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Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

The fate of weakly bound light nuclei in central collider experiments: A challenge in favor of a late continuous decoupling mechanism

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1. General remarks

Since the early days of high energy nuclear collisions, *i.e.* since about 4 decades, the abundances of produced hadrons and composite nuclei were experimentally recorded over a wide range of beam energies up to collider energies at RHIC and LHC, the latter in order to explore the QCD phase-transition dynamics, *cf. e.g.* Refs. [\[1,2](#page-4-0)]. The recently observed production rates are in part covering up to nine orders of magnitude for the various reaction products, *cf.* the recent review [\[3](#page-4-0)] compiled and published by my local colleagues. However, much to the surprise of many colleagues, including me: since the late 80*ℎ* the "statistical hadronization models" showed an amazing stamina. They provide excellent fits to this wealth of data, *cf.* Ref. [\[3,](#page-4-0) Fig. 2] and further references therein, with solely two parameters: the temperature and the baryon-chemical potential $(T, \mu_B)_{0}$, thereby assuming instantaneous global equilibration. Particularly intricate and so far unsettled seemed the production mechanism of very loosely bound nuclei, such as deuterons or the even by far more faintly bound Hypertritons, $¹$ the lat-</sup> ter with halo tails of their wave-functions extending further than the expected system size at $T_0 = 160$ MeV, *cf.* the question raised in the "Outlook" section of Ref. [\[3\]](#page-4-0). Clearly, such loosely bound nuclei are not expected to decouple at such high densities and temperatures [\[5–7\]](#page-4-0).

Let me start with some principle clarifications:

- **1:** The laws of physics are ruled by *continuous equations of motion*, ² irrespective in which kind of approximation scheme ever calculated, with corresponding stochastic interpretations in the quantum case. For the following it is furthermore important to clarify the difference between the here used notions of *FREEZING-IN* and *DECOUPLING*.
- **2: Freezing-in** defines the possibly early situations, from where on certain *in-medium properties* become about stationary and finally approximately equate to the experimentally observed values. Such conclusions can solely be stated on the basis of model considerations.
- **3: Decoupling** is a process which occurs, when matter is subjected to any type of structural changes. This concerns phase-transitions as well as the here addressed release of particles from an interacting medium. It thus defines the continuous physical process, by which the particles finally decouple from the corresponding previous phase. These processes are *individual*, since they rely *e.g.* on interaction cross sections *etc.*

The here addressed process concerns the decoupling from the interaction zone, such that from that moment on the decoupled particles can leave unperturbed as free particles ad infinitum!

- ¹ Hypertriton $\binom{\Lambda}{3}$ H] is a deuteron with a Λ -Hyperon halo with solely 130 keV binding energy and an rms-radius of about 10 fm [\[4\]](#page-4-0).
² This classifies and thus bans discontinuous methods à la Cooper-Frye, or coa
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<https://doi.org/10.1016/j.physletb.2024.139091> Received 19 August 2024; Accepted 21 October 2024

Available online 22 October 2024
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Fig. 1. Schematic plot of the decoupling probability defined in *cf.* Eq. (1b) for two different particles with particle (a) coupling stronger to the medium than particle (b), *cf.* estimate (4b) below.

- **4:** The ubiquitously used **instantaneous Freeze-out concepts**³ tentatively mingle the two above aspects and may seriously misguide conclusions about the here addressed decoupling mechanism.
- **5:** These colliding systems evolve completely adiabatically without external influence. This allows to describe their evolutions in terms of hydrodynamics assuming local thermo-chemical equilibrium along every flowline with an appropriate coarse graining procedure. 4 Collecting the properties of all fluid cells with the same local hydrodynamic properties then defines the Grand Canonical equilibrium ensemble of the measured spectra and abundances.

2. The continuous decoupling picture

The aforementioned deficiency [4](#page-0-0) can be avoided by the here proposed concept of continuous decoupling. It is common textbook wisdom *e.g.* in cosmology [\[8](#page-4-0)], here presented in the Boltzmann Equation (BE) approach⁵ assuming perturbative decoupling. Furthermore a simplified form is used for the here addressed bulk decoupling assuming spatial homogeneity in local proper time τ .

