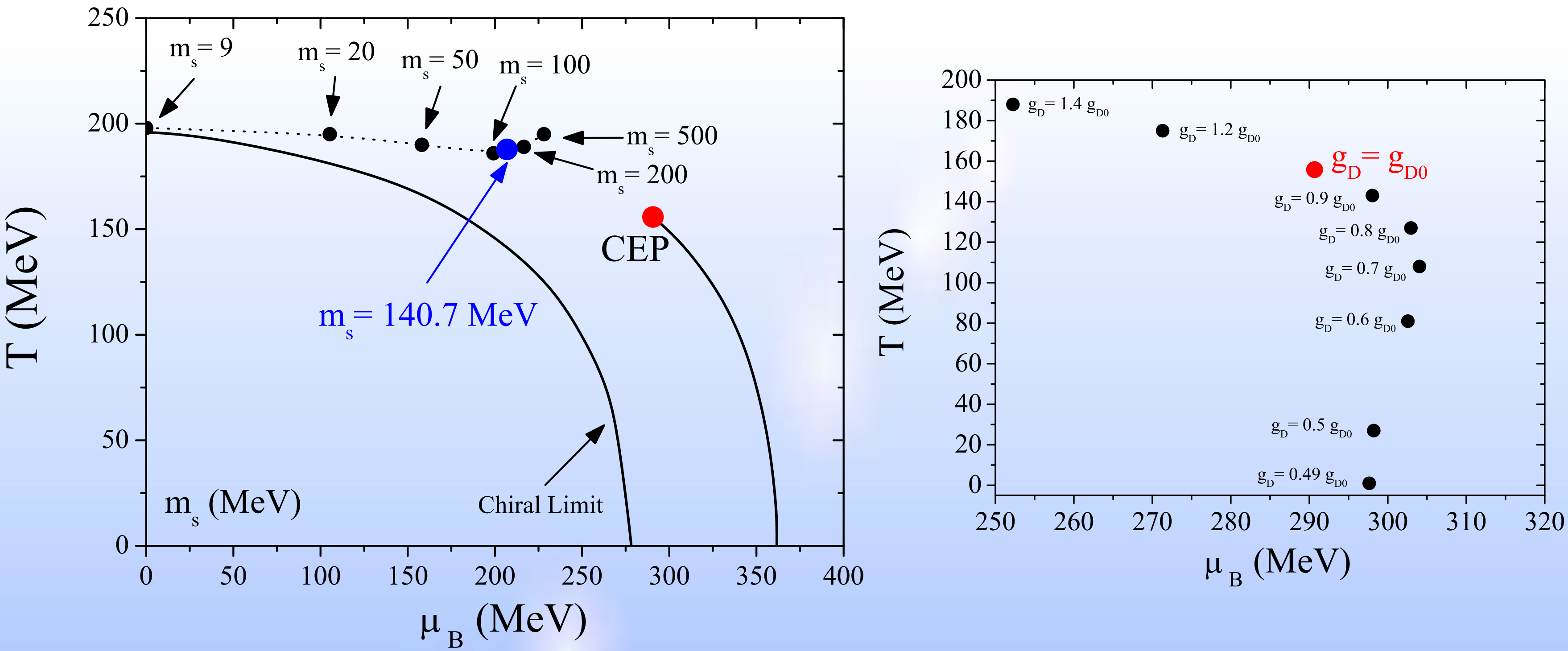
Predictive power of models, the case of the chiral CEP



Comparison of predictions for the location of the CEP in the (T, μ_B) plane (the baryonic chemical potential $\mu_B = 3\mu_{quark}$). Black points are model predictions, green one are LQCD predictions and red one are freeze-out points measured in HIC. From Stephanov, PoS LAT **2006** (2006) 024.

Different mechanisms, same output on the predictions

CEP with varying strange quark sector properties (mass and t'Hooft flavor mixing): Two microscopic mechanisms: strange quark propagation (m_s) and $U_A(1)$ anomaly (g_D) (P.Costa et al).



CEP position becomes very sensitive to m_s around the physical value of m_s ; also sensitive to the variation of the t'Hooft interaction and in the "same direction". We must look for mechanism correlated to observables that gives an effect on the CEP in the "perpendicular" direction.

We also want to have a systematic way to study those microscopic effects on predictions.

Experimental facts, chiral symmetry breaking and modelisation

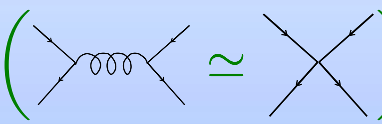
- **Only hadrons in vacuum:** **Quark confinement** in the non-perturbative regime and **asymptotic freedom** (color deconfinement) at higher energy, related to breaking of Z_{N_c} at finite temperature

- **No Wigner realization of the chiral symmetry in vacuum:** **Spontaneous chiral symmetry breaking of $SU_R(3) \times SU_L(3)$ to $SU(3)_V$**
$\Rightarrow$ octet of (almost) **Goldstone** bosons, the off-scale light pseudoscalar octet.

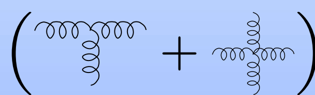
- **η' not of the Goldstone type:** **Adler–Jackiw–Bell $U_A(1)$ anomaly** breaks $U_R(N_f) \times U_L(N_f)$ to $U_V(1) \times SU_R(N_f) \times SU_L(N_f)$ ('t Hooft picture: interaction with instantons changes chirality).

$\Rightarrow$ **the PNJL chiral model** ($q = (q_u, q_d, q_s)$ are the light quark fields) :

$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma_\mu D^\mu - \hat{m})q + \frac{1}{2} g_S \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q}\, i\gamma_5 \lambda^a q)^2]$$



$$+ g_D\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\} - \mathcal{U}(\Phi[A], \bar{\Phi}[A]; T)$$



Rem: baryonic mass generation and chiral symmetry:
Even with zero bare quark mass, if the "quark condensate" $\langle \bar{q}q \rangle \neq 0 \Rightarrow$ generation of a dynamical mass $-2g_S\langle \bar{q}q \rangle$ that breaks spontaneously the chiral symmetry.
$-g_S\langle \bar{q}q \rangle \simeq 330$ MeV $\simeq M_N/3$.
When $\langle \bar{q}q \rangle \to 0$ at finite temperature/density: chiral phase transition.

PART I

Sensitivity of the CEP prediction (exact inverse problem)

A. Biguet, H.H., T. Brugière, P. Costa and P. Borgnat,

"Sensitivity of predictions in an effective model – application to the chiral critical end point position in the Nambu–Jona-Lasinio model,"

Eur. Phys. J. A 51, no. 9, 121 (2015), [arXiv:1409.0990 [hep-ph]]

Parametrisation of the NJL model

"Toy model" two flavors NJL model with scalar interactions:

$$\mathcal{L}_{NJL} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right] .$$

We have one **"a priori"**: the quark scalar-pseudo-scalar sector of the NJL model is relevant to study the chiral properties of QCD. Every conclusions gathered from this model has to be evaluated with respect to this hypothesis.

Three dimension-full "free" parameters (or loosely constrained by phenomenology):

- m_0 the quark mass around the u and d quark masses
- Λ the three-dimensional cutoff of the order of the Λ_{QCD}
- G the coupling constant, $G = g/\Lambda^2$, $g \in [1, 10]$.

(At least) three phenomenological inputs: pion properties + condensate, in vacuum

$$\begin{pmatrix} m_{\pi} \\ f_{\pi} \\ c = -\langle\bar{q}q\rangle^{1/3} \end{pmatrix} = \begin{pmatrix} 137 & \mathrm{MeV} \\ 93.0 & \mathrm{MeV} \\ 315 & \mathrm{MeV} \end{pmatrix} .$$

The inverse problem

NB: here, model $\equiv$ Lagrangian + approximations + parametrisation procedure.

- **Direct problem** $\boxed{\Lambda, m_0, G \Rightarrow m_\sigma, m_\pi, f_\pi, c, \text{CEP}}$ (or other predictions)

- **Inverse problem** $\boxed{m_\pi, f_\pi, c \Rightarrow \Lambda, m_0, G}$

For example one can minimize a merit function as a χ^2.
Remark: the value of the χ^2 is important to quantify the quality of the fit but the shape of the function (very flat or very narrow) is also an important information concerning the robustness of the fit. We will indirectly get an access to this information.

- **Here, exact inverse problem** with Hartree + Ring + Quasi-Goldstone approximation.

Luckily, with physical values for the inputs $\Rightarrow$ unique physical solution (inverse problem well posed).

Sensitivity and ill-posedness of a problem

Concern in general: parametrisation in vacuum ⇒ prediction in medium (position of the CEP) but how good is this **extrapolation** ?
Is the inverse problem **ill-posed** (no solution, no unique solution) ?

Previous studies:

How different physical sectors **qualitatively** affect the CEP position ?
To do this ⇒ variation of the parameters (thus destroying the vacuum phenomenology).

Goals: Systematic study of the variations of the whole parameter space compatible with the "true" inputs of the model (m_π, f_π, c) and assessment of the sensitivity of the extrapolation with a **quantitative** criterium.

⇒ introduction of a sensitivity coefficient with respect to the inputs.

Sensitivity definition

Infinitesimal sensitivity of a prediction based on the (statistical) propagation of an uncertainty:

Let X be a prediction depending on two inputs a and b.
Standard deviation of X (where $\sigma(a)$ and $\sigma(b)$ are deviations for the inputs with some distribution):

$$\sigma^2(X) = \left(\frac{\partial X}{\partial a}\right)^2 \sigma^2(a) + \left(\frac{\partial X}{\partial b}\right)^2 \sigma^2(b) \; .$$

Sensitivity:

$$\Sigma(X) = \lim_{\sigma \to 0} \frac{\sigma_{rel}(X)}{\sigma^I_{rel}}$$

where,

$$\sigma_{rel}(X) = \frac{\sigma(X)}{X} \quad (1)$$

$$\sigma^I_{rel} = \frac{1}{2}\left(\frac{\sigma(a)}{a} + \frac{\sigma(b)}{b}\right) \; , \quad (2)$$

and $\lim_{\sigma \to 0}$ means we take infinitesimal variations of the inputs.

In the NJL model

We choose vanishing *relative dispersion* of the inputs namely for $I = a$ or b, $\sigma(I)/I = p$ and $p \to 0$:

$$\Sigma(X) = \sqrt{\left(\frac{\partial X}{\partial m_\pi}\right)^2 \frac{m_\pi^2}{X^2} + \left(\frac{\partial X}{\partial f_\pi}\right)^2 \frac{f_\pi^2}{X^2} + \left(\frac{\partial X}{\partial c}\right)^2 \frac{c^2}{X^2}}\,.$$

We choose a uniform distribution (no a priori) for the inputs (and check that the results does not depend on this choice).

Sensitivity meaning

- Large (infinite) sensitivity $\Rightarrow$ the extrapolation is very sensitive, the predictive power is low.
  Any small but finite errors in the inputs (experimental errors as for the condensate or theoretical systematic errors because of the approximations) will damaged the prediction.
- Small sensitivity $\Rightarrow$ the prediction is robust and can be trusted **if the model itself can be trusted.**

Temperature and chemical potential CEP sensitivities

NB: parameter sensitivities small (around 3). At least the inverse problem is well-posed and one can use the model to compute predictions.

	Sensitivities		Values
Parameters	Λ	2.83	0.653 (GeV)
	m_0	4.11	0.0051 (GeV)
	$G\Lambda^2$	3.32	2.11
In-medium predictions	T_{CEP}	71.5	0.0299 (GeV)
	μ_{CEP}	1.05	0.327 (GeV)

Table 1: Sensitivities of the parameters and in-medium predictions considering infinitesimal changes of the inputs. The sensitivities of the parameters and of μ_{CEP} are close to 1. The sensitivity of the temperature coordinate of the CEP is very large. These values were computed numerically with a Monte-Carlo.

- For T_{CEP}, $\Sigma \simeq 70$: no consistent conclusion can be given within the model. Even the existence of the CEP may be questioned.
- For μ_{CEP}, $\Sigma \simeq 1$ (even lower than the parameters !).

Consequences of small but finite deviations of the inputs

Why finite variations are relevant ?

Mainly for illustration of the effect, correlations, etc, but also:

- Obvious for observables **not very well known experimentally** (as for the quark condensate).
- For the pion: experimental value very well known but **unknown systematic errors**.

1. NJL is an effective (uncontroled) model of QCD. Only our "a priori" of the correctness of the model (symmetries for example) tells us we can use it.
2. Approximations within the NJL model:
   Quasi-Goldstone approximation $\Rightarrow$ 1% of variation without it.
   Next order in $1/N_c$ (meson loop approximation) $\Rightarrow$ 5%.

