The fate of weakly bound light nuclei in central collider experiments: a challenge in favor of a late continuous decoupling mechanism

Jörn Knoll, GSI, Mai 2024

Abstract:
Arguments are presented that the reaction products of central high energy nuclear collisions up to collider energies can be understood in terms of a continuous decoupling mechanism. This includes the "late" decoupling of loosely bound light nuclei such as deuterons or faintly bound hyper-tritons.[1]

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[1] Footnotes and tiny green commends concern verbal clarifications during presentation or during the subsequent discussion.

in Memory

In Memory of Rudolf Bock †April 9, this year

As Founding Father and one of GSI's Research Directors Rudolf Bock initiated and continuously expanded our engagement in high energy nuclear collisions:

50th Anniversary of first Nuclear Beams @ BEVALAC
1974: GSI-LBL Contract (R. Bock - H. Grunder)

∼50 years of Fireball Model (1976)

G.D. Westfall, J. Gosset, P.J. Johansen, A.M. Poskanzer, W.G. Meyer, H.H. Gutbrod, A. Sandoval, R. Stock

S. Nagamiya, M.-C. Lemaire, E. Moeller, S. Schnetzer, G. Shapiro, H. Steiner, I. Tanihata (1981)

First Model descriptions

Hydrodynamics

W. Scheid, H. Müller, W. Greiner (1974)
C.Y. Wong, T.A. Welton (1974)
Y. Kitazoe, M. Sano (1975)
A.A. Amsden, F.H. Harlow, G.F. Bertsch, J.R. Nix, *full rel. 3-d Hydro.* (1976/77)

Non-Equilibrium Transport

Cascade: H.W.Bertini, T.A. Gabriel, R.T. Santoro (1974)
Hard Spheres: J.P. Bondorf, H.T. Feldmeier, S. Garpman, E.C. Halbert (1976)
Rows on Rows: J.K., J. Hüfner (1977)
Cascade: K.K. Gudima, H. Iwe, V.D. Toneev (1978)

Quark Matter at the Horizon

QM-Theory (1980)	QM-Experiment (1980)	Quark Matter 2 (1982)
Bielefeld (H. Satz)	GSI (R. Bock, R. Stock)	Bielefeld (M. Jakob, H. Satz)

General Remarks

- The fundamental Laws of Physics are continuous in space-time[2], this concerns:
  - any Restructuring of Matter (*e.g.* Phase transitions)
  - Decoupling from an interacting medium
- **Question:** How can one understand thermal two parameter fits of central high-energy nuclear collisions?

The following Definitions are used:

- **Freezing-in:** the moment, when in-medium observables become *stationary* and finally agree with the measurements;
- **Decoupling:** the moment, when particles *decouple*, such that they can *undisturbed* reach ASYMPTOTIA.
- **Presented Concept** is based on the *Boltzmann Eq.*
  - generalizations towards *QM, Finite Size, Formation-time, etc.* I published in 2008 (Non-Eq-real-time Formalism)

thanks ⇒ Dima Voskresensky & Yura Ivanov

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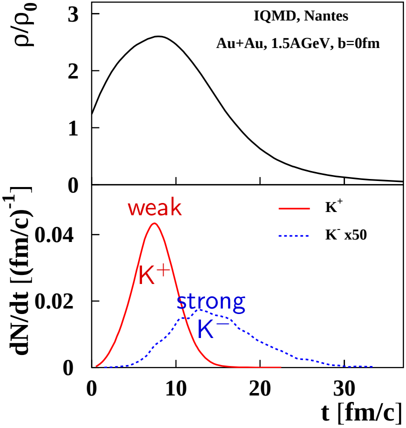
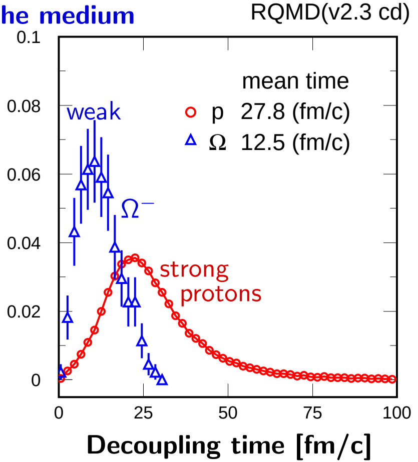
[2] This classifies discontinuous prescriptions such as "Cooper-Frye" or Coalescence methods as *inappropriate theoretical tools*.

Learning from Model Features . . .