Here we review the *individual properties* of each particle type with mass m . The individual detector yields valid for bosons and fermions then result to [\[5,7,6,8](#page-4-0)]

$$
N^{\text{dec}}(\tau) = \int \frac{\mathrm{d}^3 x \mathrm{d}^3 k}{(2\pi\hbar)^3} \int\limits_{\tau}^{\infty} \mathrm{d}\tau' F(\vec{x}, \vec{k}, \tau') \mathcal{P}^{\text{dec}}(\tau'), \tag{1a}
$$

$$
\mathcal{P}^{\text{dec}}(\tau) = \Gamma(\tau) \exp\left\{-\int\limits_{\tau}^{\infty} \Gamma(\tau') d\tau'\right\},\tag{1b}
$$

$$
\Gamma(\tau) = \left\langle \sigma_{\text{tot}} | v_{\text{rel}} | n(\tau) \right\rangle_{\text{BE}} \tag{1c}
$$

with
$$
\int_{-\infty}^{\infty} d\tau P^{\text{dec}}(\tau) \equiv 1.
$$
 (1d)

Thereby $F(\vec{x}, \vec{k}, \tau)$ and $P_{\text{dec}}(\tau)$ denote each particle's local phasespace occupation and individual decoupling probability [\[6\]](#page-4-0), *cf.* Fig. 1, retro respectively determined by a complete "ad infinitum" solution of the BE analyzed in the adiabatic coarse graining context of introductory Note 5. The local damping rates $\Gamma(\tau)$ depend on total cross-sections of the concerned particles with the surrounding medium with density $n(\tau)$ at relative velocity v_{rel} , *cf.* (1c). Furthermore each decoupling probability (1b) integrates to unity (1d) along each fluid-cell's future path assuring particle number conservation.

The individual BE rates $F(\vec{x}, k, \tau) \times \Gamma(\tau)$ in Eq. (1a) describe the local creation of the observed particles at phase-space point (\vec{x}, k) and local time τ due to transport processes with the surrounding medium. The straight escape paths to *asymptotia* are assumed to proceed in close

Fig. 2. Local adiabatic exponent $\kappa(T/m)$ for adiabatic volume evolution, $TV(T)^{\kappa(T)-1} = const$, *cf.* Eq. (2e), for monomer gases in Boltzmann-statistic conserving corresponding particle numbers for: NR particles ($m \to \infty$), relativistic nucleons (N), particles with intermediate mass of 350 MeV (middle black line) and pions (π) . For massless particles $\kappa = 4/3$.

vicinity of the corresponding local fluid cells. Along them the exponential factor (1b) then determines the particles' individual *survival probabilities*, in future not to be "kicked" off this mode along their escape paths through interactions with the surrounding medium.

Omitting the τ -dependencies the adiabatically evolved densities $n(T, \mu)$ of each relativistic particle become

$$
n(T, \mu) = \int \frac{d^3 k}{(2\pi\hbar)^3} F_{\text{eq}} = \underbrace{\left(\frac{m}{2\pi\hbar^2}\right)^{3/2} f_{\text{eq}}}{N R} \lambda_{\text{rel}}(T/m)
$$
(2a)

with
$$
f_{\text{eq}} = e^{\mu_{\text{NR}}/T}
$$
, $\lambda_{\text{rel}}(T/m) \approx 1 + \frac{2T}{m} + \frac{T^2}{2m^2}$, (2b)

$$
F^{\text{eq}} = \exp[\{\mu - (m^2 + \vec{k}^2)^{1/2}\}/T] \tag{2c}
$$

$$
p(T, \mu) = n(T, \mu)T
$$
 (even relativistic), (2d)

$$
\kappa(T) = 1 + \left(T \frac{\mathrm{d}}{\mathrm{d}T} \ln n(T, \mu(T))\right)^{-1} \tag{2e}
$$

and $\mu_{\text{NR}} = \mu - m$. Here F^{eq} denotes the relativistic thermal singleparticle occupation with pressure $p(T, \mu)$ and relativistic adiabatic index $\kappa(T/m)$, the latter displayed for various masses in Fig. 2. The underbraced part in Eq. (2a) specifies the *non-relativistic* (NR) part of the densities $n(T, \mu)$ with dimensionless *fugacity* (or chemical activity) f_{eq} . Thereby $\lambda_{rel}(T/m)$ is a convenient relativistic correction factor caused by the non-Gaussian forms of the relativistic $F^{eq}(\vec{k})$ with $\vec{k} \in \mathbb{R}^3$ and approximation (2b) valid within 0.5% precision till $T < 2m$.