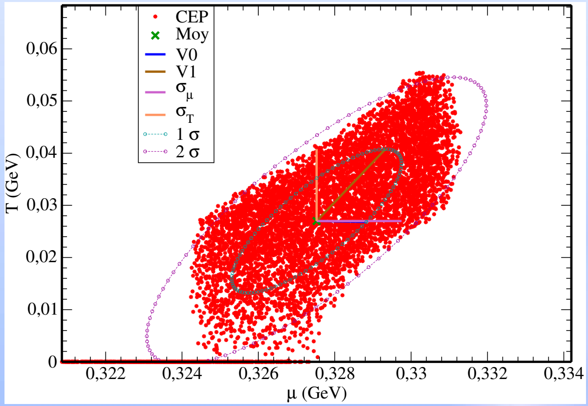
Hence it seems unreasonable to expect than the inverse problem as an accuracy better than 1%.

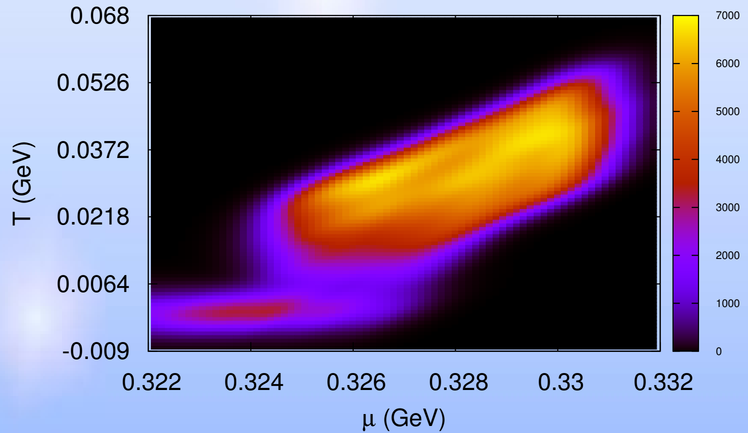
CEP unpredictability

We allow finite variation with relative dispersion $p = 1\%$ to compute the sensitivities:

$$
\begin{array}{cccccc}
m_\pi & \in & [& 135.6 & , & 138.4 &] \\
f_\pi & \in & [& 92.07 & , & 93.93 &] \\
\langle \bar{q}q \rangle^{1/3} & \in & [& -318.1 & , & -311.8 &]
\end{array}
\Rightarrow
$$

$\overline{T_{\text{CEP}}}$	0.0303 (GeV)
$\sigma(T_{\text{CEP}})$	0.0107 (GeV) (biased)
$\sigma(T_{\text{CEP}})/\overline{T_{\text{CEP}}}$	35.25 (%)
$\overline{\mu_{\text{CEP}}}$	0.3280 (GeV)
$\sigma(\mu_{\text{CEP}})$	0.0018 (GeV)
$\sigma(\mu_{\text{CEP}})/\overline{\mu_{\text{CEP}}}$	0.54 (%)

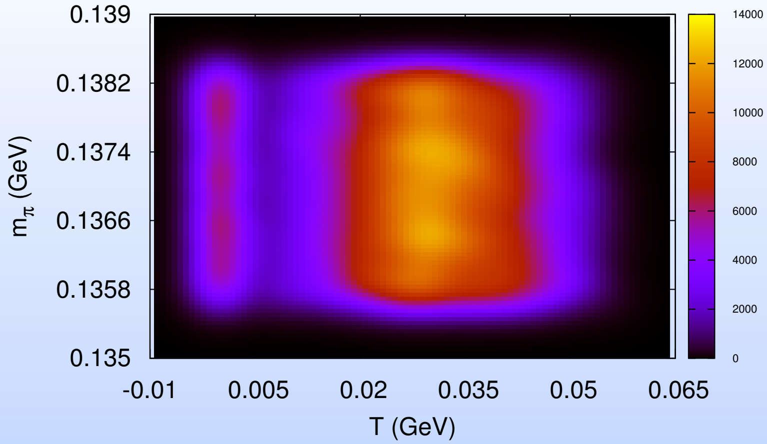


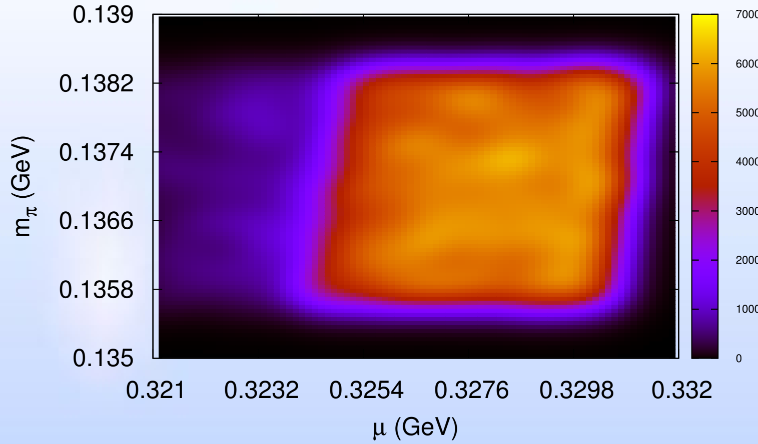


- Striking dispersion in the T direction.
- With only a 1% variation of the inputs, in 10% of the case there is no CEP ! There is no mechanism in this model that ensures the CEP must exists.
- Very stable chemical potential prediction.

Correlations

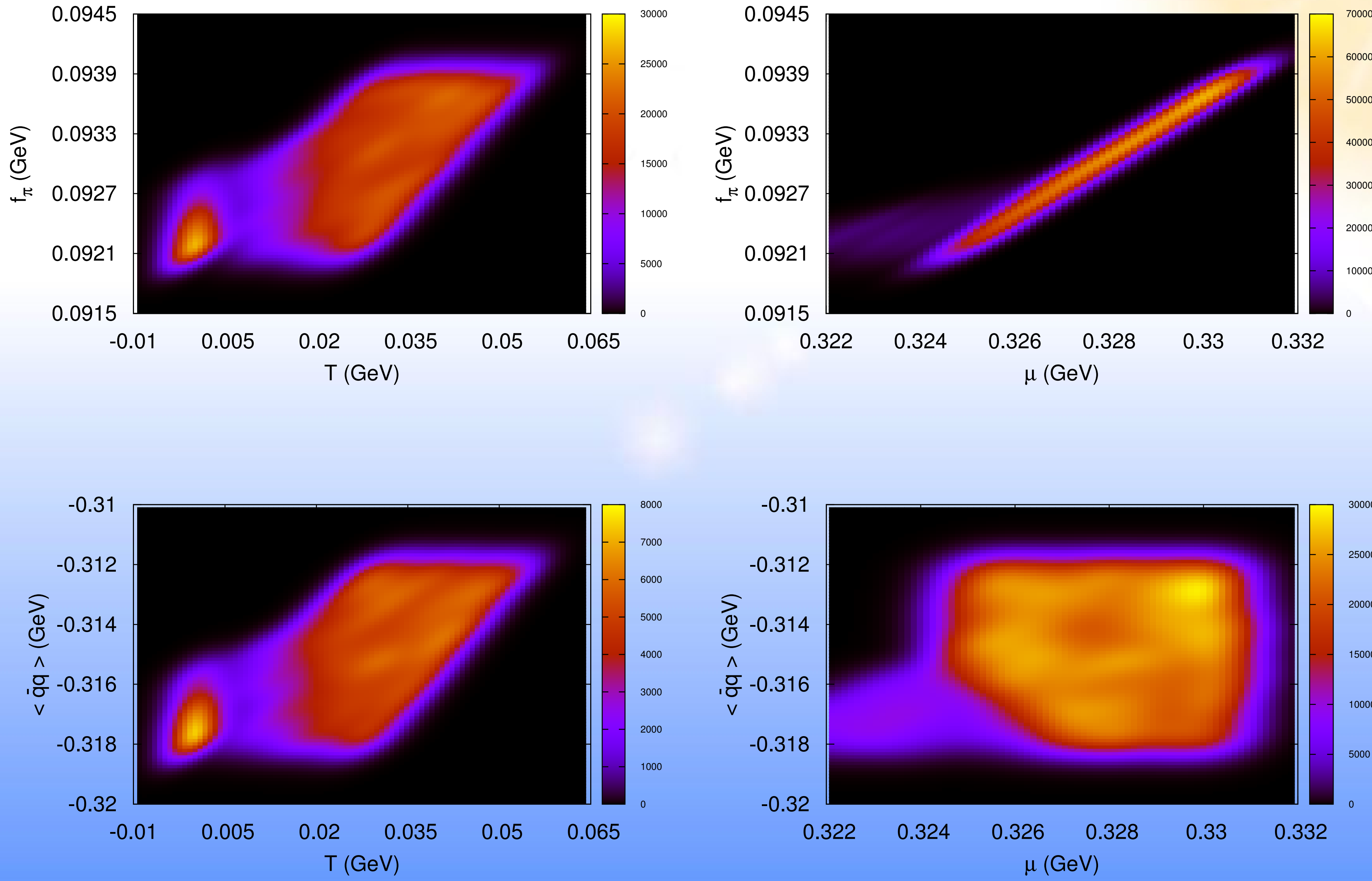
Correlation of the temperature (left) or the chemical potential (right) with the inputs. Uniform distribution, $\boldsymbol{p = 1\%}$ and $N = 20^3$. To represent the correlations we have done a scatter plot of the two datasets then reconstructed the density of points with the KDE algorithm. The color coded z-axis is then in GeV^{-2}.





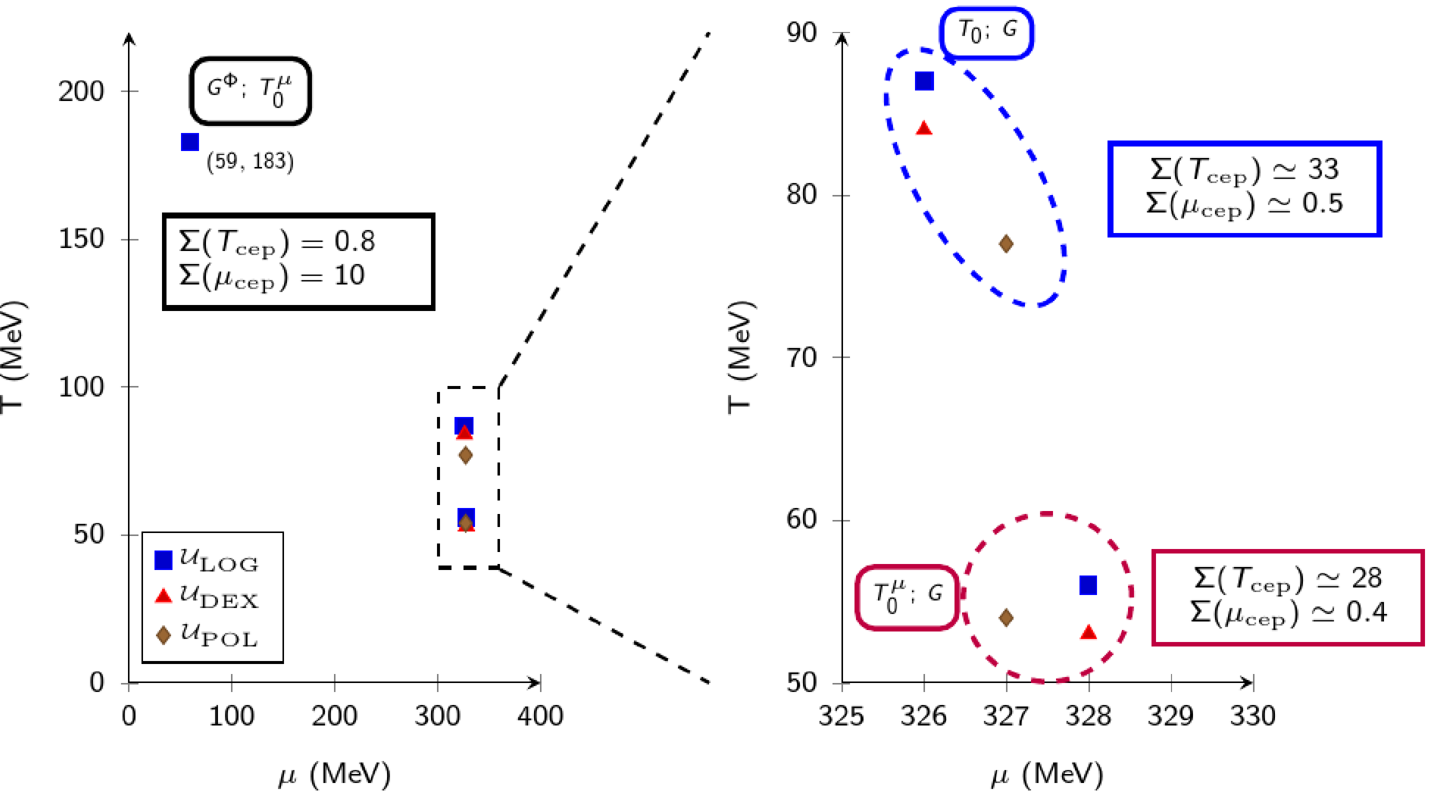
Correlation coefficients between the inputs or the model parameters with the temperature and the chemical potential at CEP with uniform distribution, $\boldsymbol{p = 0.25\%}$ (converged), and $n = 20^3$.