Coupling to the medium



[3]



IQMD calc. of K^+ & K^-;
Hartnack et al. 2007

$\Delta t_{\rm dec} \approx 10$ fm/c, $\rho_i/\rho_f \approx 5$

RQMD calc.: Ω^- & protons
van Hecke, Sorge, Xu '98

$\Delta t_{\rm dec} \approx 25$ fm/c, $\rho_i/\rho_f \approx 8$

Model assumes that Ω^- couples weakly

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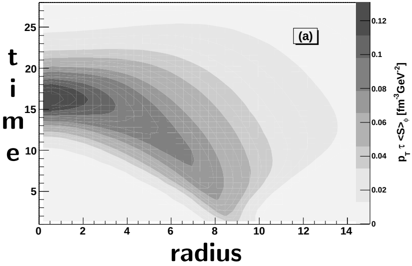
[3] This page and the following ones concern properties of the models.

Decoupling Events (momentum dependence)

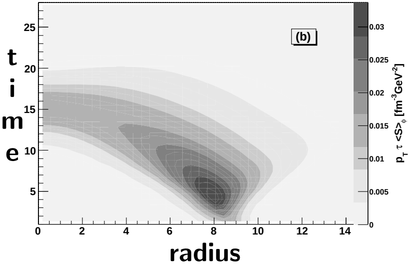
Hybrid Model: Hydro + kinetic Transport (Y. Sinyukov et al.)



pion momentum = 300 MeV/c

$v_\pi = 0.9\,c$

(volume decoupling)



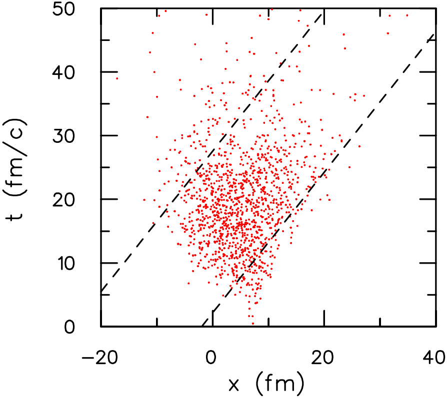
pion momentum = 700 MeV/c

$v_\pi = 0.98\,c$

(surface decoupling)

Decoupling Events & HBT radii

Hybrid Model: Hydro + kinetic Transport (S. Pratt)



HBT radii:

$$R^2_{\rm out} = \langle (x - vt)^2 \rangle$$
$$R^2_{\rm out} \neq \langle x^2 \rangle + v^2 \langle t^2 \rangle$$

$$R^2_{\rm side} = \langle y^2 \rangle$$

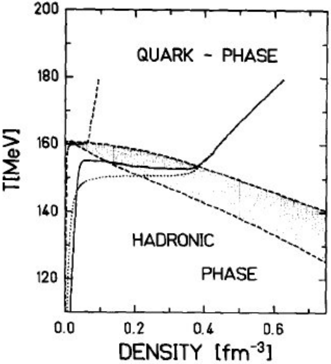
pion momentum = 300 MeV/c

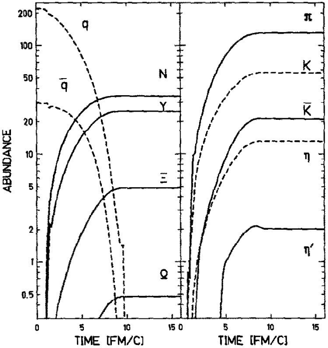
HBT-radii compatible with RHIC events: $R_{\rm out}/R_{\rm side} \approx 1.2$

Phase transition scenario

Flavor Kinetics: QGP → hadronic matter

- 1st order phase transition; H.-W.Barz, B.Friman, H.Schulz
- phase conversion by chemical reactions, & J.K. NPA(1988)
- driven by chemical potentials in compliance with detailed balance.





→ latent heat stabilizes T during phase transition;
→ hadrons are produced during the entire phase transition;
→ phase transition duration ∼5 fm/c;
→ volume changes by factor ∼10;
→ resulting chem. abundance close to chem. equilibrium.

