the On-Shell Effective Field Theory and the Chiral Kinetic Equation

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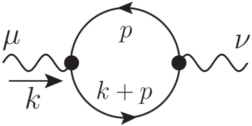
Motivation: HTLs and Kinetic Theory

Hard thermal loops

- Perturbative QED (and QCD) plasma at finite T
- Soft photons, with momenta $k \sim eT$
- 1-loop polarization function (HTLs), same order as bare inverse propagator, $i\Pi_{\mu\nu} \sim e^2 T^2$

$$i\Pi^{\mu\nu}(k) =$$



- Dominated by hard fermion modes $p \sim T$ in the loop

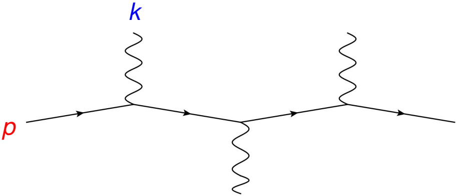
HTL effective action of QED

$$\mathcal{S} = -\frac{m_D^2}{4} \int_{x,v} F_{\alpha\mu} \frac{v^\alpha v^\beta}{(v \cdot \partial)^2} F^\mu_\beta \quad , \qquad m_D^2 = e^2 T^2/3$$

(Braaten, Pisarski 1990) (Frenkel, Taylor 1990)(...)

Fermion kinetic theory

- Alternative approach: *kinetic theory for fermions (w/o collisions)*
- On-shell fermion momentum p sets the **hard scale**
- **Soft momenta** k due to interactions with gauge fields
- Separation of scales $p \gg k$



At high T the kinetic theory for hard fermions $p \sim T$ and soft gauge fields $k \sim gT$ leads to the HTL effective action
(Blaizot, Iancu 1994)(Kelly, Liu, Luchessi, Manuel 1994)

On-shell effective field theory

Goal

QED with $m = 0$ fermions + antifermions

(Almost) on-shell fermion

$$q^\mu = Ev^\mu + k^\mu$$

$$v^\mu = (1, \mathbf{v}), \quad v_\mu v^\mu = 0$$

(Almost) on-shell antifermions

$$q^\mu = -E\tilde{v}^\mu + k^\mu$$

$$\tilde{v}^\mu = (1, -\mathbf{v}), \quad \tilde{v}_\mu \tilde{v}^\mu = 0$$

- Energy of the on-shell fermion, E
- Residual momentum, k^μ ($\ll Ev^\mu$)

(Manuel and JMT-R, 2014)

Derivation of effective Lagrangian

Dirac field decomposed in

- +/- energy components (close to $+E$ and $-E$)
- Particle/antiparticle modes

$$\psi_{v,\tilde{v}} = e^{-iEv\cdot x}\left(P_v\,\chi_v(x) + P_{\tilde{v}}\,H^{(1)}_{\tilde{v}}(x)\right) + e^{iE\tilde{v}\cdot x}\left(P_{\tilde{v}}\,\xi_{\tilde{v}}(x) + P_v\,H^{(2)}_v(x)\right)$$

$$P_v = \frac{1}{2}\not{v}\not{u} \quad , \qquad P_{\tilde{v}} = \frac{1}{2}\not{\tilde{v}}\not{u}$$

are projectors along v and $\tilde{v}$, with $u = (v+\tilde{v})/2$ is the reference frame.

In the QED Lagrangian

$$\mathcal{L} = \sum_{E,\mathbf{v}} \mathcal{L}_{E,\mathbf{v}}$$

we integrate out far off-shell modes: $H^{(1)}_{\tilde{v}}$ and $H^{(2)}_v$

Lagrangian in an arbitrary frame (Carignano, Manuel, JMTR 2018)

$$\mathcal{L}_{E,v} + \mathcal{L}_{-E,\tilde{v}} = \bar{\chi}_v(x)\left(i\,v\cdot D + i\not{D}_\perp \frac{1}{2E + i\tilde{v}\cdot D} i\not{D}_\perp\right) \not{v}\,\chi_v(x)$$
$$+ \bar{\xi}_{\tilde{v}}(x)\left(i\,\tilde{v}\cdot D + i\not{D}_\perp \frac{1}{-2E + iv\cdot D} i\not{D}_\perp\right) \not{v}\,\xi_{\tilde{v}}(x)$$