Correlations of		T_{CEP}	μ_{CEP}
with inputs	m_π	0.033	0.098
	f_π	0.725	0.985
	$\langle \bar{q}q \rangle$	0.677	0.117
with parameters	m_0	0.845	0.629
	Λ	0.960	0.612
	$G\Lambda^2$	0.998	0.792



0.0945
0.0939
0.0933
0.0927
0.0921
0.0915
f_π (GeV)
-0.01
0.005
0.02
0.035
0.05
0.065
T (GeV)
30000
25000
20000
15000
10000
5000
0
0.322
0.324
0.326
0.328
0.33
0.332
μ (GeV)
70000
60000
50000
40000
30000
20000
10000
-0.31
-0.312
-0.314
-0.316
-0.318
-0.32
$<\bar{q}q>$ (GeV)
8000
7000
6000
5000
4000
3000
2000
1000

Sensitivity with finite temperature constraints



$\rightarrow$ G^Φ et T_0 : Pas de CEP $\forall\ \mathcal{U}$
$\rightarrow$ G et T_0 ou T_0^μ : CEP instable vs. inputs mais stable vs. $\mathcal{U}$
$\rightarrow$ G^Φ et T_0^μ : CEP stable vs. inputs mais instable vs. $\mathcal{U}$

$\Sigma(T_{\text{CEP}})_{\text{PNJL}} < \Sigma(T_{\text{CEP}})_{\text{NJL}}$: Contraintes LQCD $T \neq 0$ et $\mu = 0$

$\rightarrow$ More constraints helps.

$\rightarrow$ $G(\Phi)$ surprisingly stable but:

- the form of $G(\Phi)$ (or even its relevance) is not well established, there is no good inputs to fix its coefficient. The a priori one can have on $G(\Phi)$ is not very good (in particular, no microscopic explanation).
- CEP is unstable vs. the potential choice (CEP do not exist in some cases !).

$\Rightarrow$ more systematic approach to the Polyakov loop models: using all data we have and study the differences with different form of the potential.

PART II

Parametrisation of the NJL model based on vacuum data

Nicolas Baillot, HH, Rainer Stiele, Pedro Costa

Overconstrained parametrisation based on vacuum observables

For the moment: parametrisation from vacuum observables (and effect on the CEP prediction) for an **exact inverse problem (same number of inputs and parameters)** in a toy model (but interesting for the understanding of chiral physics).

$\Rightarrow$ **Now more inputs than parameters (over-constrained inverse problem).** Requires minimizing a merit function (e.g. χ^2) and we use Monte-Carlo with Markov chain (MCMC $\rightarrow$ we get the posterior probabilities).
The inputs may be:

- mesonic mass spectrum only (NJL or QM at this approximation and without diquarks does not really describe well the hadronic spectrum)
- mesonic decay constants
- the quark condensate

Our main knowledge about the NJL model is that it is **good at reproducing pseudo-scalar pseudo-Golsdtone boson properties and also generate the correct chiral physics.**
$\Rightarrow$ we must have as inputs pseudo-scalar Goldstone bosons but also some quantities related to the chiral condensates scale (pseudo-Goldstone bosons masses $\ll$ quark masses so they cannot constraint well this scale).
This prior will be reflected in the choice of our inputs. For example in the exact inverse problem we directly take as an input the chiral condensate (the sigma and scalar mesons could do but not really well described by the NJL model).

What can we learn from data: Bayes theorem

Very schematically:

- On the one hand: theory (ab initio calculation): theory $\Rightarrow$ predictions.
- On the other hand: data $\Rightarrow$ observables, hopefully mostly model independant (e.g. hadronic spectrum $\Rightarrow$ condensate via sum rules, extraction of the temperature from HIC data, etc).

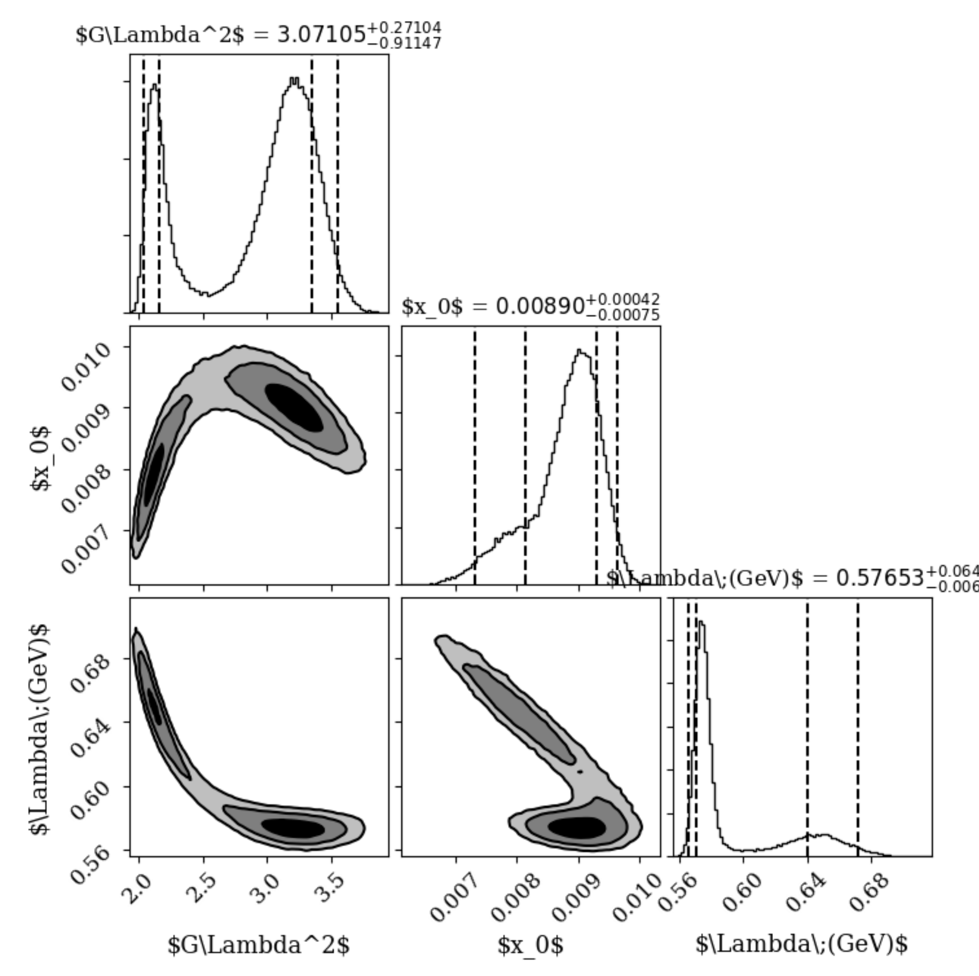
Meeting in the middle ?

$\Rightarrow$ build posterior probability from data and model i.e. constraints on the model based on a prior estimate. The uncertainty in the constraints propagate to uncertainty in the prediction.

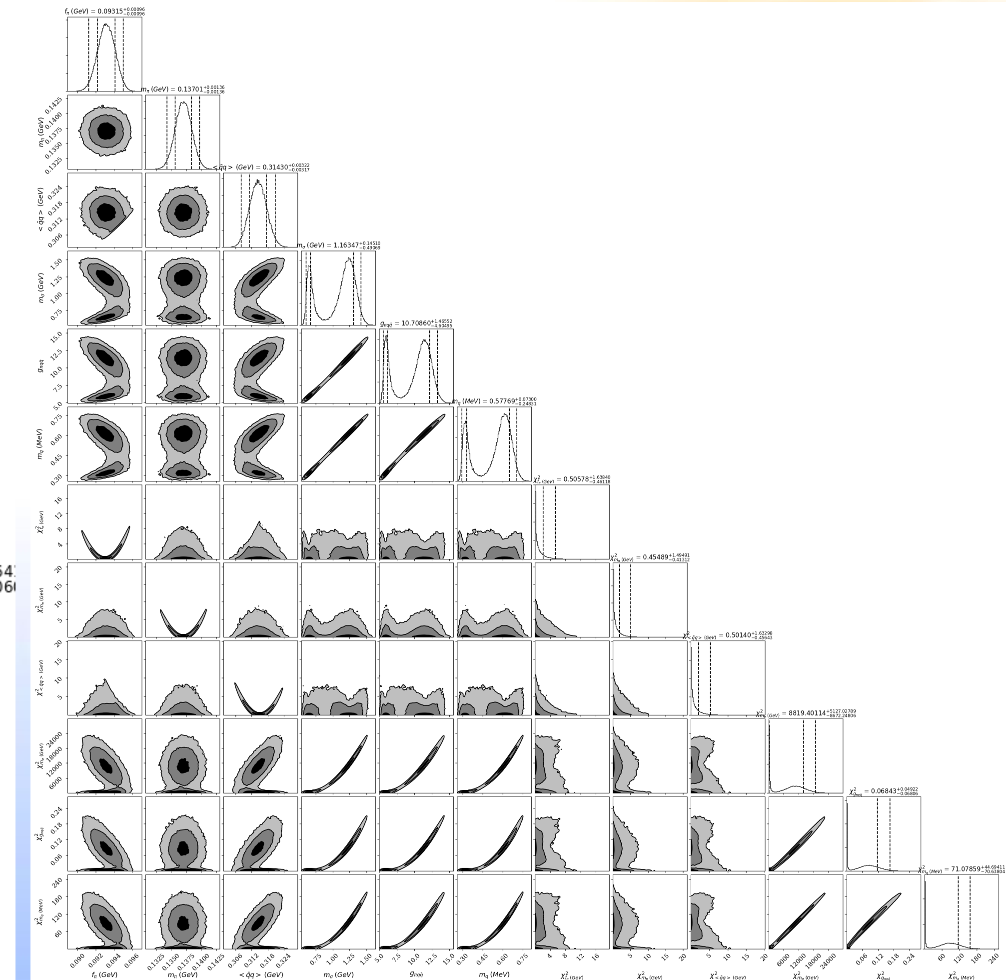
Illustration of the method: $SU(2)$ exactly constrained

Exact inverse problem phenomenologic inputs:

