Dileptons in HSD 2.5

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HSD home-page:
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1 Dilepton channels

The dilepton (e^+e^- or $\mu^+\mu^-$) spectra in HSD are calculated perturbatively with the time integration method. For the details of the dilepton implementation see our review [1] and also Refs. [2, 3, 4, 5, 6, 7, 8, 9]. The time integration is perfomed over the actual dilepton emission rate during the full reaction time (contrary to the 'spontaneous decay' assumption which counts the dilepton radiation only at freeze-out). [1]

Table 1: **Dilepton channels in HSD 2.5**

i	Dilepton channel	
1	Dalitz decay of π^0:	$\pi^0 \to \gamma e^+e^-$
2	Dalitz decay of η:	$\eta \to \gamma e^+e^-$ (or $\mu^+\mu^-$, also for channels below)
3	Dalitz decay of ω:	$\omega \to \pi^0 e^+e^-$
4	Dalitz decay of Δ:	$\Delta \to N e^+e^-$
5	direct decay of ω:	$\omega \to e^+e^-$
6	direct decay of ρ:	$\rho \to e^+e^-$
7	direct decay of ϕ:	$\phi \to e^+e^-$
8	direct decay of J/Ψ:	$J/\Psi \to e^+e^-$
9	direct decay of Ψ':	$\Psi' \to e^+e^-$
10	Dalitz decay of η':	$\eta' \to \gamma e^+e^-$
11	pn bremsstrahlung:	$pn \to pne^+e^-$
12	$\pi^\pm N$ bremsstrahlung:	$\pi^\pm N \to \pi N e^+e^-$, where $N = p$ or n

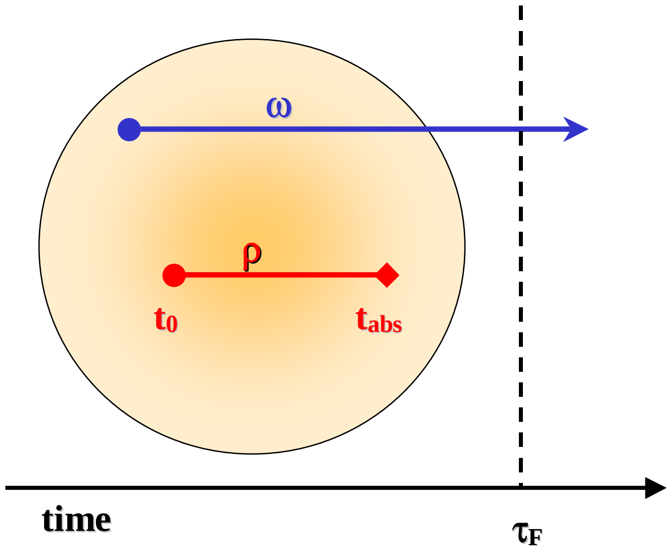
All branching ratios, electromagnetic partial and total decay widths are taken from the PDG [10].

1. The pn and $\pi^\pm N$ bremstrahlungs are calculated in the soft-photon approximation (SPA). Only elastic pn and $\pi^\pm N$ collisions are accounted in the bremsstrahlung (i.e. $pn \to pne^+e^-$, $\pi^\pm N \to \pi N e^+e^-$). We stress that the SPA approximation might be considered as an upper limit for the bremsstrahlung contribution (especially for πN !). The bremsstrahlung channels are switched off for $E_{lab} \geq 6$ GeV since it is very questionable to use the SPA at high energies.

2. The channel $\rho \to e^+e^-$ includes the dilepton radiation by all rho mesons produced in baryon-baryon, meson-baryon or meson-meson (e.g. $\pi^+\pi^-$ annihilation) collisions. The same holds for the other mesons – $\rho, \eta, \omega, \phi, J/\Psi, \Psi'$.

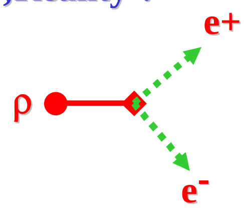
[1] Useful link: HSD-home page - talk at HADES Collaboration Meeting (Feb. 2005).