High T-Case (Thermal Fit of ALICE Data)

$\Uparrow v_\perp \approx 0.7c \Leftarrow$ from $\vec{p}$-spectra

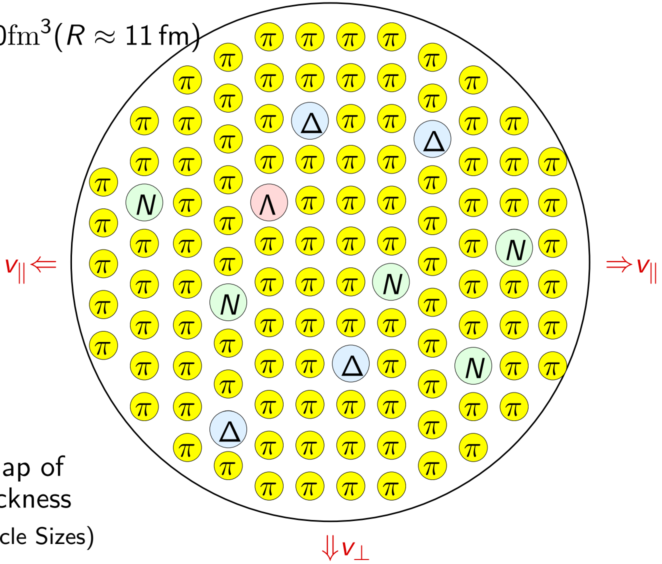
$T = 160\,\mathrm{MeV}$

$V \approx 5300\mathrm{fm}^3 (R \approx 11\,\mathrm{fm})$

$N_\pi \approx 700$

$N_N \approx 30$

$N_\Delta \approx 30$



Central Slap of
2 fm thickness

(r.m.s Particle Sizes)

These sketches displays the spatial situation in the respective local rest-frames with r.m.s. sizes of the particles (except for the Δ-resonances, which are supposed to be bigger).

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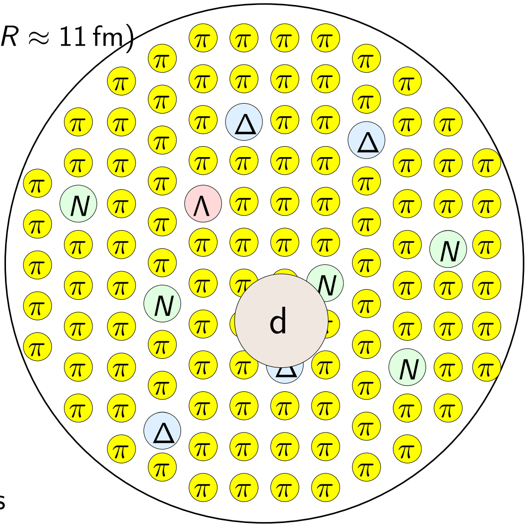
$v_\parallel \Leftarrow$



$\Rightarrow v_\parallel$

$\Downarrow v_\perp$

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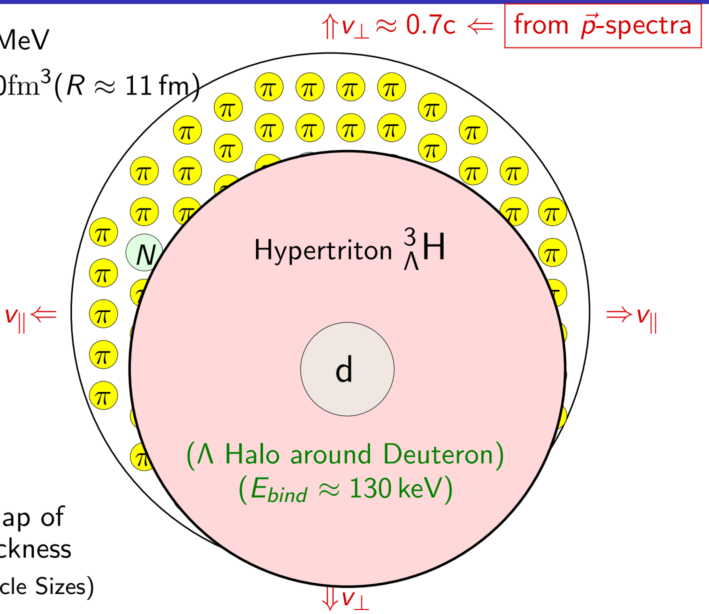


$\Uparrow v_\perp \approx 0.7c \Leftarrow$ from $\vec{p}$-spectra

$v_\parallel \Leftarrow$

$\Rightarrow v_\parallel$

Hypertriton ${}^3_\Lambda\mathrm{H}$

(Λ Halo around Deuteron)
($E_{bind} \approx 130\,\mathrm{keV}$)

$\Downarrow v_\perp$

Central Slap of
2 fm thickness

(r.m.s Particle Sizes)

These sketches displays the spatial situation in the respective local rest-frames with r.m.s. sizes of the particles (except for the Δ-resonances, which are supposed to be bigger).