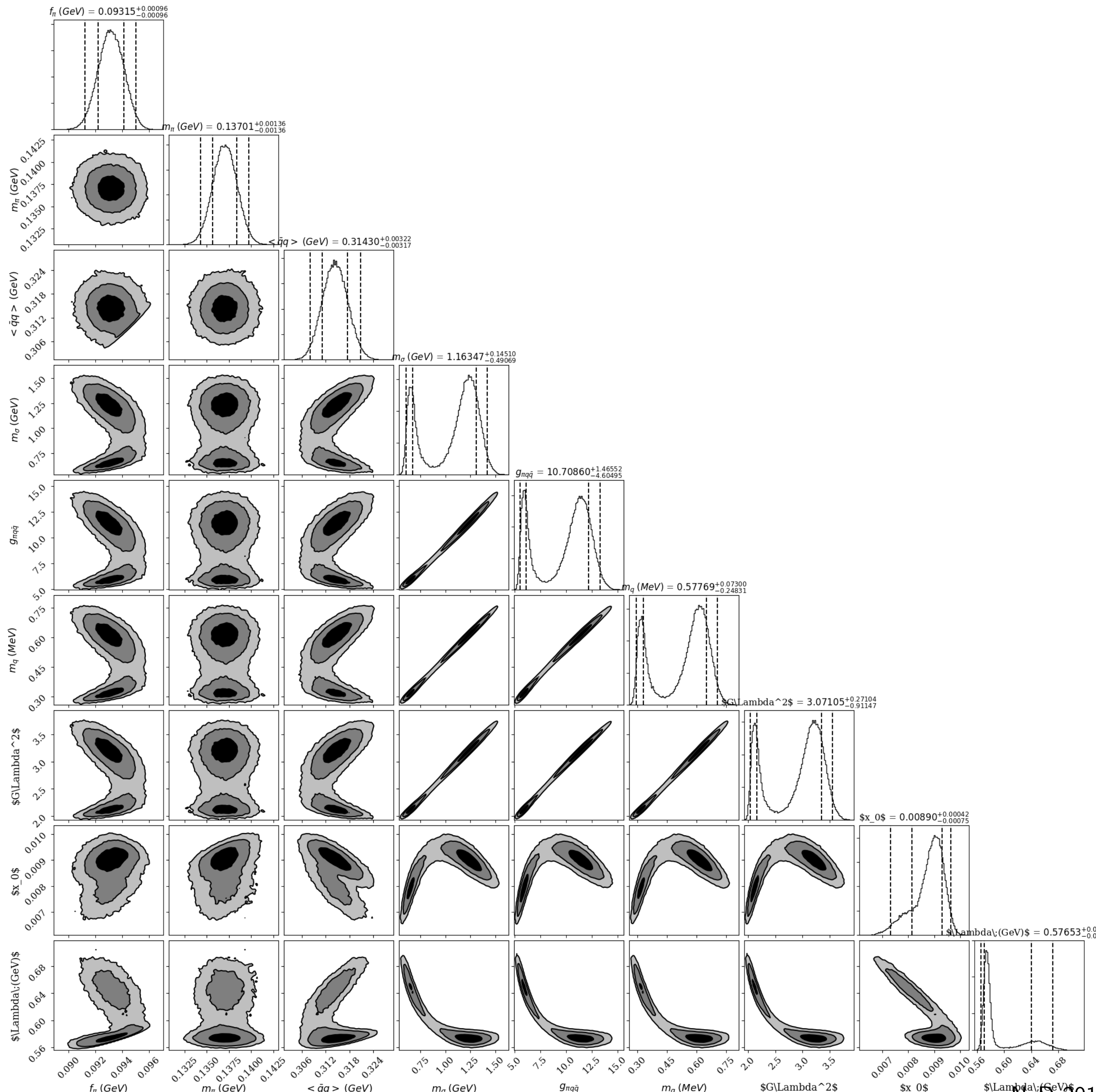
Constraint	central	standard deviation
$f_\pi \ (GeV)$	0.093	0.01
$m_\pi \ (GeV)$	0.137	0.01
$< \bar{q}q > \ \ (GeV)$	0.316	0.02



Parameters



Predictions/χ^2



Predictions / parameters Notice how the input phenomenology is perfectly reproduced **but the sigma prediction** is bimodal with a second peak at 1.2 GeV.

Illustration of the method: $SU(2)$ exactly constrained

Main features:

data	expected	obtained	χ^2	χ^2_{min}
$f_\pi\ (GeV)$	0.093$\pm$0.01%	[0.09 : 0.09]	[0.04 : 2.14]	0.0
$m_\pi\ (GeV)$	0.137$\pm$0.01%	[0.14 : 0.14]	[0.04 : 1.95]	0.0
$< \bar{q}q >\ (GeV)$	0.315$\pm$0.01%	[0.31 : 0.32]	[0.04 : 2.13]	0.0
$m_\sigma\ (GeV)$	0.6$\pm$0.01%	[0.67 : 1.31]	[147.15 : 13946.43]	0.0
$g_{\pi q \bar{q}}$	6.0$\pm$3.0%	[6.1 : 12.17]	[0.0 : 0.12]	0.0
$m_q\ (MeV)$	0.313436$\pm$0.1%	[0.33 : 0.65]	[0.44 : 115.77]	0.0

- The χ^2 is perfectly converged (diagonal plots inputs / χ^2)
- We see that the inverse problem is exact. We have a prior on the inputs (1%) and the MCMC find $\chi^2 = 0$. The standard deviation of the parameter is in fact zero.
- There is two minimum of the χ^2. The MCMC has found a problem in the model: there is an unphysical solution with $m \simeq \Lambda$ (m should be $\simeq \Lambda/2$).
- Correlations can be seen and understood e.g. x_0 with m_π (m_0 fixes the mass of the approximate Goldstone boson π).

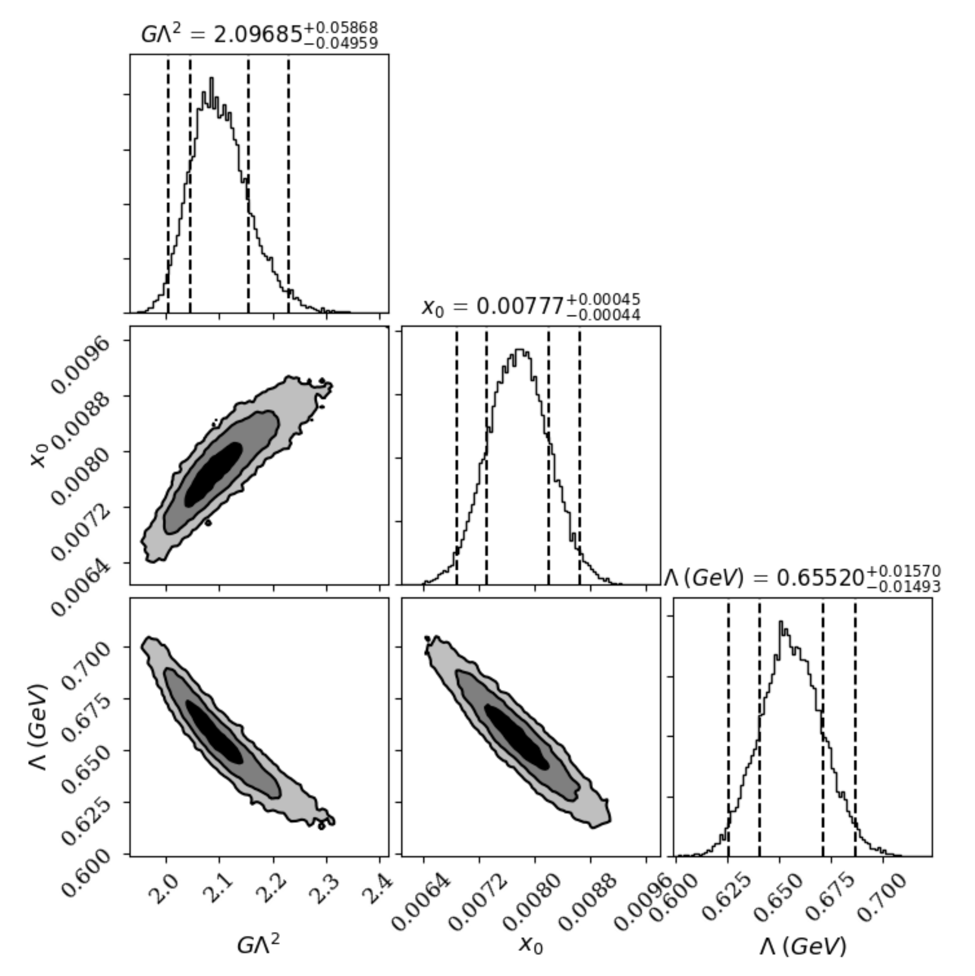
The non-unicity of minimum of the χ^2 is a problem because it means a bifurcation of the prediction of the model that looses its predictive power.

Either the model really have two parametrisations giving sensible, physical results or ...

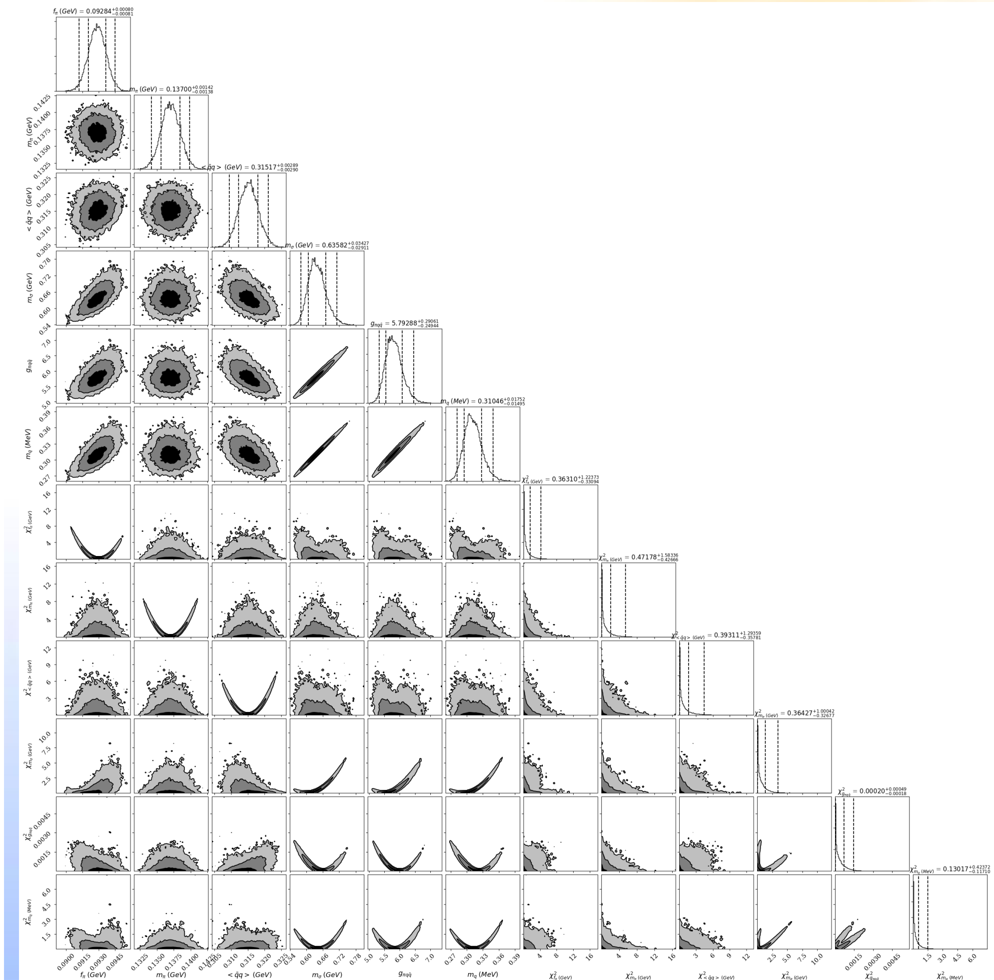
Illustration of the method: $SU(2)$ with a physical constraint

In fact the two sets of solution to the inverse problem is known (one is unstable) and more prior can remove this solution.

We add to our prior knowledge that albeit the sigma is not well describe by the model, it should be around 600 MeV but with a large uncertainty $\simeq$ 100 MeV (or it is equivalent to ask for the mass m to be around $M_N/3 \simeq 300$ MeV).



Parameters



Predictions/χ^2

One can see that everybody is very well constrained, even m_0 and this will not be the case in $SU(3)$.

One can read the accepted parametrisation but also consider correlations e.g. m_{quarks} with m_σ (of course in this simple case it was already known) but other are less obvious ($g_{\pi qq}/m_\sigma$?)