Time integration method for dileptons



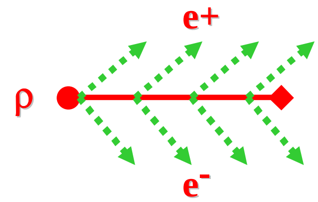
τ_F – final time of computation in the code
t_0 – production time
t_{abs} – absorption (or hadronic decay) time

‚Reality':



only ONE e+e- pair with probability ~ Br(ρ->e+e-)=4.5·10^{-5}

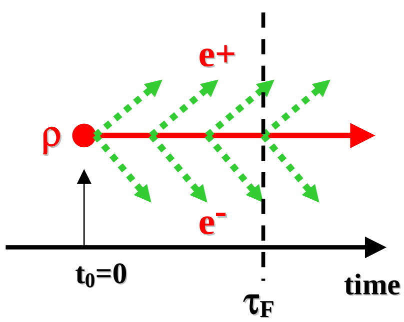
‚Virtual' – time int. method:



Calculate probability P(t) to emit e+e- pair at each time t and integrate P(t) over time!

ρ: $t_0 < t < t_{abs}$
ω: $t_0 < t <$ infinity

The time integration method for dileptons in HSD



Dilepton emission rate:

$$\frac{dN(\rho\to e^+e^-)}{dt}(t) = -\frac{1}{\gamma\tau} N_\rho Br(\rho\to e^+e^-), \qquad \tau = \frac{\hbar c}{\Gamma_{tot}}$$

$$N(\rho\to e^+e^-)(t) = N_0 e^{-\frac{t}{\gamma\tau}}$$

$$\int_0^\infty dt N(t) = \int_0^{\tau_F} dt N(t) + \int_{\tau_F}^\infty dt N(t)$$

$$\text{numerically}: dt \to \Delta t, \qquad \frac{dN}{dt} \to \frac{1}{\Delta t}[N(t+\Delta t) - N(t)]$$

Dilepton invariant mass spectra:

$$\frac{dN^{\rho\to e^+e^-}}{dM} = \sum_{time}^{\tau_F} \Gamma^{\rho\to e^+e^-}(M) \cdot \frac{\Delta t}{\gamma(\hbar c)} \cdot \frac{1}{\Delta M} + Br^{\rho\to e^+e^-}(M) \cdot e^{-\frac{\Gamma_{tot}\tau_F}{\gamma(\hbar c)}} \cdot \frac{1}{\Delta M}$$

$0 < t < \tau_F$ $\qquad\qquad$ $\tau_F < t <$ infinity

❖ The time integration method allows to account for the in-medium dynamics of vector mesons!

2 Dilitz decays $A \to B\ l^+l^-$

Here l^+l^- are electron e^+e^- or muon $\mu^+\mu^-$ pairs, i.e. $m_l = m_e = 0.511 \cdot 10^{-3}$ GeV or $m_l = m_\mu = 0.105658389$ GeV.

2.1 Dalitz decay $\pi^0 \to \gamma\ l^+l^-$

$$\frac{d\Gamma^{\pi^0\to\gamma\ l+l-}}{dM} = \frac{4\alpha}{3\pi}\frac{\Gamma^{\pi^0\to\gamma\gamma}}{M}\left(1-\frac{4m_l^2}{M^2}\right)^{1/2}\left(1+\frac{2m_l^2}{M^2}\right)\left(1-\frac{M^2}{m_\pi^2}\right)^3 |F^{\pi^0\to\gamma\gamma}(M)|^2, \tag{1}$$

where

$$\begin{aligned}
&F^{\pi^0\to\gamma\gamma}(M) = 1 + B_{\pi^0}M^2, \quad B_{\pi^0} = 5.5\ \text{GeV}^{-2}\\
&\Gamma^{\pi^0\to\gamma\gamma} = 7.8\cdot 10^{-9}\ \text{GeV}\\
&\Gamma^{tot}_{\pi^0} \simeq \Gamma^{\pi^0\to\gamma\gamma}\\
&Br^{\pi^0\to\gamma\gamma} = 0.988
\end{aligned} \tag{2}$$