Instantaneous decoupling

Instantaneous Decoupling:

$$\frac{\mathrm{d}^3 N}{\mathrm{d}p^3} = \int \frac{\mathrm{d}^3x\,\mathrm{d}\tau}{(2\pi\hbar)^3}\, \underbrace{F(\vec{x},\underset{\Uparrow}{\vec{p}},\tau)}_{\text{in Eq.: KMS}}\, \underbrace{\delta(\tau-\tau_{\text{freeze}})}_{\text{Hypersurface}}$$

Detector Yields

In-Medium Properties

- **What is odd about it?**

Instantaneous decoupling

Instantaneous Decoupling:

[Meter] [Femto-Meter]

$$\frac{d^3N}{dp^3} \cancel{=} \int\frac{d^3x\,d\tau}{(2\pi\hbar)^3}\underbrace{F(\vec{x},\underset{\Uparrow}{\vec{p}},\tau)}_{\text{in Eq.: KMS}}\underbrace{\delta(\tau-\tau_{\text{freeze}})}_{\text{Hypersurface}}$$

violates Continuity of Equations of Motion!

Detector Yields

In-Medium Properties

- **What is odd about it?**
- **There is no Control, whether the particles can reach ASYMPTOTIA!**
- **Why can then data be fitted (Spectra, Abundances)?**
- **How to cure?**

Individual Continuous Decoupling

Continuous Decoupling: (in Boltzmann Eq. picture)

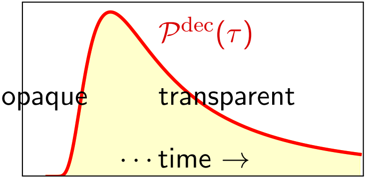
$$\Gamma(\vec{x},\vec{p},\tau) = \sigma(p_{\mathrm{rel}})\,\rho(\vec{x},\tau)\,v_{\mathrm{rel}}$$

$$\frac{\mathrm{d}^3N}{\mathrm{d}p^3}=\int\frac{\mathrm{d}^3x\,\mathrm{d}\tau}{(2\pi\hbar)^3}\,\overbrace{\underset{\substack{\Uparrow\\ \overbrace{\text{in Eq.: KMS}}}}{F(\vec{x},\vec{p},\tau)}\,\Gamma(\vec{x},\vec{p},\tau)}^{\Gamma_{\mathrm{gain}}(\vec{x},\vec{p},\tau)}\times\underbrace{\overbrace{\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,\mathrm{d}\tau'\right]}^{P_{\mathrm{survival}}(\vec{x},\vec{p},\tau)}}_{\mathcal{P}^{\mathrm{dec}}(\tau)}$$

Detector Yields

In-Medium Properties

$$\int_0^\infty \mathrm{d}\tau\,\underbrace{\Gamma(\tau)\exp\left\{-\int_\tau^\infty \mathrm{d}\tau'\,\Gamma(\tau')\right\}}_{\mathcal{P}^{\mathrm{dec}}(\tau)} = 1$$



Individual Continuous Decoupling

Continuous Decoupling: $\Gamma(\vec{x},\vec{p},\tau)=\sigma(p_{\mathrm{rel}})\,\rho(\vec{x},\tau)v_{\mathrm{rel}}$
(in Boltzmann Eq. picture)

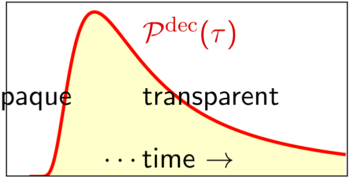
$$\frac{\mathrm{d}^3N}{\mathrm{d}p^3}=\int\frac{\mathrm{d}^3x\mathrm{d}\tau}{(2\pi\hbar)^3}\overbrace{\underbrace{F(\vec{x},\vec{p},\tau)}_{\substack{\Uparrow\\ \text{in Eq.: KMS}}}\,\Gamma(\vec{x},\vec{p},\tau)}^{\Gamma_{\mathrm{gain}}(\vec{x},\vec{p},\tau)}\times\overbrace{\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,\mathrm{d}\tau'\right]}^{P_{\mathrm{survival}}(\vec{x},\vec{p},\tau)}$$

$$\underbrace{\Gamma(\vec{x},\vec{p},\tau)\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,\mathrm{d}\tau'\right]}_{\mathcal{P}^{\mathrm{dec}}(\tau)}$$