Frame vector in the LRF is $u^\mu = (1,0,0,0)$.

$$E = u\cdot p\,; \quad p^\mu = E v^\mu$$

- Inspired by similar procedure in other EFTs e.g. HQET and HDET
- In OSEFT, hard scale is **dynamical**, no medium (yet)
- At finite T it reproduces HTL (plus higher orders in $1/T$) (Manuel, Soto, Stetina 2016)
- We considered the Abelian version (some similarities with SCET)

OSEFT in arbitrary frame

After integrating all the modes far from the mass shell,

$$q^\mu = Ev^\mu + k^\mu \quad , \qquad E = u \cdot p$$

with

$$Ev^\mu \gg k^\mu$$

$1/E$ expansion of OSEFT (only for particles)

$$\begin{aligned} \mathcal{L}_{E,v} &= \bar{\chi}_v(x) \left(i\, v \cdot D + i \not{D}_\perp \frac{1}{2E + i\tilde{v} \cdot D} i \not{D}_\perp \right) \not{v}\, \chi_v(x) \\ &= \bar{\chi}_v(x) \left(iv \cdot D - \frac{\not{D}_\perp^2}{2E} - \frac{i \not{D}_\perp (i\tilde{v} \cdot D) i \not{D}_\perp}{4E^2} + \cdots \right) \not{v}\, \chi_v(x) \end{aligned}$$

Important! Physical energy is $E_q = u \cdot q$. E dependence must disappear.

Comment about Lorentz invariance

Lorentz symmetry

- $v^\mu, \tilde{v}^\mu$ break 5 Lorentz generators: $v_\mu M^{\mu\nu}, \tilde{v}_\mu M^{\mu\nu}$; $M \in SO(3,1)$.
- These transformations encoded in freedom to decompose q^μ

$$q^\mu = E v^\mu + k^\mu$$

- Lorentz invariance $\to$ **"reparametrization invariance"** (Manohar, Mehen, Pirjol, Stewart 2002)

RI transformations

$$\text{(I)} \begin{cases} v^\mu & \to & v^\mu + \lambda_\perp^\mu \\ \tilde{v}^\mu & \to & \tilde{v}^\mu \end{cases} \quad \text{(II)} \begin{cases} v^\mu & \to & v^\mu \\ \tilde{v}^\mu & \to & \tilde{v}^\mu + \epsilon_\perp^\mu \end{cases} \quad \text{(III)} \begin{cases} v^\mu & \to & (1+\alpha) v^\mu \\ \tilde{v}^\mu & \to & (1-\alpha)\tilde{v}^\mu \end{cases}$$

where $\lambda_\perp^\mu, \epsilon_\perp^\mu, \alpha$ are 5 infinitesimal parameters such that $v \cdot \lambda_\perp = v \cdot \epsilon_\perp = \tilde{v} \cdot \lambda_\perp = \tilde{v} \cdot \epsilon_\perp = 0$.

Knowing this...

	Type I	Type II	Type III
v^μ	$v^\mu + \lambda_\perp^\mu$	v^μ	$v^\mu(1+\alpha)$
$\tilde{v}^\mu$	$\tilde{v}^\mu$	$\tilde{v}^\mu + \epsilon_\perp^\mu$	$\tilde{v}^\mu(1-\alpha)$
E	$E + \frac{1}{2}\lambda^\perp \cdot p$	$E + \frac{1}{2}(\epsilon_\perp \cdot p)$	$E(1+\alpha) - \alpha(\tilde{v}\cdot p)$
D_μ	$D_\mu + iE\lambda_\mu^\perp + \frac{i}{2}(\lambda_\perp \cdot p)v^\mu$	$D_\mu + \frac{i}{2}(\epsilon_\perp \cdot p)v_\mu$	$D_\mu + 2i\alpha E\, v_\mu - i\alpha(\tilde{v}\cdot p)v_\mu$
$\chi_v(x)$	$\left(1 + \frac{1}{4}\not{\lambda}_\perp \not{\tilde{v}}\right) \chi_v(x)$	$\left(1 + \frac{1}{2}\not{\epsilon}_\perp \frac{1}{2E+i\tilde{v}\cdot D} i\not{D}_\perp\right) \chi_v(x)$	$\chi_v(x)$