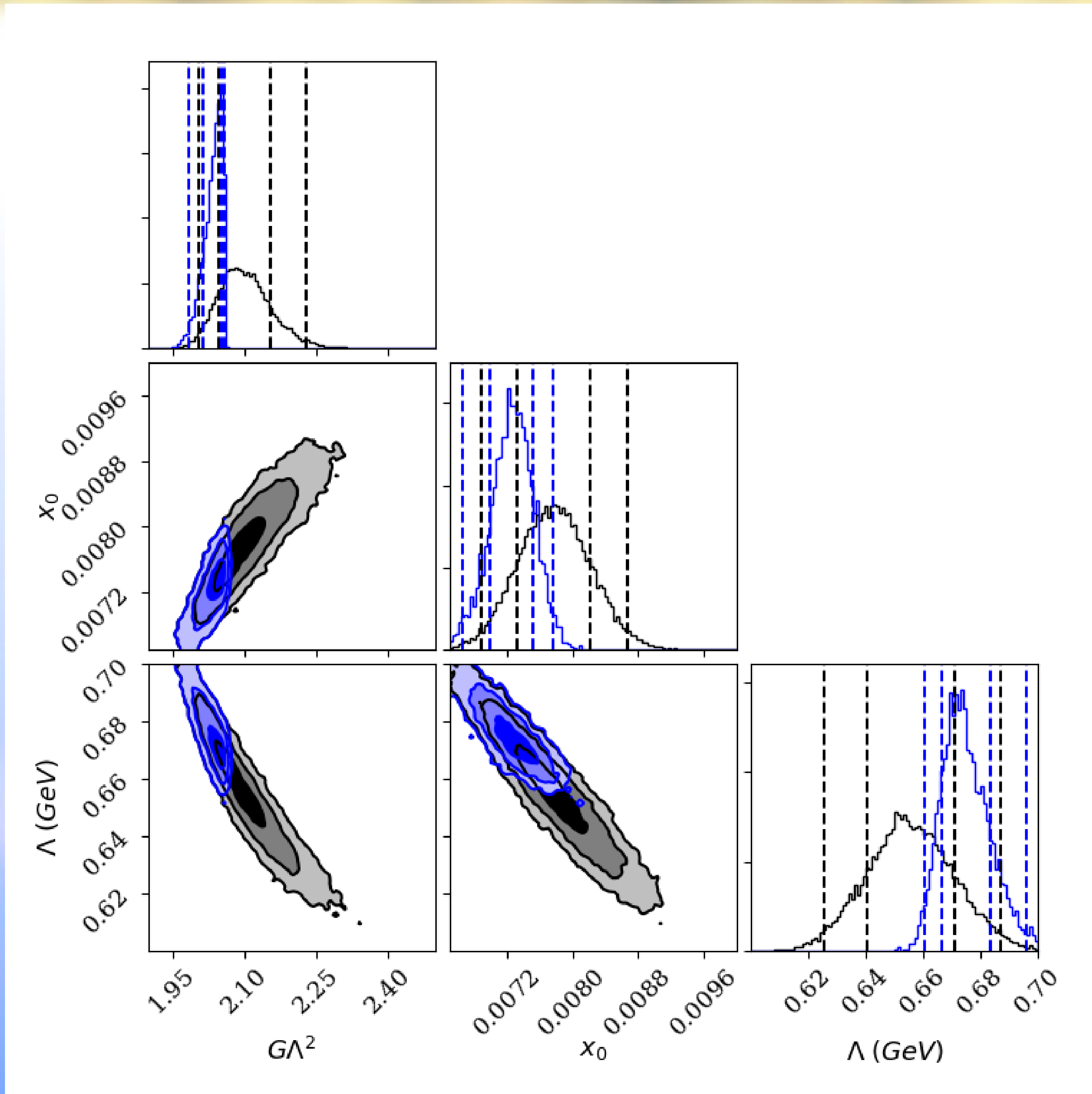
Illustration of the method: $SU(2)$ with a physical constraint

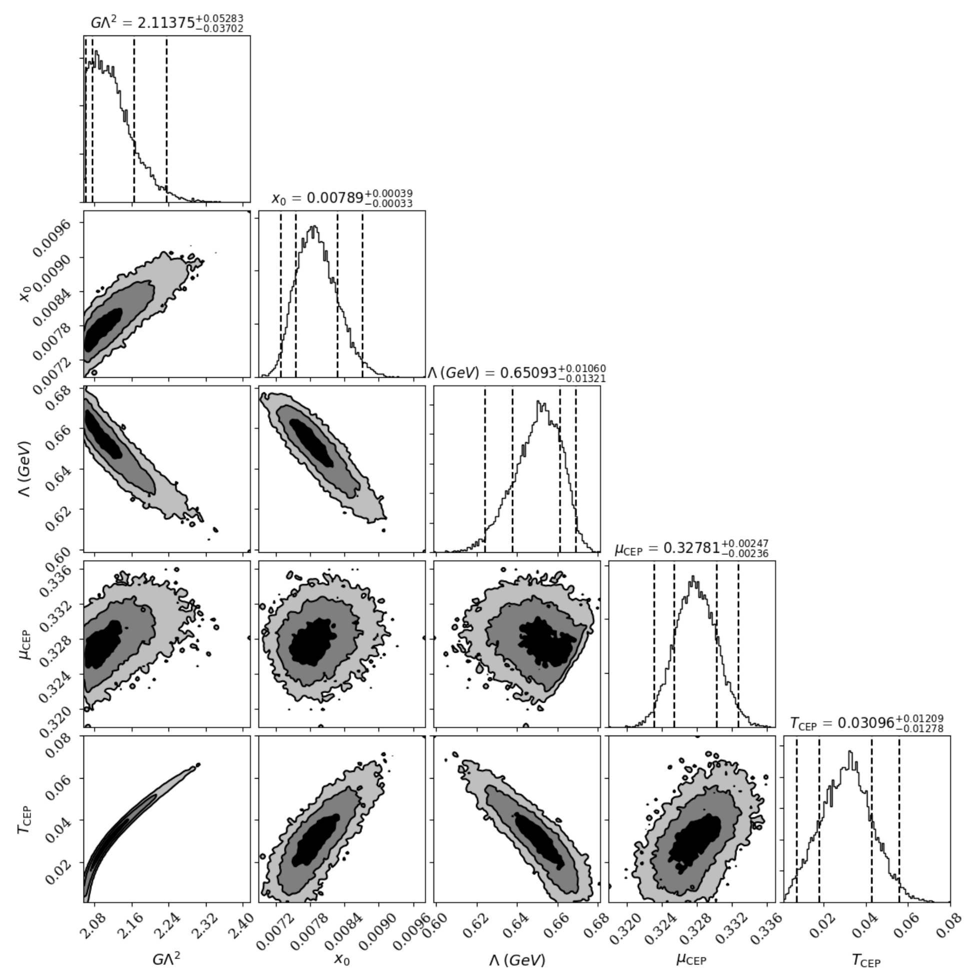
parameter	prior	posterior
$G\Lambda^2$	[0.0 : 15.0]	[2.05 : 2.16]
x_0	[0.0 : 0.1]	[0.01 : 0.01]
$\Lambda\ (GeV)$	[0.3 : 2.0]	[0.64 : 0.67]

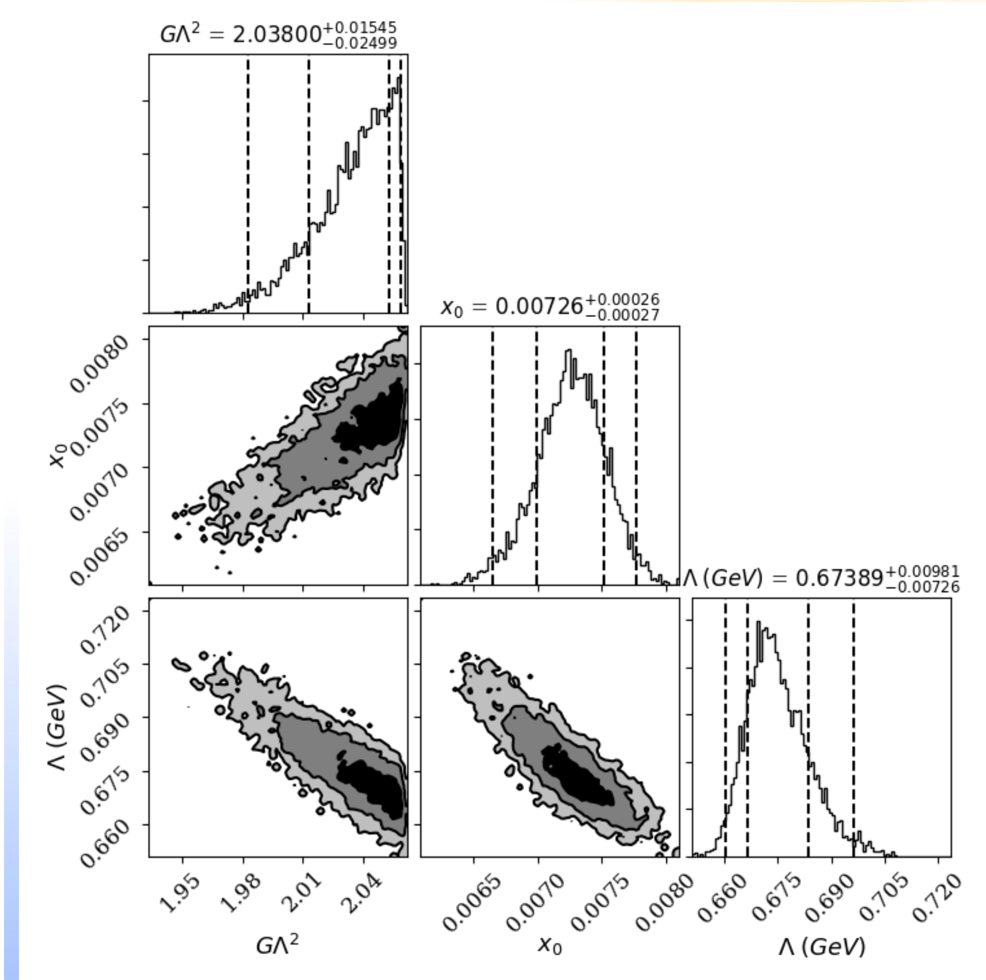
data	expected	obtained	χ^2	χ^2_{min}
$f_\pi\ (GeV)$	0.093$\pm 0.01\%$	[0.09 : 0.09]	[0.03 : 1.59]	0.0
$m_\pi\ (GeV)$	0.137$\pm 0.01\%$	[0.14 : 0.14]	[0.05 : 2.06]	0.0
$< \bar{q}q >\ (GeV)$	0.315$\pm 0.01\%$	[0.31 : 0.32]	[0.04 : 1.69]	0.0
$m_\sigma\ (GeV)$	0.6$\pm 0.1\%$	[0.61 : 0.67]	[0.04 : 1.36]	0.0
$g_{\pi q\bar{q}}$	6.0$\pm 3.0\%$	[5.54 : 6.08]	[0.0 : 0.0]	0.0
$m_q\ (MeV)$	0.313436$\pm 0.1\%$	[0.3 : 0.33]	[0.01 : 0.55]	0.0

Illustration of the method: $SU(2)$ CEP prediction

Separation of the parameter space between sets with or without CEP (blue).







Parameters / CEP

Parameters / No CEP

Notice the correlations and very large uncertainty on the CEP

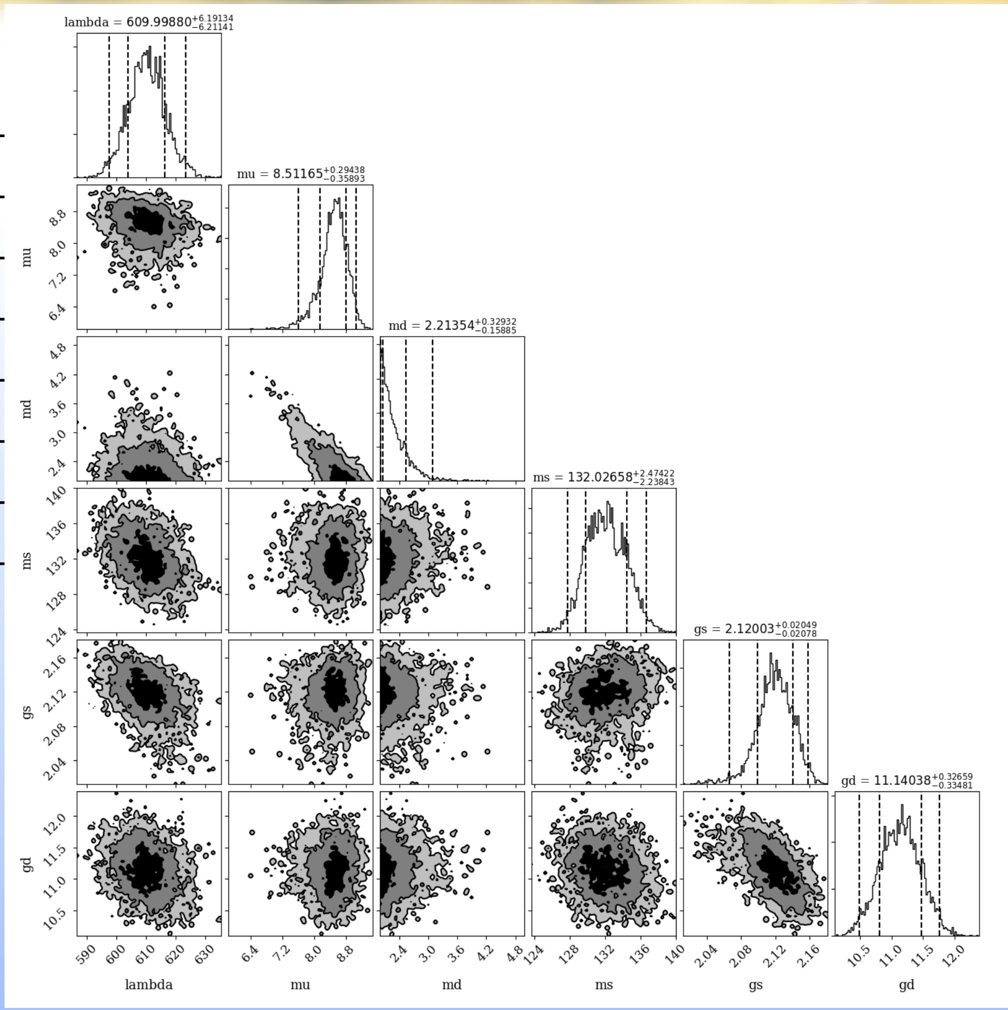
Parametrisation based on vacuum SU(3) full mesonic spectrum

Preliminary

parameter	prior	posterior
lambda	[300.0 : 2000.0]	[603.79 : 616.19]
mu	[2.0 : 10.0]	[8.15 : 8.81]
md	[2.0 : 10.0]	[2.05 : 2.54]
ms	[100.0 : 180.0]	[129.79 : 134.5]
gs	[0.0 : 3.0]	[2.1 : 2.14]
gd	[10.0 : 15.0]	[10.81 : 11.47]

Parametrisation seems fine but in fact large χ^2 all over the place.