Detector Yields

$\boxed{\textbf{In-Medium Properties}}$

$$\int_0^\infty\mathrm{d}\tau\,\underbrace{\Gamma(\tau)\exp\{-\int_\tau^\infty\mathrm{d}\tau'\Gamma(\tau')\}}_{\mathcal{P}^{\mathrm{dec}}(\tau)}=1$$



- Determines: Survival Probability to reach ASYMPTOTIA
- Individual: Each particle has it own $\boxed{\text{Decoupling-Window}}$
- Γ depends on observable: $\sigma=\sigma_{\mathrm{tot}}$ for Spectra;
  $\sigma=\sigma_{\mathrm{inel}}$ for Abundances
- Generalizations: Near-zone Interactions, Finite size, Formation-Time & QM Effects ⇒ J.K. (2008).

Individual Continuous Decoupling

Continuous Decoupling: $\Gamma(\vec{x},\vec{p},\tau)=\sigma(p_{\rm rel})\,\rho(\vec{x},\tau)v_{\rm rel}$
(in Boltzmann Eq. picture)

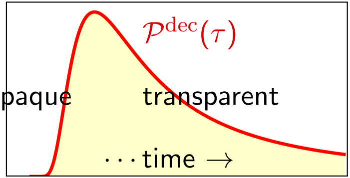
$$\frac{\mathrm{d}^3N}{\mathrm{d}p^3}=\int\frac{\mathrm{d}^3x\mathrm{d}\tau}{(2\pi\hbar)^3}\overbrace{\underbrace{\underset{\Uparrow}{F(\vec{x},\vec{p},\tau)}}_{\text{in Eq.: KMS}}\Gamma(\vec{x},\vec{p},\tau)}^{\Gamma_{\rm gain}(\vec{x},\vec{p},\tau)}\overset{\times}{}\overbrace{\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,\mathrm{d}\tau'\right]}^{P_{\rm survival}(\vec{x},\vec{p},\tau)}$$

$$\underbrace{\Gamma(\vec{x},\vec{p},\tau)\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,\mathrm{d}\tau'\right]}_{\mathcal{P}^{\rm dec}(\tau)}$$

Detector Yields

In-Medium Properties

$$\int_0^\infty\mathrm{d}\tau\,\underbrace{\Gamma(\tau)\exp\{-\int_\tau^\infty\mathrm{d}\tau'\Gamma(\tau')\}}_{\mathcal{P}^{\rm dec}(\tau)}=1$$



maximum at: $\left[\dot{\Gamma}(\tau)+\Gamma^2(\tau)\right]_{\tau_{\rm max}}=0,$ with $\mathcal{P}^{\rm dec}(\tau_{\rm max})\approx\Gamma(\tau_{\rm max})/\mathrm{e}$

uncertainty relation: $\boxed{\Delta\tau_{\rm dec}\approx\frac{\mathrm{e}}{\Gamma(\tau_{\rm max})}}$

$$\boxed{\begin{array}{ccc}\Gamma_i & /\Gamma_{\rm max}/ & \Gamma_f\\ \mathrm{e}^{\mathrm{e}/2}/ & 1 & /\mathrm{e}^{-\mathrm{e}/2}\end{array}}$$

Individual Continuous Decoupling

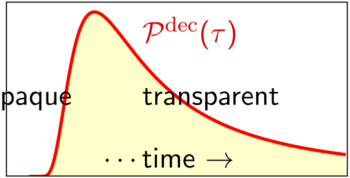
Continuous Decoupling: $\Gamma(\vec{x},\vec{p},\tau)=\sigma(p_{\rm rel})\,\rho(\vec{x},\tau)\,v_{\rm rel}$
(in Boltzmann Eq. picture)

$$\frac{d^3N}{dp^3}=\int\frac{d^3x\,d\tau}{(2\pi\hbar)^3}\;\overset{\Gamma_{\rm gain}(\vec{x},\vec{p},\tau)\qquad\times\qquad P_{\rm survival}(\vec{x},\vec{p},\tau)}{\underset{\underset{\underbrace{\text{in Eq.: KMS}}}{\Uparrow}}{F(\vec{x},\vec{p},\tau)}\,\underbrace{\Gamma(\vec{x},\vec{p},\tau)\,\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,d\tau'\right]}_{\mathcal{P}^{\rm dec}(\tau)}}$$