...we can check that

$$\mathcal{L}_{E,v} \xrightarrow{\text{Type I, Type II, Type III}} \mathcal{L}_{E,v}$$

to **ALL** orders in $1/E$ expansion (Carignano, Manuel, JMTR 2018)

The OSEFT Lagrangian is Lorentz invariant,
with some transformations realized via RI transformations. ▸ Symmetries

Chiral Kinetic Equation

Schematic procedure: details in (Kadanoff, Baym 1962) (Elze, Heinz 1989) (Botermans, Malfilet 1990) (Blaizot, Iancu 2002):

1. Real-time formalism in Keldysh-Schwinger basis
2. Focus on vector/axial components of fermions (antifermions analogous)

$$G_{E,v}(x,y) = \langle \bar{\chi}_v(y) \frac{\tilde{\not{v}}}{2} \chi_v(x) \rangle$$

3. Apply Wigner transform and gradient expansion $\partial_X \ll \partial_s$

Wigner function

$$G_{E,v}(X,k) = \int d^4s\, e^{ik\cdot s}\, G_{E,v}\left(X+\frac{s}{2}, X-\frac{s}{2}\right)\, U\left(X-\frac{s}{2}, X+\frac{s}{2}\right)$$

$$X = \frac{1}{2}(x+y) \quad , \qquad s = x-y \quad , \qquad U = P\exp[-ie\int_\gamma dx^\mu A_\mu(x)]$$

4. Neglect collision terms $\rightarrow$ quasiparticle picture

Kadanoff-Baym equations provide 2 sets of equations:

1. "Sum equations" (constraint)
2. "Difference equations" (kinetic equation)

These equations are computed order by order in the expansion in $1/E$ of OSEFT Lagrangian

$$
\begin{aligned}
\mathcal{O}_x^{(0)} &= i\, v \cdot D \, \frac{\tilde{\not{v}}}{2} \\
\mathcal{O}_x^{(1)} &= -\frac{1}{2E} \left(D_\perp^2 + \frac{e}{2} \sigma_\perp^{\mu\nu} F_{\mu\nu} \right) \frac{\tilde{\not{v}}}{2} \\
\mathcal{O}_{x,\mathrm{LFR}}^{(2)} &= \frac{1}{8E^2} \left[\not{D}_\perp \,, \left[i\tilde{v} \cdot D \,, \not{D}_\perp \right] \right] \frac{\tilde{\not{v}}}{2} \\
&- \frac{1}{8E^2} \left\{ D_\perp^2 + \frac{e}{2} \sigma_\perp^{\mu\nu} F_{\mu\nu}, \left(iv \cdot D - i\tilde{v} \cdot D \right) \right\} \frac{\tilde{\not{v}}}{2}
\end{aligned}
$$

Sum equations $\rightarrow$ Dispersion relation

In terms of physical variables

Dispersion relation

$$q^2 - eS_\chi^{\mu\nu} F_{\mu\nu} = 0$$

where spin tensor is

$$S_\chi^{\mu\nu} = \chi \frac{\epsilon^{\alpha\beta\mu\nu} u_\beta q_\alpha}{2E_q} \quad , \qquad \chi = \pm 1 \quad \text{(chirality)}$$

In the LRF

$$E_q = \pm |\mathbf{q}| \left(1 - e\chi \frac{\mathbf{B} \cdot \mathbf{q}}{2\mathbf{q}^2} \right)$$

(Son, Yamamoto 2012), (Manuel, JMTR 2013), (Chen, Son, Stephanov, Yee, Yin 2014) (...)

Difference equations $\to$ Transport equation

In terms of physical variables

Chiral Kinetic Equation

$$\left[\frac{q^\mu}{E_q} - \frac{e}{2E_q^2} S^{\mu\nu}_\chi F_{\nu\rho} \left(2u^\rho - \frac{q^\rho}{E_q}\right)\right] \Delta_\mu \; G^\chi_{E,v}(X,q) = 0$$

where

$$E_q = q \cdot u \quad ; \qquad \Delta^\mu \equiv \frac{\partial}{\partial X_\mu} - eF^{\mu\nu}(X) \frac{\partial}{\partial q^\nu}$$

And

$$G^\chi_{E,v}(X,q) = 2\pi \delta_+(Q^\chi) f^\chi(X,q) \; ; \qquad \delta_+(Q^\chi) = \theta(E_q) E_q \delta(q^2 - eS^{\mu\nu}_\chi F_{\mu\nu})$$