2.2 Dalitz decay $\eta \to \gamma\ l^+l^-$

$$\frac{d\Gamma^{\eta\to\gamma\ l+l-}}{dM} = \frac{4\alpha}{3\pi}\frac{\Gamma^{\eta\to\gamma\gamma}}{M}\left(1-\frac{4m_l^2}{M^2}\right)^{1/2}\left(1+\frac{2m_l^2}{M^2}\right)\left(1-\frac{M^2}{m_\eta^2}\right)^3 |F^{\eta\to\gamma\gamma}(M)|^2, \tag{3}$$

where

$$\begin{aligned}
&F^{\eta\to\gamma\gamma}(M) = \left(1-\frac{M^2}{\Lambda_\eta^2}\right)^{-1} \quad \Lambda_\eta = 0.72\ \text{GeV}\\
&\Gamma^{\eta\to\gamma\gamma} = 4.6\cdot 10^{-7}\ \text{GeV}\\
&\Gamma^{tot}_{\eta} = 1.18\cdot 10^{-6}\ \text{GeV}\\
&Br^{\eta\to\gamma\gamma} = 0.3933
\end{aligned} \tag{4}$$

2.3 Dalitz decay $\omega \to \gamma\ l^+l^-$

$$\begin{aligned}
\frac{d\Gamma^{\omega\to\pi^0\ l+l-}}{dM} &= \frac{2\alpha}{3\pi}\frac{\Gamma^{\omega\to\pi^0\gamma}}{M}\left(1-\frac{4m_l^2}{M^2}\right)^{1/2}\left(1+\frac{2m_l^2}{M^2}\right)\\
&\times\left[\left(1+\frac{M^2}{(m_\omega^2-m_\pi^2)}\right)^2 - \frac{4m_\omega^2 M^2}{(m_\omega^2-m_\pi^2)^2} - \right]^{3/2} |F^{\omega\to\pi^0 l^+l^-}(M)|^2,
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
&|F^{\omega\to\pi^0 l^+l^-}(M)|^2 = \frac{\Lambda_\omega^4}{(\Lambda_\omega^2-M^2)^2+\Lambda_\omega^2\Gamma_\omega^2} \quad \Lambda_\omega = 0.65\ \text{GeV}, \quad \Gamma_\omega = 0.075\ \text{GeV}\\
&\Gamma^{\omega\to\pi^0\gamma} = 7.17\cdot 10^{-4}\ \text{GeV}\\
&\Gamma^{tot}_{\omega} = 8.44\cdot 10^{-3}\ \text{GeV}\\
&Br^{\omega\to\pi^0\gamma} = 0.085
\end{aligned} \tag{6}$$

2.4 Dalitz decay $\eta' \to \gamma\ l^+l^-$

$$\frac{d\Gamma^{\eta'\to\gamma\ l+l-}}{dM} = \frac{4\alpha}{3\pi}\frac{\Gamma^{\eta'\to\gamma\gamma}}{M}\left(1-\frac{4m_l^2}{M^2}\right)^{1/2}\left(1+\frac{2m_l^2}{M^2}\right)\left(1-\frac{M^2}{m_{\eta'}^2}\right)^3|F^{\eta'\to\gamma\gamma}(M)|^2, \tag{7}$$

where

$$\begin{aligned}
&|F^{\eta'\to\gamma\gamma}(M)|^2 = \frac{\Lambda_{\eta'}^4}{(\Lambda_{\eta'}^2-M^2)^2+\Lambda_{\eta'}^2\Gamma_{\eta'}^2} \quad \Lambda_{\eta'} = 0.75 \text{ GeV} \quad \Gamma_{\eta'} = 0.14 \text{ GeV}\\
&\Gamma^{\eta'\to\gamma\gamma} = 4.28\cdot 10^{-6} \text{ GeV}\\
&\Gamma_{\eta'}^{tot} = 0.202\cdot 10^{-3} \text{ GeV}\\
&Br^{\eta'\to\gamma\gamma} = 0.0212\\
&m_{\eta'} = 0.95778 \text{ GeV}
\end{aligned} \tag{8}$$