In each local rest-frame entropy conservation together with particle number conservation further require

$$
\sigma(T,\mu)V(T) = \left(1 + T\frac{\partial}{\partial T}\right)n(T,\mu)V(T)
$$
\n(3a)

$$
= \underbrace{n(T,\mu)V(T)}_{N=const} \bigg\{ \underbrace{\frac{\mu_{\text{NR}}}{T} + \frac{5}{2} + \phi(T/m)}_{\sigma(T,\mu)/n(T,\mu)=const} \bigg\},
$$
 (3b)

$$
\frac{\mu_{\text{NR}}(T)}{T} = \ln f(T) = \ln f(T_0) - \phi\left(\frac{T_0}{m}\right) + \phi\left(\frac{T}{m}\right),\tag{3c}
$$

$$
\phi\left(\frac{T}{m}\right) := T\frac{\partial}{\partial T}\ln\lambda_{\text{rel}}(T/m) = \frac{\frac{2T}{m} + \frac{T^2}{m^2}}{1 + \frac{2T}{m} + \frac{T^2}{2m^2}}.
$$
\n(3d)

Relation (3c) defines the adiabatic courses of the chemical potentials with relativistic corrections $\phi(T/m)$ defined in (3d). The corresponding leading terms are the standard NR forms, like the Sackur–Tetrode formula [\[9,10\]](#page-4-0) for the single particle entropy (3b).

The decoupling probability attains its maximum at

$$
\left[\frac{\mathrm{d}}{\mathrm{d}\tau}\Gamma(\vec{k},\tau) + \Gamma^2(\vec{k},\tau)\right]_{\tau_{\text{max}}} = 0.
$$
 (4a)

The following toy-model

$$
\Gamma(\tau) = \Gamma_0 \left(\frac{\tau_0}{\tau}\right)^3 \Rightarrow \Gamma^2(\tau_{\text{max}}) = \frac{3}{\Gamma_0 \tau_0^3} = \frac{1}{\tau_{\text{max}}^2},\tag{4b}
$$

³ Discontinuous freeze-out schemes are implicitly used, if calculated inmedium spectra are *equated* with the asymptotically measured spectra (they thus bypass the decoupling process).

⁴ Spatially expanding systems require a volume growth adapted coarse graining procedure, such that *e.g.* the mean number of baryons per fluid cell is approximately conserved.

⁵ Generalizations of the decoupling scheme to include non-local and quantum effects are possible *cf. e.g.* my 2008 paper [\[6](#page-4-0)].

$$
\Delta \tau^{\text{dec}} \approx \frac{1}{\mathcal{P}^{\text{dec}}(\tau_{\text{max}})} \approx \frac{e}{\Gamma(\tau_{\text{max}})},\tag{4c}
$$

clarifies the dependencies on the initial $\Gamma_0 := \Gamma(\tau_0)$ [\[6,8](#page-4-0)] with the *simple rule of thumb*: The stronger Γ_0 , the later and broader the decoupling window.

On the left side of each curve in Fig. [1](#page-1-0) the interaction rates along the escape paths are such high, that those particles cannot undisturbed decouple: *i.e.* the exponential survival probabilities in ([1b](#page-1-0)) tend to zero: *Those parts of the medium are opaque for the detectors*!

The general rule along the escape paths: 1∕Γ determines the mean free lifetime to remain in the present mode. In the Quantum sense, *cf.* [\[6](#page-4-0), [11\]](#page-4-0), the spectral functions are correspondingly broadened⁵, unless the particles can freely decouple. With decreasing $\Gamma(\tau)$ the medium becomes successively transparent and the $P_{\text{dec}}(\tau)$ attain forms similarly to those displayed in Fig. [1.](#page-1-0)

For multi-particle systems total entropy along with the overall conservation laws and detailed balance have to be fulfilled, which may lead to complicated evolutions. Still, the above decoupling concept does allow for *entirely analytical analyses* with far reaching consequences.