CKE in the Local Rest Frame

In the LRF

$$\left(\Delta_0 + \hat{\mathbf{q}}^i \left(1 + e\chi \frac{\mathbf{B}\cdot\hat{\mathbf{q}}}{2q^2} \right) \Delta_i + e\chi \frac{\epsilon^{ijk} E^j \hat{q}^k - B^i_{\perp,\mathbf{q}}}{4q^2} \Delta_i \right) f_v^{\chi}(X,\mathbf{q}) = 0$$

$$\Delta^{\mu} \equiv \frac{\partial}{\partial X_{\mu}} - eF^{\mu\nu}(X) \frac{\partial}{\partial q^{\nu}}$$

CKE in the Local Rest Frame

In the LRF

$$\left(\Delta_0 + \hat{\mathbf{q}}^i \left(1 + e\chi \frac{\mathbf{B}\cdot\hat{\mathbf{q}}}{2q^2}\right)\Delta_i + e\chi \frac{\epsilon^{ijk}E^j\hat{q}^k - B^i_{\perp,\mathbf{q}}}{4q^2}\Delta_i\right) f_v^\chi(X,\mathbf{q}) = 0$$

$$\Delta^\mu \equiv \frac{\partial}{\partial X_\mu} - eF^{\mu\nu}(X)\frac{\partial}{\partial q^\nu}$$

which differs from other authors in subleading terms

$$\left(\Delta_0 + \hat{\mathbf{q}}^i \left(1 + e\chi \frac{\mathbf{B}\cdot\hat{\mathbf{q}}}{2q^2}\right)\Delta_i + e\chi \frac{\epsilon^{ijk}E^j\hat{q}^k}{2q^2}\Delta_i\right) f^\chi(X,\mathbf{q}) = 0$$

(Hidaka, Pu, Yang 2017)(Son, Yamamoto 2012)

It can be explained due to the different fermion fields (Lin, Shukla 2019)

$$G_v = \langle \bar{\chi}_v(y)\frac{\tilde{\not{v}}}{2}U(y,x)\chi_v(x)\rangle \qquad vs \qquad G = \langle\bar{\psi}(y)U(y,x)\psi(x)\rangle$$

Chiral anomaly

Fermion current

The fermion current

$$j^\mu(x) = -\frac{\delta \mathcal{S}_{OSEFT}}{\delta A_\mu(x)}$$

is computed up to order $1/E_q^2$ in the original variables

$$\begin{aligned} j^\mu(X) &= e \sum_{\chi=\pm} \int \frac{d^4 q}{(2\pi)^3} \, 2\theta(E_q)\, \delta(q^2 - e S_\chi^{\mu\nu} F_{\mu\nu}) \\ &\times \left\{ q^\mu - \frac{e}{2E_q} S_\chi^{\mu\nu} F_{\nu\rho} (2u^\rho - \frac{q^\rho}{E_q}) + \mathcal{O}\left(\frac{1}{E_q^3}\right) \right\} f^\chi(X,q) \end{aligned}$$

To compute the chiral current we introduce chiral fields $A_5^\mu, F_5^{\mu\nu}$

$$j_5^\mu(x) = -\frac{\delta \mathcal{S}_{OSEFT}}{\delta A_\mu^5(x)}$$

Chiral anomaly equation

Using the kinetic equation in the LRF and integrating

$$\partial_\mu j_5^\mu(X) = e^2 \chi \frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2} \frac{1}{6} \lim_{|\mathbf{q}|\to 0} [f^R(|\mathbf{q}|) - f^L(|\mathbf{q}|)]$$

Summing both fermions and antifermions in a plasma in equilibrium

$$f^\chi(|\mathbf{q}|) = \frac{1}{e^{(|E_q|-\mu_\chi)/T} + 1} , \qquad \tilde{f}^\chi(|\mathbf{q}|) = \frac{1}{e^{(|E_q|+\mu_\chi)/T} + 1}$$

we obtain

Consistent axial anomaly

$$\partial_\mu j_5^\mu(X) = \partial_\mu [j_R^\mu(X) - j_L^\mu(X)] = \frac{1}{3} \frac{e^2}{2\pi^2} \mathbf{E}\cdot\mathbf{B}$$