2.5 Dalitz decay $\Delta \to N\ l^+l^-$

$$\frac{d\Gamma^{\Delta\to N\ l+l-}}{dM} = \frac{2\alpha}{3\pi}\frac{\Gamma_0(M,M_\Delta)}{M}, \tag{9}$$

$$\begin{aligned}
&\Gamma_0(M,M_\Delta) = \frac{\lambda^{1/2}(M^2,m_N^2,M_\Delta^2)}{16\pi M_\Delta^2}\cdot m_N\cdot[2m_T(M,M_\Delta)+m_L(M,M_\Delta)]\\
&m_L(M,M_\Delta) = (efg)^2\frac{M_\Delta^2}{9m_N}M^2\cdot 4(M_\Delta-m_N-q_0), \quad e^2 = 4\pi\alpha, \quad g = 5.44\\
&m_T(M,M_\Delta) = (efg)^2\frac{M_\Delta^2}{9m_N}\left[q_0^2(5M_\Delta-3(q_0+m_N))-M^2(M_\Delta+m_N+q_0)\right]\\
&f = -1.5\frac{M_\Delta+m_N}{m_N((m_N+M_\Delta)^2-M^2)}\\
&q_0 = (M^2+p_f^2)^{1/2}\\
&p_f^2 = \frac{(M_\Delta^2-(m_N+M)^2)(M_\Delta^2-(m_N-M)^2)}{4M_\Delta^2}\\
&\lambda(M^2,m_N^2,M_\Delta^2) = M^4+m_N^4+M_\Delta^4-2(M^2m_N^2+M^2M_\Delta^2+m_N^2M_\Delta^2).
\end{aligned} \tag{10}$$

Here M_Δ is the current mass of the Δ-resonance - calculated in HSD according to the spectral function with the total width from Ref. [12] (cf. also Ref. [11]) :

$$\Gamma_\Delta^{tot}(M_\Delta) = \Gamma_R\frac{M_{\Delta 0}}{M_\Delta}\cdot\left(\frac{q}{q_r}\right)^3\cdot F^2(q) \tag{11}$$

$$\begin{aligned}
&F(q) = \frac{\beta_r^2+q_r^2}{\beta^2+q^2}\\
&q^2 = \frac{(M_\Delta^2-(m_N+m_\pi)^2)(M_\Delta^2-(m_N-m_\pi)^2)}{4M_\Delta^2}\\
&q_r^2 = 0.051936, \quad \beta_r^2 = 0.09\\
&\Gamma_R = 0.11 \text{ GeV}, \quad M_{\Delta 0} = 1.232 \text{ GeV}
\end{aligned} \tag{12}$$

3 Direct decay of vector mesons $V \to\ l^+l^-$

The dilepton decay width of vector meson V with the mass M (calculated in HSD according to the spectral function) is

$$\Gamma^{V\to l^+l^-}(M) = C_V \frac{m_V^4}{M^3}, \tag{13}$$

where $C_V = \frac{\Gamma^{V\to l^+l^-}(m_V)}{m_V}$, m_V is the pole mass of the vector meson V.

For broad resonances such as ρ meson, the branching ratio to dileptons depends on the mass M:

$$Br^{V\to l^+l^-}(M) = \frac{\Gamma_{V\to l^+l^-}(M)}{\Gamma^V_{tot}(M)}. \tag{14}$$

Here the total width of the ρ meson is

$$\Gamma^\rho_{tot}(M) \quad \simeq \quad \Gamma_{\rho\to\pi\pi} = \Gamma_0 \left(\frac{m_V}{M}\right)^2 \left(\frac{q}{q_V}\right)^3 \tag{15}$$

$$q = \frac{(M^2 - 4m_\pi^2)^{1/2}}{2}, \quad q_V = \frac{(m_V^2 - 4m_\pi^2)^{1/2}}{2}$$

For narrow resonances such as $\omega, \phi, J/\Psi, \Psi'$ a constant total width and branching ratio are used: $\Gamma^V_{tot} \equiv \Gamma_{tot}(m_V)$, $Br_0^{V\to l^+l^-} \equiv Br^{V\to l^+l^-}(m_V)$.