Alternatively, the continuous decoupling mechanism can be demonstrated in transport simulations, which sample the last interaction events of each particle type, *cf. e.g.* Fig. 6 in Ref. [\[12](#page-4-0)], which correspond to the $N(\tau)$ -distributions defined in Eq. [\(1a\)](#page-1-0). The individuality of decoupling processes was already demonstrated about two decades ago in various transport calculations, *cf. e.g.* [\[13](#page-4-0), Fig. 4], [\[14](#page-4-0), Fig. 23], [\[15](#page-4-0), Fig. 11], [\[16,](#page-4-0) Fig. 2].

3. Evolution steps of central nuclear collisions

Here the essential evolution steps from the hadronization stage until the late decoupling of the composite nuclei, the main focus of this note, are briefly summarized.

3.1. The hadronization stage

This stage concerns the conversion from the QCD phase to the hadronic phase. Also this phase-transition is continuous, during which all hadrons are produced and thereby continuously decouple from the QCD phase in the here addressed sense. In "our" Flavor Kinetic model [\[1,2](#page-4-0)] the decoupling process lasted about 5 fm/c or even shorter, *cf. e.g.* Fig. 1 in [\[1\]](#page-4-0). The created hadrons were produced in approximate chemical equilibrium.⁶

Mesons, that couple weakly to the hadronic medium, have the chance to be *early messengers of the hadronization stage*. Thereby heavy mesons, like charm mesons, are burdened with mass-thresholds, *e.g.* $\propto \exp(-m_{c\bar{c}}/T)$, which drop fast with *T*.

Baryon–anti-baryon annihilation: At collider energies this process is shown by data to quickly cease, $cf.$ the in-medium⁷ results [\[17\]](#page-4-0), such that from then on baryons and anti-baryons evolve essentially decoupled from one another with individual adiabatic $\{T, \mu_B(T)\}$ -courses. For reactions at lower energies, see for instance the in-medium calculations of Refs. [\[18,19\]](#page-4-0).

Fig. 3. Pi-*N***-Delta Entanglement** here estimated in a simple equilibrium model with constant pion fugacity f_{π} . It shows the two stages of the dynamics. Above $T \approx 60$ MeV the $\pi N\Delta$ -cycles hinder the formation of composite nuclei; below this temperature pions decouple from the baryons and the baryon evolution reaches the NR-decoupling stage with $\kappa \approx 5/3$.

3.2. Collective flow

Collective flow is generated from the moment of highest compression on. Depending on bombarding energy this may include its creation during the QCD-phase and the entire hadronization process presumably then generating the major amount of flow [\[1\]](#page-4-0). Since pressure gradients cease fast with the dilution of the system, the collective flow may quickly *freeze in*, in accordance with introductory Note [2](#page-0-0).

A confirming sign of collective flow isthat the mean-square momenta $\langle k^2 \rangle$ of the observed particles' momentum spectra scale with the square of their masses, *i.e.* $\langle k^2 \rangle \propto m^2$ such that $\langle v^2 \rangle \approx \langle k^2/(m^2 + k^2) \rangle \approx const.$

The mean transverse flow velocity can be deduced *e.g.* from the maxima of the transverse momentum spectra, which scale linear in p_{\perp} at low momenta. For all NR-particles measured in Ref. [\[20\]](#page-4-0) these maxima are occurring around $\langle p_{\perp} \rangle \approx m$ determining the mean transverse collective flow velocity to $\langle v_{\perp} \rangle \approx 0.7$ c.

Assuming *e.g.* a flow saturation around $T \approx 150$ MeV allows to estimate the maxima of the decoupling windows. For the model calculations of Ref. [\[12,](#page-4-0) Fig. 6] the maximum of Deuterons can be determined to occur at around $1/8^{th}$ of the initial density within a T-range of $[70 \rightarrow 40]$ MeV, here roughly analyzed with adiabatic indices within the range κ ∈ [1.45 → 5/3], *cf*. Fig. [2](#page-1-0).