- The 1/3 is carried by the "consistent" version of the anomaly (as opposed to the "covariant anomaly") back-up
- Expected from the definition of the current as a functional derivative (Landsteiner 2016)(Fujikawa 2017)

Chiral magnetic effect

The 3D vector current (after q_0 integration) up to $\mathcal{O}(1/E_q)$ in the LRF

$$j^i(X) = e \sum_{\chi=\pm} \int \frac{d^3q}{(2\pi)^3} \left(\frac{q^i}{E_q} + \frac{S_\chi^{ij} \Delta_j}{E_q} \right) f^\chi(X,q)$$

and we use equilibrium distribution

$$f_{eq}^\chi(E_q) = f_{eq}^\chi(|\mathbf{q}|) - e\chi \frac{\mathbf{B} \cdot \mathbf{q}}{2|\mathbf{q}|^2} \frac{df_{eq}^\chi(|\mathbf{q}|)}{d|\mathbf{q}|}$$

The current

$$j^i = e^2 \sum_\chi \chi \int \frac{d^3q}{(2\pi)^3} \left[\frac{(B^j \hat{q}^i \hat{q}^j - B^i)}{2|\mathbf{q}|} \frac{df_{eq}^\chi(|\mathbf{q}|)}{d|\mathbf{q}|} - \frac{B^j \hat{q}^j \hat{q}^i}{2|\mathbf{q}|} \frac{df_{eq}^\chi(|\mathbf{q}|)}{d|\mathbf{q}|} \right]$$

Incorporating particles and antiparticles of both chiralities we integrate

CME in equilibrium

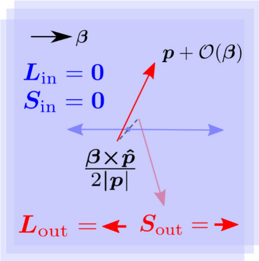
$$\mathbf{j}(X) = e^2 \left(\frac{2}{3} + \frac{1}{3} \right) \frac{\mu_5}{4\pi^2} \mathbf{B}(X)$$

(Son, Yamamoto 2012) (Manuel, JMTR 2013) (Kharzeev, Stephanov, Yee 2016)

Distribution function under Lorentz transformations

Lorentz transformation of distribution function

- Modified Lorentz transformation for chiral fermions
  (Chen, Son, Stephanov, Yee, Yin 2014) (Duval, Horvathy 2014)
- Transformation implies "side-jump" (displacement in boosted frame)
  (Stone, Dwivedi, Zhou 2015) (Chen, Son, Stephanov 2015)



(fig. from Chen, Son, Stephanov, Yee, Yin 2014)

- Distribution function f is not scalar
  (Chen, Son, Stephanov 2015) (Hidaka, Pu ,Yang 2016)

Transformations giving "side jump" are contained in RI transformations
How does the distribution function behave under those?

RI transformation of f

How do the RI transformations act on $f \equiv f^{\chi}_{E,v}(X,k)$ (**particle case**)?

2 of them have no effect:

$$f \xrightarrow{\text{Type I}} f$$

$$f \xrightarrow{\text{Type III}} f$$

But Type II:

$$f \xrightarrow{\text{Type II}} f - \frac{1}{2E_q} S^{\mu\nu}_{\chi} \epsilon^{\perp}_{\nu} \Delta_{\mu} f + \mathcal{O}\left(\epsilon^2_{\perp}, \frac{1}{E_q^2}\right)$$

Using $\epsilon^{\mu}_{\perp}/2 = u'^{\mu} - u^{\mu}$

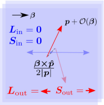
Transformation of f

$$f \xrightarrow{\text{Type II}} f - \frac{\chi \epsilon^{\alpha\beta\mu\nu} u'_{\nu} u_{\beta} q_{\alpha}}{2(q\cdot u')(q\cdot u)} \Delta_{\mu} f + \mathcal{O}\left(\epsilon^2_{\perp}, \frac{1}{E_q^2}\right)$$

which coincides with
(Chen,Son, Stephanov 2015)(Hidaka, Pu, Yang 2016)(Gao, Liang, Wang,Wang 2018)

Conclusions and Outlook

- **OSEFT**: systematic approach for the physics of nearly on-shell massless fermions/antifermions
- It can be used to derive a transport equation, anomaly equation, CME...
  See also (Manuel, Soto, Stetina 2016)(Carignano, Manuel, Soto 2017)
- Transparent study of Lorentz transformations e.g. **"side jumps"**