Table 2: The parameters for dilepton decay of vector mesons used in HSD 2.5

meson V	electron pair: e^+e^-	muon pair $\mu^+\mu^-$
ρ-meson	$Br^{\rho\to e^+e^-}(m_\rho) = 4.49 \cdot 10^{-5}$ $\Gamma^{\rho\to e^+e^-}(m_\rho) = 6.77 \cdot 10^{-6}$ GeV $C^{\rho\to e^+e^-}(m_\rho) = 8.814 \cdot 10^{-6}$	$Br^{\rho\to\mu^+\mu^-}(m_\rho) = 4.6 \cdot 10^{-5}$ $\Gamma^{\rho\to\mu^+\mu^-}(m_\rho) = 6.9 \cdot 10^{-6}$ GeV $C^{\rho\to\mu^+\mu^-}(m_\rho) = 8.96 \cdot 10^{-6}$
ω-meson	$Br^{\omega\to e^+e^-}(m_\omega) = 7.07 \cdot 10^{-5}$ $\Gamma^{\omega\to e^+e^-}(m_\omega) = 0.6 \cdot 10^{-6}$ GeV $C^{\omega\to e^+e^-}(m_\omega) = 0.767 \cdot 10^{-6}$	$Br^{\omega\to\mu^+\mu^-}(m_\omega) = 8.06 \cdot 10^{-5}$ GeV $\Gamma^{\omega\to\mu^+\mu^-}(m_\omega) = 0.68 \cdot 10^{-6}$ $C^{\omega\to\mu^+\mu^-}(m_\omega) = 0.863 \cdot 10^{-6}$
ϕ-meson	$Br^{\phi\to e^+e^-}(m_\phi) = 2.91 \cdot 10^{-4}$ $\Gamma^{\phi\to e^+e^-}(m_\phi) = 1.297 \cdot 10^{-6}$ GeV $C^{\phi\to e^+e^-}(m_\phi) = 1.27 \cdot 10^{-6}$	$Br^{\phi\to\mu^+\mu^-}(m_\phi) = 3.7 \cdot 10^{-4}$ $\Gamma^{\phi\to\mu^+\mu^-}(m_\phi) = 1.649 \cdot 10^{-6}$ GeV $C^{\phi\to\mu^+\mu^-}(m_\phi) = 1.618 \cdot 10^{-6}$
J/Ψ-meson	$Br^{J/\Psi\to e^+e^-}(m_{J/\Psi}) = 5.93 \cdot 10^{-2}$ $\Gamma^{J/\Psi\to e^+e^-}(m_{J/\Psi}) = 5.26 \cdot 10^{-6}$ GeV $C^{J/\Psi\to e^+e^-}(m_{J/\Psi}) = 1.698 \cdot 10^{-6}$	$Br^{J/\Psi\to\mu^+\mu^-}(m_{J/\Psi}) = 5.88 \cdot 10^{-2}$ $\Gamma^{J/\Psi\to\mu^+\mu^-}(m_{J/\Psi}) = 5.12 \cdot 10^{-6}$ GeV $C^{J/\Psi\to\mu^+\mu^-}(m_{J/\Psi}) = 1.652 \cdot 10^{-6}$
Ψ'-meson	$Br^{\Psi'\to e^+e^-}(m_{\Psi'}) = 8.8 \cdot 10^{-3}$ $\Gamma^{\Psi'\to e^+e^-}(m_{\Psi'}) = 2.12 \cdot 10^{-6}$ GeV $C^{\Psi'\to e^+e^-}(m_{\Psi'}) = 0.575 \cdot 10^{-6}$	$Br^{\Psi'\to\mu^+\mu^-}(m_{\Psi'}) = 1.03 \cdot 10^{-2}$ $\Gamma^{\Psi'\to\mu^+\mu^-}(m_{\Psi'}) = 2.853 \cdot 10^{-6}$ GeV $C^{\Psi'\to\mu^+\mu^-}(m_{\Psi'}) = 0.774 \cdot 10^{-6}$

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