3.3. The pion-nucleon-delta entanglement

A special role plays the overwhelming entourage of pions. With p-wave cross-sections of 16 fm² [\[21](#page-4-0)] each pion is eager to form Δ-resonances with the nucleons in continuous creation and decay cycles till approximately $T \approx 60$ MeV, *cf.* Fig. 3 and *e.g.* Ref. [\[12,](#page-4-0) Figs. 6 & 8]. These short-term cycles will seriously obstruct the formation of bound nuclei.

A particular influence of pions on the nuclear bound state abundances is shown *e.g.* by the recent transport approach of Ref. [\[22,](#page-4-0) Fig. 3c] for the Triton yields. The relatively early stabilization of the baryon abundances and their ratios were shown in transport models with pointlike particles *e.g.* in Refs. [\[23–25](#page-4-0)], indicating their early freezing-in already during this high temperature phase in the sense of introductory Note [2](#page-0-0).

Still, sooner or later along with the cease of the Δ-resonance cycles (the precise occurrence is relatively unimportant) the pions will no longer significantly interact with the baryons. This separates the pion dominated high temperature phase with its own adiabatic features from the subsequent decoupling stage of the baryons, which from then on will evolve with their own adiabatic courses, *cf.* Fig. 3, and as discussed below in Sec. [3.5](#page-3-0).

⁶ For years our Flavor Kinetic model was the only phase-transition model that *preserved detailed balance and the Onsager relations*, an essential requirement for reliable predictions, thanks to a recommendation by Gordon Baym, to formulate the rate equations be driven in terms of *chemical potentials* rather than by the commonly used particle densities.

⁷ The term "in-medium" is used for calculations, which determine the inmedium properties *e.g.* till some temperature, without explicitly caring about the here discussed decoupling of the observed particles.

3.4. Bound states in matter

Bound states are the most vulnerable objects in matter environments. First, their sizes cause large cross sections with the surrounding particles. The correspondingly large damping rates Γ generally push their decoupling windows towards lower matter densities than those of their respective constituents.

Secondly, the surrounding matter influences the binding properties of bound states. In the here discussed low energy nuclear physics context the problem was investigated by our German colleagues [\[26,27](#page-4-0)] accounting for Pauli-blocking effects. The relativistic mean field considerations [\[28](#page-4-0), Fig. 1], however, showed no significant effect along the here relevant adiabatic NR-parkour.

Still, the phenomenon is by far more general: in dense matter the bound-state wave-functions can no longer spatially extend to infinity but are rather spatially restricted. Such spatial restrictions cause a spatial squeezing of the bound-state wave-functions inducing an increase of the kinetic energy part of their total bound-state energy. The latter has the effect, that above a certain density the states are no longer bound. Together with the collisional broadening these states then form resonances, whose continuous spectral functions then have pole positions above the bound-states' nominal masses. In particular such geometric effects can *e.g.* significantly affect the Hypertriton's halo-tail for temperatures $T \ge 5$ MeV.⁸ Again the correct individual values are less important.

The on-shell formation of bound states requires a simultaneous interaction with a third party, here properly provided by the BE-rate $\Gamma(\tau)F(\vec{x}, \vec{k}, \tau)$ in Eqs. ([1](#page-1-0)).

Classical transport calculations can still approximately determine the integrated spectral strength of bound states. Thereby their real boundstate nature will be restored at the moment of their last interaction before reaching infinity, as *e.g.* recorded in Ref. [\[12](#page-4-0), Fig. 6].

3.5. Decoupling of light nuclei

The above conceptual considerations in Sects. [3.3](#page-2-0) and 3.4 suggest that the decoupling of light nuclei has to occur by far later than so far generally anticipated [\[3](#page-4-0)], namely along an adiabatic course, where the pions essentially do no longer interact with the (anti-)baryons. Rather than using the thermal fit parameters $(T_0, \mu_B(T_0))$ let us focus on the fact that the fitted *fugacities of all particles* can be interpreted as to be *dynamically constant*!