Detector Yields

In-Medium Properties

$$\int_0^\infty d\tau\,\underbrace{\Gamma(\tau)\exp\{-\int_\tau^\infty d\tau'\,\Gamma(\tau')\}}_{\mathcal{P}^{\rm dec}(\tau)}=1$$



maximum at: $\left[\dot{\Gamma}(\tau)+\Gamma^2(\tau)\right]_{\tau_{\rm max}}=0,$ with $\mathcal{P}^{\rm dec}(\tau_{\rm max})\approx\Gamma(\tau_{\rm max})/{\rm e}$

(Toy Model)

$$\Gamma=\Gamma_0\left(\frac{\tau_0}{\tau}\right)^3\Rightarrow\boxed{\Gamma^2(\tau_{\rm max})=\frac{3}{\Gamma_0\tau_0^3}=\frac{1}{\tau_{\rm max}^2}}\quad\boxed{T(\tau_{\rm max})=\tau_0\left(\frac{\tau_0}{\tau_{\rm max}}\right)^{3(\kappa-1)}}$$

Individual Continuous Decoupling

Continuous Decoupling: $\Gamma(\vec{x},\vec{p},\tau)=\sigma(p_{\rm rel})\,\rho(\vec{x},\tau)\,v_{\rm rel}$

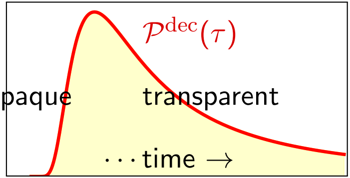
(in Boltzmann Eq. picture)

$$\frac{d^3N}{dp^3}=\int\frac{d^3x\,d\tau}{(2\pi\hbar)^3}\,\overbrace{\underset{\overbrace{\text{in Eq.: KMS}}}{\underset{\Uparrow}{F(\vec{x},\vec{p},\tau)}}\,\mathrlap{\underbrace{\phantom{\Gamma(\vec{x},\vec{p},\tau)\,\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,d\tau'\right]}}_{\mathcal{P}^{\rm dec}(\tau)}}\Gamma(\vec{x},\vec{p},\tau)}^{\Gamma_{\rm gain}(\vec{x},\vec{p},\tau)}\;\overset{\times}{}\;\overbrace{\exp\left[-\int_\tau^\infty\Gamma(\vec{x}(\vec{p},\tau'),\vec{p},\tau')\,d\tau'\right]}^{P_{\rm survival}(\vec{x},\vec{p},\tau)}$$

Detector Yields

In-Medium Properties

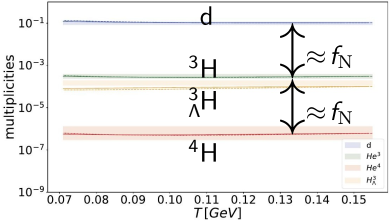
$$\int_0^\infty d\tau\,\underbrace{\Gamma(\tau)\,\exp\{-\int_\tau^\infty d\tau'\,\Gamma(\tau')\}}_{\mathcal{P}^{\rm dec}(\tau)}=1$$



- **Individuality:**
- **The stronger the coupling:**
  ⇒ the later and broader the Decoupling-Window!
- **What is the common “Denominator” that allows Fits with solely two parameters?**
- **the Solution rests on Wisdom from 200 Years ago!**

Model Data

Neidig *et. al.* U. Frankfurt

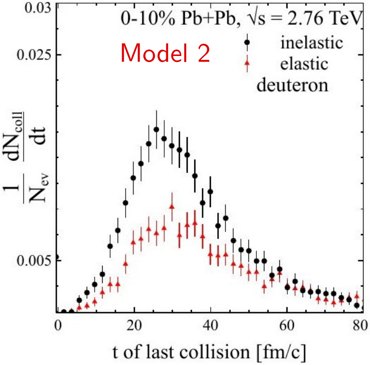


⟵ time ⟵

Model 1

(to the authors' Surprise)

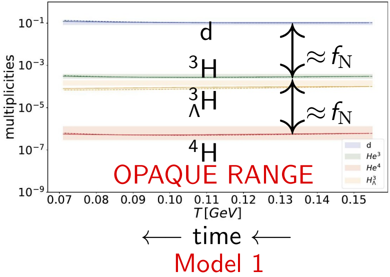
Oliinychenko *et. al.* LBL & GSI



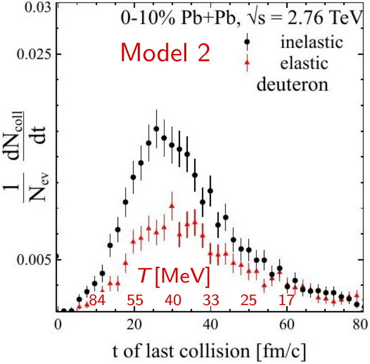
- Let's resolve the surprise and combine the Wisdom of both Models;