Globally bad fit **but** pseudoscalars are better than the scalar and this information was not given to the MCMC. The MCMC discovers (as we know) that NJL model do not handle well the large splitting of mass in the scalar sector.



data	expected	obtained	χ^2	χ^2_{min}
m_u	367.0$\pm0.01\%$	[458.91 : 466.28]	[627.21 : 731.73]	389.39
m_d	367.0$\pm0.01\%$	[450.18 : 457.42]	[513.72 : 607.08]	310.71
m_s	549.0$\pm0.01\%$	[620.87 : 628.94]	[171.4 : 212.05]	99.15
uu	241.0$\pm0.01\%$	[252.03 : 256.44]	[20.94 : 41.04]	4.51
dd	241.0$\pm0.01\%$	[251.26 : 255.63]	[18.13 : 36.86]	3.22
ss	257.0$\pm0.01\%$	[262.28 : 267.02]	[4.22 : 15.2]	0.0
$m_\pi *$	134.97$\pm0.01\%$	[134.52 : 137.12]	[0.06 : 2.67]	0.0
$f_p i*$	93.0$\pm0.01\%$	[95.92 : 97.62]	[9.85 : 24.65]	0.27
$m_\sigma *$	475.0$\pm0.2\%$	[903.52 : 919.05]	[20.35 : 21.85]	16.87
θ_s	16.0$\pm0.01\%$	[16.85 : 17.92]	[28.02 : 143.71]	0.0
$m_K *$	497.6$\pm0.01\%$	[495.3 : 503.13]	[0.03 : 1.42]	0.0
$f_K *$	113.0$\pm0.02\%$	[97.47 : 99.38]	[36.34 : 47.24]	22.48
$m_\kappa *$	1430.0$\pm0.01\%$	[1183.45 : 1194.32]	[271.63 : 297.26]	250.48
$m_\eta *$	547.9$\pm0.01\%$	[515.01 : 520.57]	[24.87 : 36.04]	12.31
G_{qq}	2.0$\pm0.01\%$	[2.99 : 3.08]	[2433.65 : 2920.74]	1565.37
G_{ss}	-3.0$\pm0.01\%$	[-4.84 : -4.69]	[3163.76 : 3750.87]	2402.41
θ_p	-5.7$\pm0.01\%$	[-7.26 : -6.18]	[71.55 : 744.74]	0.0
$m_{\eta'} *$	957.8$\pm0.01\%$	[983.98 : 1001.08]	[7.47 : 20.42]	0.42

Parametrisation based on vacuum SU(3) pseudoscalar only

Preliminary

parameter	prior	posterior
$lambda$	[300.0 : 2000.0]	[683.93 : 708.87]
m_{ud}	[2.0 : 10.0]	[4.57 : 4.86]
m_s	[100.0 : 180.0]	[129.95 : 135.78]
g_s	[0.0 : 3.0]	[1.52 : 1.57]
g_d	[10.0 : 15.0]	[13.86 : 14.75]

Concerning the χ^2 the extra-diagonal plot show some interesting correlations as the f_π with the three condensates, the f_0 with M_s, the η with η', the η' with M_s.

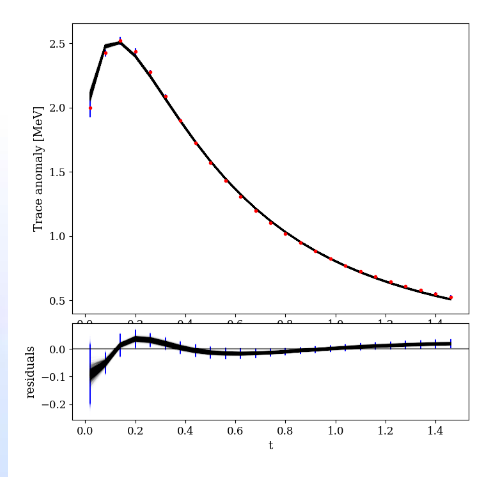
We can confirm or learn new features of the model by examining these correlations. For example for the last one, without the η' that is not a quasi-Golsdtone boson there would be (almost) nothing to fix the strange mass since the strange mass scale in the other pseudo-scalar is mixed with light quark condensate.

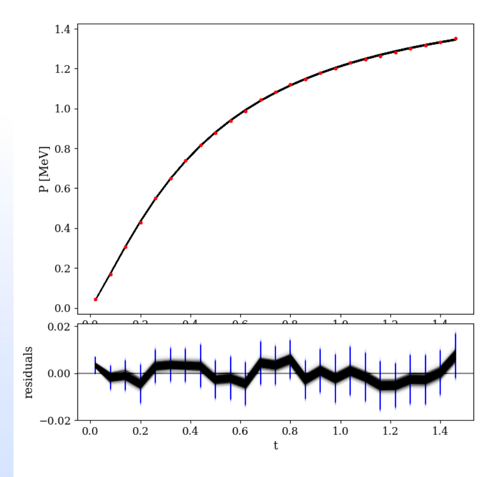
data	expected	obtained	χ^2	χ^2_{min}
m_u	$367.0 \pm 0.01\%$	[299.62 : 309.64]	[244.29 : 337.07]	20.3
m_d	$367.0 \pm 0.01\%$	[299.62 : 309.64]	[244.29 : 337.07]	20.3
m_s	$549.0 \pm 0.01\%$	[482.03 : 494.56]	[98.34 : 148.82]	4.14
uu	$241.0 \pm 0.01\%$	[257.88 : 264.04]	[49.04 : 91.39]	8.86
dd	$241.0 \pm 0.01\%$	[257.88 : 264.04]	[49.04 : 91.39]	8.86
ss	$257.0 \pm 0.01\%$	[282.67 : 290.58]	[99.79 : 170.77]	22.58
$m_\pi *$	$134.97 \pm 0.01\%$	[133.8 : 136.4]	[0.04 : 1.9]	0.0
$f_p i *$	$93.0 \pm 0.01\%$	[94.79 : 96.4]	[3.69 : 13.4]	0.0
m_σ	$475.0 \pm 0.2\%$	[720.0 : 720.0]	[6.65 : 6.65]	1.24
θ_s	$16.0 \pm 0.01\%$	[16.71 : 17.59]	[19.92 : 98.53]	0.0
$m_K *$	$497.6 \pm 0.01\%$	[503.64 : 511.51]	[1.47 : 7.82]	0.0
$f_K *$	$113.0 \pm 0.02\%$	[101.68 : 103.81]	[16.54 : 25.11]	7.91
m_κ	$1430.0 \pm 0.01\%$	[1000.49 : 1012.68]	[851.66 : 902.14]	694.77
$m_\eta *$	$547.9 \pm 0.01\%$	[526.42 : 531.97]	[8.45 : 15.37]	1.28
G_{qq}	$2.0 \pm 0.01\%$	[1.59 : 1.71]	[207.59 : 410.09]	0.0
G_{ss}	$-3.0 \pm 0.01\%$	[-3.04 : -2.85]	[1.03 : 28.9]	0.0
θ_p	$-5.7 \pm 0.01\%$	[-3.71 : -2.63]	[1213.05 : 2892.94]	0.0
$m_{\eta'} *$	$957.8 \pm 0.01\%$	[956.96 : 971.65]	[0.06 : 2.13]	0.0

Parametrisation OK for pseudoscalar but of course, scalars are off.

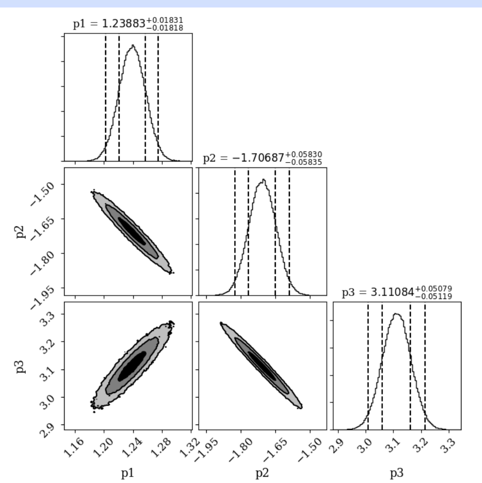
More constraints: finite temperature gauge sector

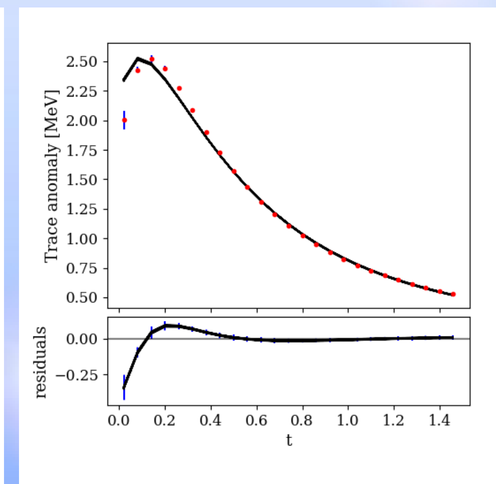
Example: polynomial potential, data from M. Caselle, A. Nada, and M. Panero, Phys. Rev. D 98 no. 5, (2018) 054513)

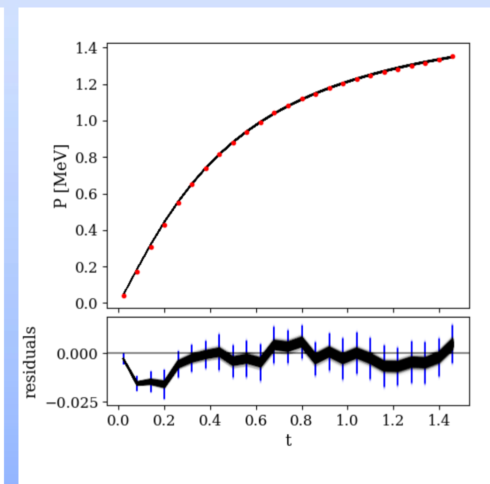


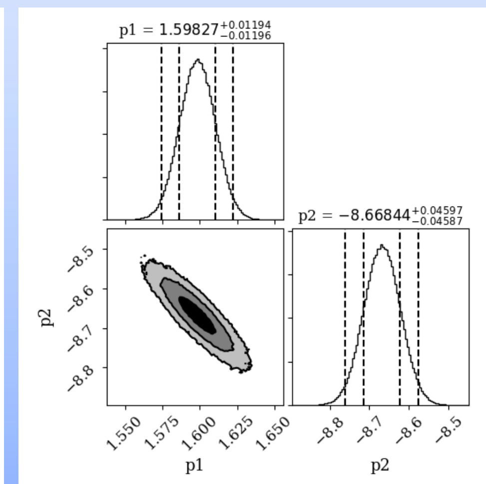


Parametrisation of different Polyakov potential **but with the same pure gauge lattice data** and results with PQM and PNJL model $\Rightarrow$ tentative to be a more systematic in the definition of Polyakov potential and explore **the model space**.