Upon looking at the comparison of the experimentally observed so called primordial abundances of the light nuclei, *cf.* the dashed line in Ref. [\[3,](#page-4-0) Fig. 2] one states an approximate linear behavior between the logarithm of the individual fugacities $f(m)$ versus mass m. With reference to the nucleon with mass m_N the observed data show

$$
f(m) \approx f(m_N)^{m/m_N}.
$$
 (5a)

How can the above fugacity-systematic ever comply with the *individuality* of a continuous decoupling process along which temperatures continuously drop?

Well, this Gordian knot was cut 200 years ago.⁹ Therefore my *sole* interpretation and conclusion is, that these light nuclei decouple by far later, *cf.* Fig. [3,](#page-2-0) along common NR-adiabatic courses. As a key feature in this context, not only *entropy* and *abundances* of NR-particles are preserved along NR-adiabates but also their *fugacities*, *cf.* Eqs. ([3](#page-1-0)) with NR-property $\phi(T/m) \equiv 0$ in ([3c\)](#page-1-0).

At the here discussed decoupling stages the inter-particle distances between the baryons are such large that an ideal gas equation of state, dominated by the nucleons, is well justified for all baryons with $\kappa = 5/3$ valid for 3 translation degrees of freedom. Their NR-evolutions then proceed in accordance with Poisson's adiabatic laws (1823). The corresponding decoupling yields of the (composite) baryons [\(1a\)](#page-1-0) then simplify to

$$
N^{\text{dec}}(m) = \int \frac{d^3 x d^3 k}{(2\pi \hbar)^3} e^{-\epsilon_{\text{NR}}(k)/T} \int_{0}^{\infty} d\tau P^{\text{dec}}(\tau) f(m)
$$
(5b)

$$
= \left(\frac{mT}{2\pi \hbar^2}\right)^{3/2} V(T) f(m) \propto m^{3/2} f(m),
$$
(5c)

⏟⏞⏞⏞⏞⏞⏞⏞⏞⏞⏟⏞⏞⏞⏞⏞⏞⏞⏞⏞⏟ adiabaticly constant

$$
\ln f(m) = \mu_{\rm NR}(T, m)/T, \ \varepsilon_{\rm NR}(k) = k^2/(2m) \tag{5d}
$$

with Poisson's adiabatic relation under-braced in line (5c). The latter further determines the correct NR mass-dependence as $N_m^{\text{dec}} \propto$ $m^{3/2} f(m)$, *cf.* Eq. ([1a](#page-1-0)). For the above steps both, the constancy of the $f(m)$, *cf.* Eq. [\(3c\)](#page-1-0) with $\phi(T/m) \equiv 0$, together with unity integral ([1d\)](#page-1-0) over $\mathcal{P}^{\text{dec}}(\tau)$ were used.

The fact that the mass systematic of the measured fugacities (5a) could be fitted by one common set of (T, μ) further implies that nucleons and composite nuclei are in *chemical equilibrium* to one another. The NRconstancy of all $f(m)$ further confirms this feature maintained along the entire decoupling courses.

Thus, it is *neither* important, when and during which later time span the measured composite baryons decouple! *Nor* is there any need to determine their individual decoupling windows: The Laws of Nature will just properly care in accordance with Eqs. (5b) *ff*!

The presented analysis uses independent particle concepts under circumstances, where they are well at place.

Notably, neither the individual decoupling windows nor thus the decoupling values of T or adiabatic index κ are directly experimentally measurable.

A final clarification: All in-medium properties such as the above discussed thermo-chemical equilibration are by no means conceptually affected by the decoupling features respectively used on the *r.h.s.* of Eqs. [\(1a\)](#page-1-0) or (5b). The latter are encoded via \mathcal{P}^{dec} , whose individual decoupling windows are solely retrospectively clarified at "*post-completion*" of the system's entire global evolution, *i.e.* at $\tau \to \infty$ via the relations discussed in Sect. [2](#page-1-0).

3.6. Entropy

In my view the here discussed experiments preferentially determine the entropy per particle rather than the hadronization temperature.