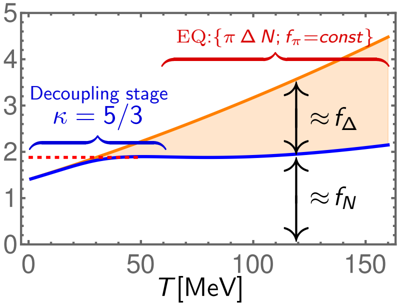
Model Data

Neidig *et. al.* U. Frankfurt



Oliinychenko *et. al.* LBL & GSI





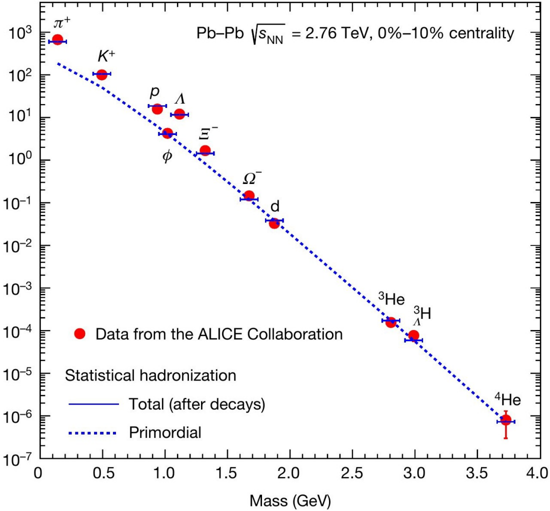
Pions dominate Entropy

$$N_\pi = \underbrace{f_\pi T^{3/2}}_{\text{NR}} V(T) \underbrace{\lambda_{\rm rel}(T/m_\pi)}_{K_2(\dots)\dots}$$

$$@T = 160\,\text{MeV: } \lambda_{\rm rel}(\pi) \approx 4$$

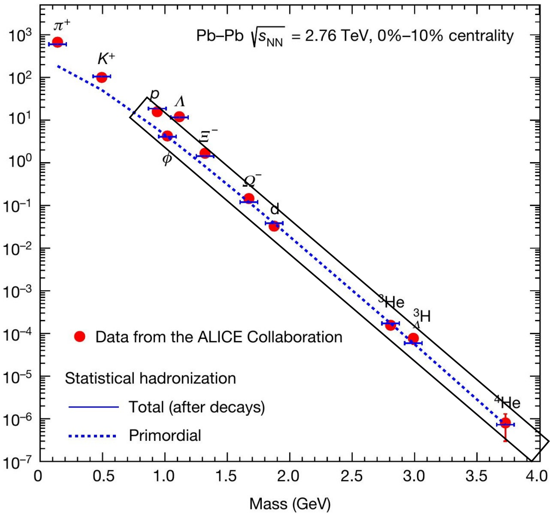
$T(t)$ determined from model 1 via known $V(t)$ and constant N_π (here with $f_\pi = const$).

ALICE Data



- **Local Environments allow Grand Canonical Concepts;**

ALICE Data



- **Local Environments allow Grand Canonical Concepts;**
- **How can this Systematics comply with the Individuality of the Decoupling process?**

Data Systematic

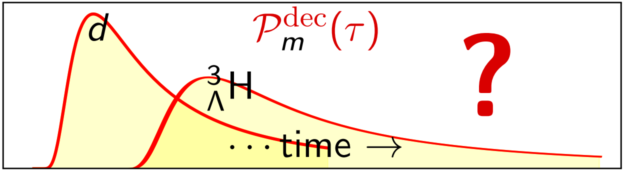
What do the data tell us?