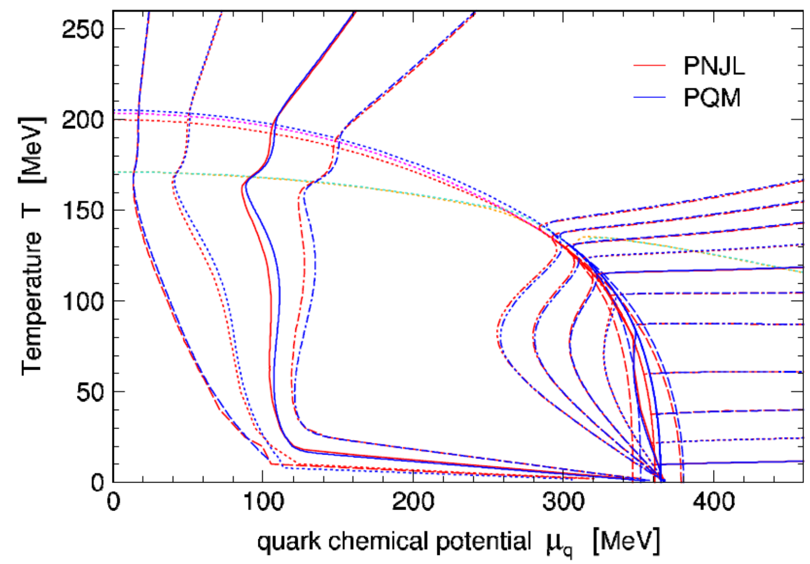




To continue

- We already have results for Wuppertal-Budapest data
- Other parametrisations (log with higher order T-dependent term
- Mixing data from different group together (resampling needed).
- Study what are the feature in the phase diagram (CEP, isentropics) that are stable (same results up to the uncertainties propagated from the vaccuum and pure gauge data) when using different models (namely PNJL and PQM).
- Effects of higher order approximations (beyond mean field) on the stability of features.

Example : isentropics



W. Alberico, A. Beraudo, R. C. Pereira, P. Costa, H. Hansen, M. Motta and RS, *in preparation*

This program is a kind of exploration of the model space of the effective chiral quark models (NJL/QM) and background gauge field approximation (Poly / Log / Log$_n$). Unstable features represent physical observable that need more microscopic understanding and more constraint.

As a partial conclusion

We have to be careful in the choice of our prior to impose enough physical constraints. The analysis of the posterior distribution is important to increase our confidence in the model (e.g. we found again the unphysical $SU(2)$ solution).

The more priors we have (if we can trust the model to reproduced them) the better.

We are trying to be more systematic in our approach to include as much as possible of the known and well understood domain.

PART III

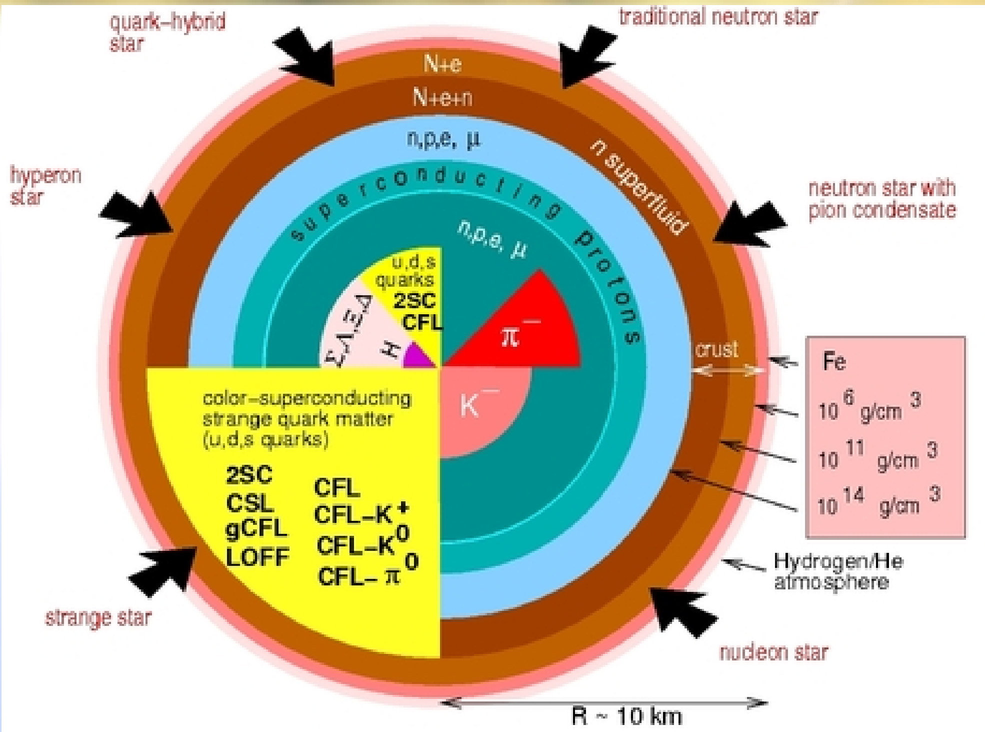
Perspectives: compact stars as a laboratory for dense matter

(and very very heavy “ion” collisions)

HH, Jérome Margueron, Guy Chanfray, Jean-Francois Coupechoux, Alexandre Arbey

Compact stars

Lot of effort to constrain nuclear EoS from **new compact star observations**.
Data driven effort possible with the advance of observations, in particular **multimessenger astronomy**.
Particularly interesting event: kilonova AT2017gfo corresponding to the merging of two compact stars and observed as gravitational waves (GW170817) in LIGO/Virgo interferomer.
New satellite: NICER: Neutron Star Interior Composition Explorer



(Physics department, San Diego state university)

Hydrostatic equilibrium and influence of a second body

Equilibrium, TOV (1939)

The mass M and radius R are completly determine by equation of state $P(\rho)$ (pressure as a function of the density) and the hydrostatic equilibrium general relativistic Tolman-Oppenheimer-Volkoff equation (TOV ; 1939) with conditions $R = r(P = 0)$ and $M = m(R)$:

$$\frac{dP}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P}{c^2}\right]\left[m(r) + 4\pi r^3 \frac{P}{c^2}\right]\left[1 - \frac{2Gm(r)}{rc^2}\right]^{-1}$$

With a second body, Thorne and Campolattaro (1967)

One can defines the tidal polarizability (related to the linear approximation of the quadrupole moment: $Q \simeq \lambda \mathcal{E} \simeq \lambda \partial^2 \Phi$) that probe for the **internal structure**:

$$\lambda = \frac{2}{3} k_2(R) \left(\frac{c^2 R}{GM}\right)^5$$

where k_2 is $l = 2$ tidal Love number

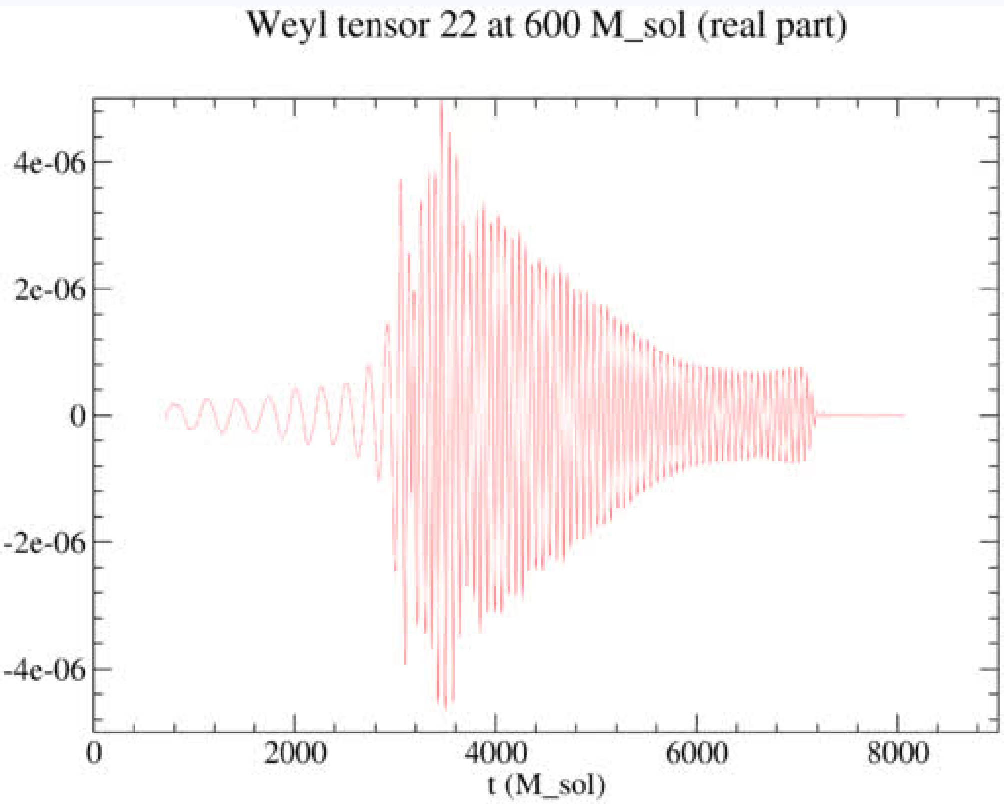
It can be extracted from gravitational wave (GW) signal of binary neutron stars (BNS) mergers. Estimation for is GW170817 0 to 800.

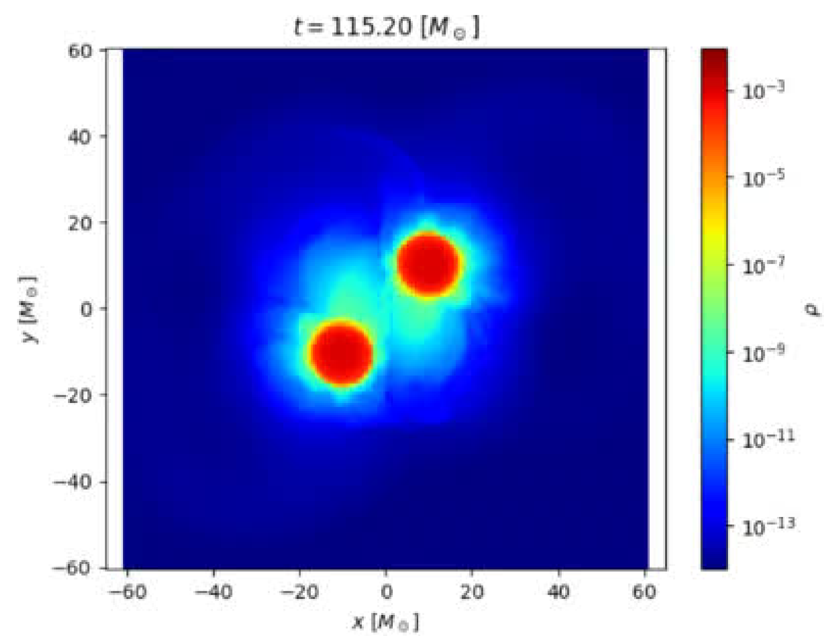
BNS merger simulation and gravitational wave

First calculation at IPNL (EoS LS220) :

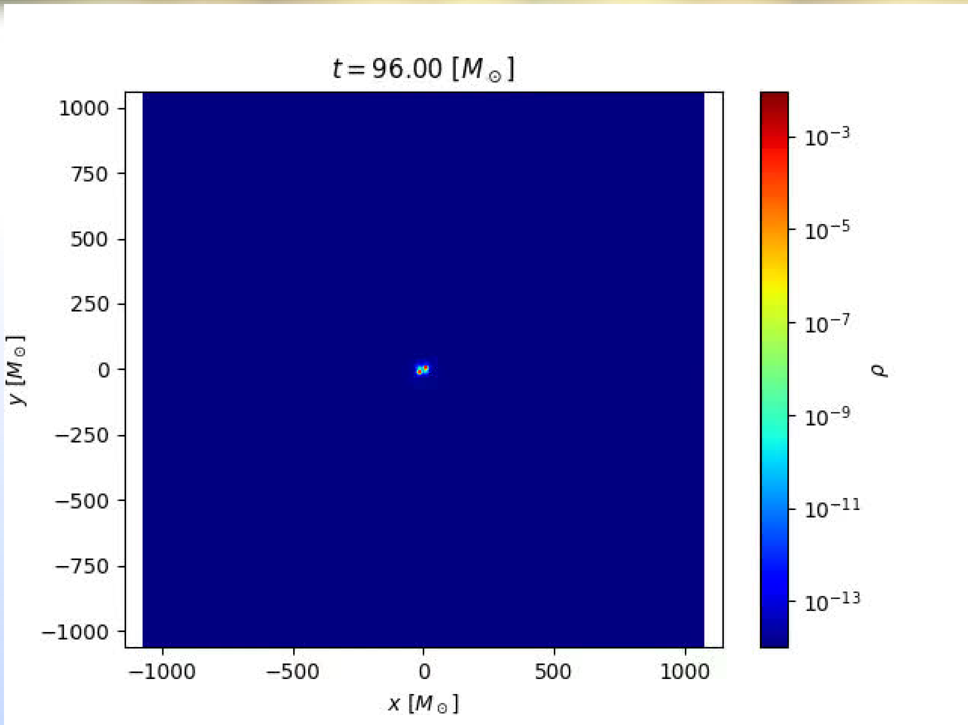
Code WhiskyTHC, David Radice, Princeton : (Templated hydrodynamics code for general relativity)