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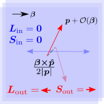
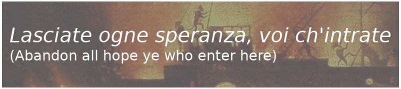


- **Outlook**: Effects of mass (Weickgennant, Sheng, Speranza, Wang, Rischke 2019)
- **Outlook**: Understanding “side jumps” by **computing collision terms**



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The OSEFT Lagrangian is Lorentz invariant,
with some transformations realized via RI transformations.
(Carignano, Manuel, JMTR 2018)

For $v^\mu = (1,0,0,1)$ and $\tilde{v}^\mu = (1,0,0,-1)$:

1. Type I: $Q_1^- \equiv J_1 - K_2, Q_2^+ = J_2 + K_1$
2. Type II:$Q_1^+ \equiv J_1 + K_2, Q_2^- = J_2 - K_1$
3. Type III: K_3

Notice that Type I (Type II) leaves invariant v^μ $(\tilde{v}^\mu)$. They are elements of the **Wigner's little group** associated to v^μ $(\tilde{v}^\mu)$.

1. Wigner's Little Group of v^μ: $(Q_1^+, Q_2^-, J_3) \in SE(2)$
2. Wigner's Little Group of $\tilde{v}^\mu$: $(Q_1^-, Q_2^+, J_3) \in SE(2)$

go back

Chiral anomaly equation

Apply $\partial_\mu = \Delta_\mu + F_{\mu\nu}\partial_q^\nu$, and make use of the CKE to cancel the first term. For one chirality χ:

$$\partial_\mu j^\mu_\chi(X) = \frac{1}{2}\left[\partial_\mu j^\mu(X) + \chi\partial_\mu j_5^\mu(X)\right]$$
$$= e^2 \int \frac{d^4q}{(2\pi)^3}\left\{q^\mu - \frac{e}{2E_q}S_\chi^{\mu\nu}F_{\nu\rho}\left(2u^\rho - \frac{q^\rho}{E_q}\right)\right\}$$
$$\times \quad F_{\mu\lambda}\frac{\partial}{\partial q^\lambda}\left[f^\chi(X,q)\,2\theta(E_q)\,\delta(q^2 - eS_\chi^{\mu\nu}F_{\mu\nu})\right]$$

In the LRF, only surface terms remain. Integrate around a sphere of radius R, and take the limit $R \to 0$ (Stephanov, Yin 2012)

$$\partial_\mu j^\mu_\chi(X) = -e^2\chi \lim_{R\to 0}\left(\int \frac{d\mathbf{S}_R}{(2\pi)^3}\cdot \mathbf{E}\,\frac{\hat{\mathbf{q}}\cdot\mathbf{B}}{4R^2}f^\chi(|\mathbf{q}|=R)\right.$$
$$\left. - \int \frac{d\mathbf{S}_R}{(2\pi)^3}\cdot\frac{\hat{\mathbf{q}}}{4R^2}\mathbf{E}\cdot\mathbf{B}f^\chi(|\mathbf{q}|=R)\right)$$
$$= e^2\chi\frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2}\frac{1}{6}f^\chi(|\mathbf{q}|=0)$$

In the presence of chiral gauge fields we have

Chiral current nonconservation

$$\partial_\mu j_5^\mu(X) = \frac{1}{3}\frac{e^2}{2\pi^2}\left(\mathbf{E}\cdot\mathbf{B} + \mathbf{E}_5\cdot\mathbf{B}_5\right)$$

Vector current anomaly

$$\partial_\mu j^\mu(X) = \frac{1}{3}\frac{e^2}{2\pi^2}\left(\mathbf{E_5}\cdot\mathbf{B} + \mathbf{E}\cdot\mathbf{B}_5\right)$$

To get a conserved current one adds Bardeen counterterms to the quantum action

$$\int d^4x\, \epsilon^{\mu\nu\rho\lambda} A_\mu A_\nu^5 \left(c_1 F_{\rho\lambda} + c_2 F^5_{\rho\lambda}\right)$$

with the choice $c_1 = \frac{1}{12\pi^2}$ and $c_2 = 0$

(Landsteiner 2016) Go back

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