Fugacities of Nuclei with mass m obbey:

$$\boxed{f_m = \exp\left[(\mu_m - m)/T\right] = const_m!}$$

with systematcs: $\boxed{f_m = (f_{\mathrm{N}})^{m/m_{\mathrm{N}}}}$

How to comply with Individuality?:



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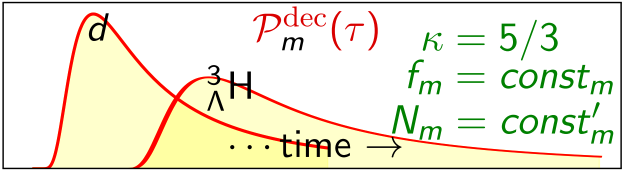
$$\boxed{f_m = \exp\left[(\mu_m - m)/T\right] = const_m!}$$ ✓

with systematcs: $\boxed{f_m = (f_\mathrm{N})^{m/m_\mathrm{N}}}$ ✓ **Chem. Eq.**

All Conditions are fulfilled along non-relativistic Adiabates



$$\kappa = 5/3$$
$$f_m = const_m$$
$$N_m = const'_m$$

$$\boxed{N_m = \underbrace{\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} V}_{\text{adiabaticly const.}} f_m}$$

$$\boxed{f_m \equiv \int \mathrm{d}\tau f_m \mathcal{P}_m^\mathrm{dec}(\tau)}$$

S.D. Poisson (1823), N.L.S. Carnot (1824),
J.P. Joule (1850), W.J.M. Rankine (1866), ...

Summary and Discussion

Freezing-in versus Decoupling of central Collider Experiments:

- All *Model Fits* of abundances and Momentum spectra confirm their **early Freezing-in** soon after *Hadronization* ;
- the **proper Continuous Decoupling** is individual and requires an *unperturbed way* out of the collision zone:
  - weakly interacting probes decouple earlier than strongly interacting ones;
  - the Decoupling of Nuclides depends on their spatial sizes.

- How can then the thermal Model Fits be understood?
  Well, it comes about an intricate Conspiracy, where:

a) below $T \approx 60\text{MeV}$ (*i.e.* once the cycles of Δ-Formation have ceased) the Evolutions converge to NR-Adiabates (with adiabatic index $\kappa = 5/3$)

b) along these Adiabates Entropy, NR-fugacities and Particle Numbers are conserved (Wisdom from 1823/24) and

c) as approximate Nambu-Goldstone Particles the Number of Pions are approximately conserved!

Discussion

Robustly determined Observable

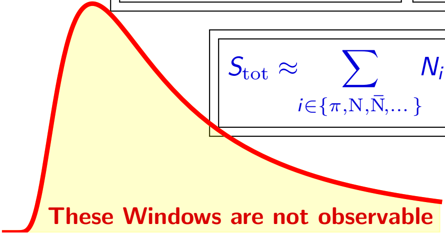
Fugacities

$$f_m = \left(f_{m_\mathrm{N}}\right)^{m/m_\mathrm{N}}; \quad f_\pi\,\mathbf{?}$$

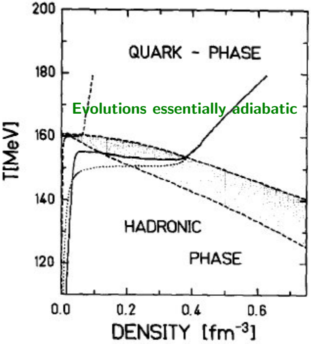
Entropy Balances

$$S/N \approx 5/2 - \ln f$$

$$S_\mathrm{tot} \approx \sum_{i\in\{\pi,\mathrm{N},\bar{\mathrm{N}},\dots\}} N_i\left(5/2 - \ln f_i\right)$$


These Windows are not observable

- Composite Nuclei decouple late
- Some Pions decouple early,
  the rest below $T \approx 60\,\mathrm{MeV}$
- Entropy may look far back in time:
  e.g. even till the moment of
  highest compression



Evolutions essentially adiabatic

Thank You

Robustly determined Observable

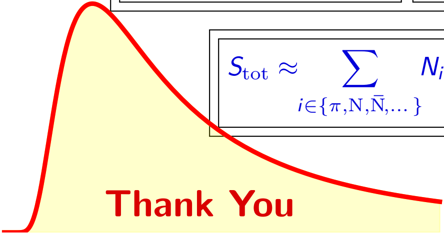
Fugacities

$$f_m = \left(f_{m_\mathrm{N}}\right)^{m/m_\mathrm{N}}; \quad f_\pi \,\mathbf{?}$$

Entropy Balances

$$S/N \approx 5/2 - \ln f$$

$$S_\mathrm{tot} \approx \sum_{i\in\{\pi,\mathrm{N},\bar{\mathrm{N}},\dots\}} N_i\left(5/2 - \ln f_i\right)$$



Thank You

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