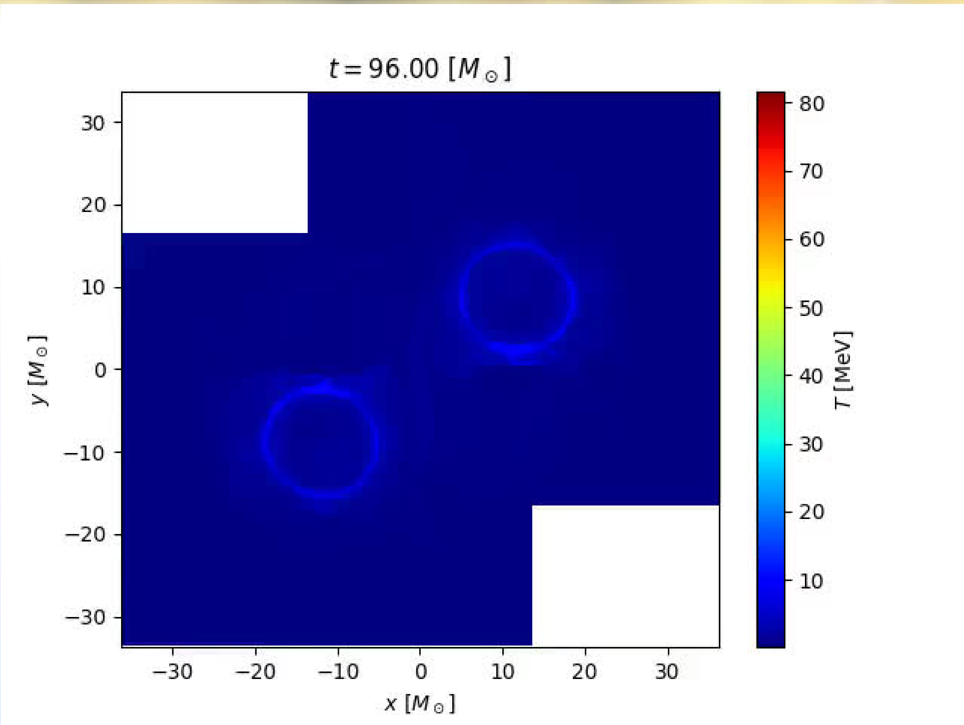
https://www.astro.princeton.edu/~dradice/whiskythc.html





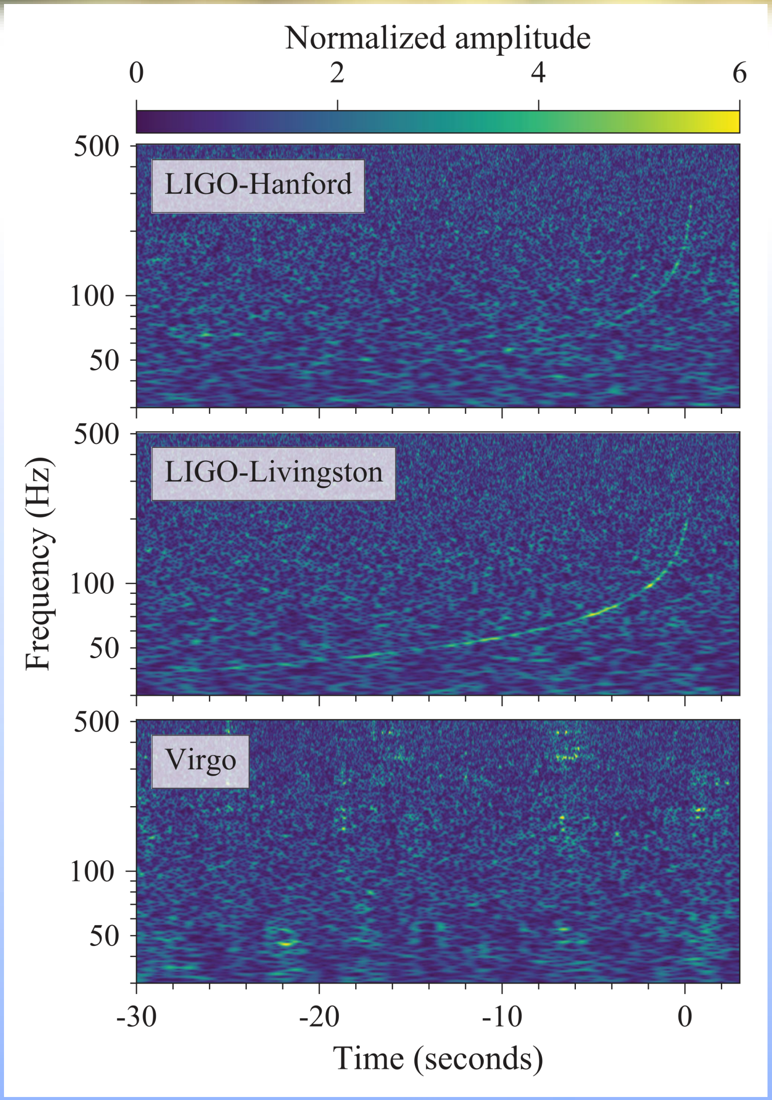
Density / temperature

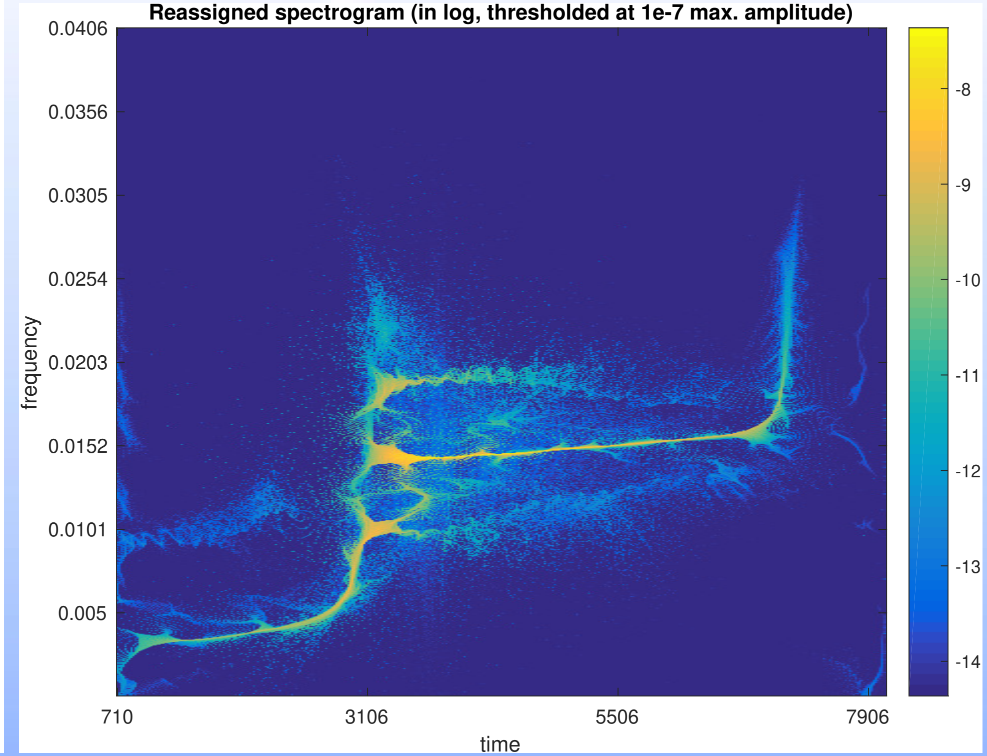




Notice the temperature. May be quark degrees of freedom are relevant (if the feature persists with more up to date EoS) ?

Spectrogramme





(With P. Borgnat, LP ENSL)

Neutron star phenomenology and the CEP: proof of concept

Can compact stars phenomenology constrain QCD critical properties (or the other way around, depending on who will be measured first with enough accuracy) ?

Proof of concept in the most favorable setup:

- NJL $SU(2)$: only the light quark condensate, that have a very important variation when varying the density, contrarly to the strange condensate (even if it known that pure quark star contains strangeness if they exists).
- Tidal deformability observable (probe the core of the matter)
- Pure quark star with no nucleonic EoS for the crust (maximising the chance to have large deformation).

If with this very favorable setup we do not have a positive result it does not look good for this research direction ... But there are other constrains to look at (e.g. relevance of quark matter during the merging due to the high temperature)

If we have a positive result, it does not mean it will survive once we adopt a more realistic description of the dense matter (strangeness, nucleonic crust, etc).

Correlations CEP - Neutron star phenomenology

Very preliminary result

Correlations for a central density $4n_{sat}$:

	R	M	k_2	Λ
T_{CEP}	0.7	0.9	0.1	0.1
μ_{CEP}	0.6	0.8	0.1	0.1

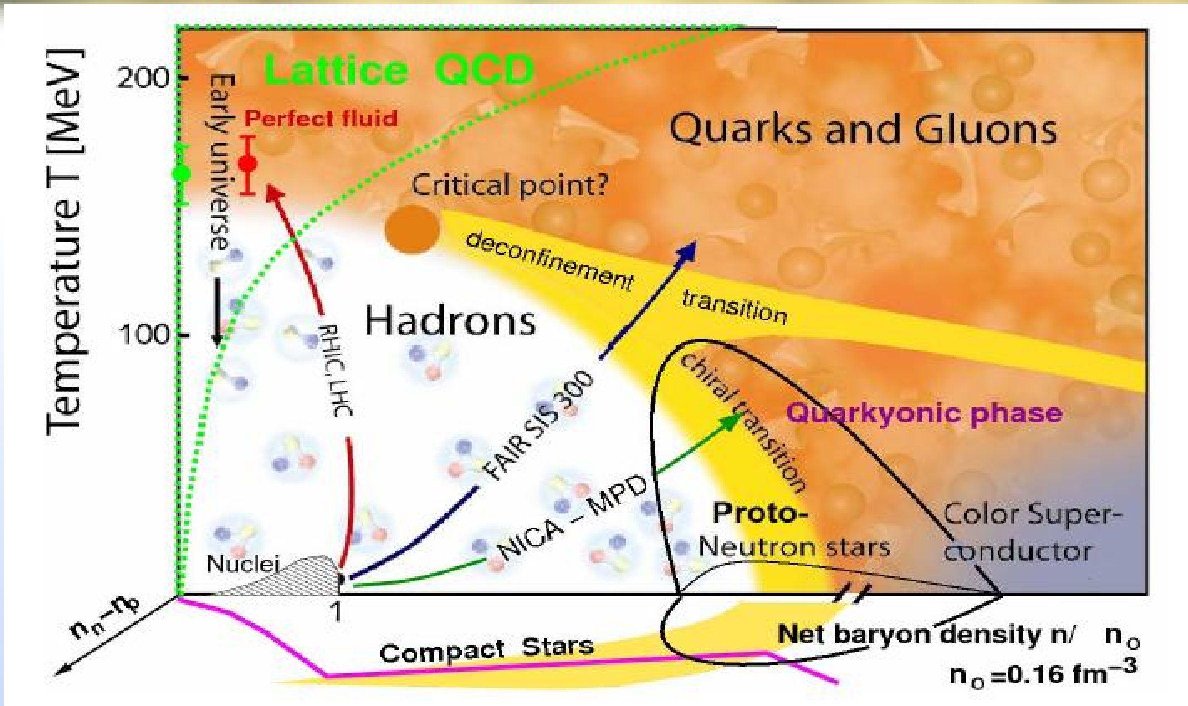
Correlations for a central density $3n_{sat}$:

	R	M	k_2	Λ
T_{CEP}	0.9	0.9	0.4	0.4
μ_{CEP}	0.7	0.9	0.3	0.4

No strong correlation with the tidal deformability but with mass and radius.

If this result persists with a more realistic description of the quark matter then: if we are sure we have measured this parameter for a compact star with a quark core then we may be able to give a more accurate prediction of the CEP ; or if we have a measure of the CEP we may be able to decide if compact star observation are compatible with the presence of a quark core.

Conclusions



The construction of the phase diagram of QCD is in progress with the help of effective models (and other approaches).

(Massimo Di Toro)

Our aim: build quark EoS from effective model for the whole phase diagram (also linked with nuclear EoS) based on the best knowledge we have with other approaches ⇒ compact star and hot and dense phases phenomenology.

Provide effective models a la PQM/PNJL with parametrisation carefully crafted, grounded in experimental data (HIC / Compact star) and LQCD data (finite T) to offer the best possible extrapolation at all T and μ.