Since for NR-particles relativistic correction ϕ vanishes, their entropy per particle can be obtained from the under-braced Sackur-Tetrode factor $[9,10]$ $[9,10]$ in Eq. ([3b\)](#page-1-0) defining the running chemical potential ([3c\)](#page-1-0) for non-relativistic baryons. This then implies the constancy of their $f(m)$, *cf.* Eq. [\(3c\)](#page-1-0), and thus allows to determine the entropy of any created (anti-)baryon. This strategy agrees with the one suggested by Siemens and Kapusta [\[31\]](#page-4-0) concerning the d/p -ratio some 40 years ago, here presented in the continuous decoupling picture.

The determination of the entropy of the dominating pions is more subtle and definitely model dependent. As Nambu-Goldstone bosons [\[32,33\]](#page-4-0) their number is approximately conserved, *cf. e.g.* [\[17](#page-4-0)]. Assuming that only a minority of pions is involved in Δ-formation cycles allows to obtain their entropy per particle via Eq. ([3b\)](#page-1-0) through the experimentally determined pion fugacity $f(m_\pi, T_\pi)$ still for an appropriately to be chosen temperature T_{π} .

The total entropy, which discloses information about the early situation of entropy saturation around the moment of highest compression,

⁸ Thanks to both a geometrical and a rough Beth-Uhlenbeck estimate [\[29\]](#page-4-0) exploiting the entanglement between bound state properties and phase-shifts by my local colleague B. Friman.

⁹ With discoveries of those thermodynamic processes, investigated *e.g.* by S. D. Poisson (1823), N. L. S. Carnot [\[30\]](#page-4-0) (1824) and others, whose properties were classified as *adiabatic* a few decades later.

is then determined through the sum of the baryon parts (dominated by nucleons) plus the meson part (dominated by pions).

4. Summary

The here presented analytical considerations resolve dynamical inter-plays that can barely be disentangled by mere numerical simulations. They explain the for a long time puzzling features of freeze-out and decoupling mechanisms of central high energy nuclear collisions. The success rests on the general concept that any transition process is *individual and has its own time frame*.

None the less certain observables, like the abundances of particles and their ratios *cf. e.g.* Refs. [23–25], can already approximately be stabilized relatively early, here referred to as *freezing-in*. Since pressure gradients quickly cease, also the collective flow field may stabilize correspondingly, this way also predetermining the momentum spectra in the sense of introductory Note [2.](#page-0-0)

Still, all experimentally observed reaction products will *definitely decouple later*. Thereby these processes resolve the latest in-medium situations, which the observed particles have encountered. They determine their abundances and momenta notably during their respective decoupling stages in a model-independent way. Any extrapolation to earlier evolution situations would require appropriate model estimates.

How is it then possible, that all abundances can simply be described by solely two parameters, although the decoupling processes proceed individually?

This rests on a well hidden intricate interplay between the early high temperature phase dominated by pions and the later low temperature evolution. During the latter the here addressed baryon sector evolves essentially decoupled from the pions with its special well known adiabatic properties of non-relativistic particles, namely preserving their fugacities along the entire decoupling parkour. The latter assures that the particular decoupling yields are independent of the temporal appearances and widths of the corresponding decoupling windows. Therefore the formalism gives a physically sound and rigorous explanation of the observed wealth of data. It naturally provides the chance for the undisturbed production of the faintly bound (Anti-)Hypertritons. The latter rests on a very late and sufficiently dilute local decoupling scenario with local bound-state formation then within the realm of standard low energy nuclear physics even at sub-picometer scales.

As explained in Sect. [3.6,](#page-3-0) the method allows for a robust determination of the entropy content carried by the baryons and anti-baryons. Concerning the high temperature sector there is a need to clarify the non-equilibrium aspects of pions and mesons along the entanglement process sketched in Sect. [3.3](#page-2-0) by appropriate model studies, *e.g.* using the techniques and decoupling tools of Sect. [2](#page-1-0).

In 1907 G. W. Lewis [34] has shown that fugacities (*activities* in chemistry) are measurable observables, *e.g.* through a thermochemical contact with a calibrated reference system. In this context, the fugacities determined in these central collision experiments are then straight messengers of the particles' physical in-medium properties during their proper individual decoupling stages.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

I thank my (local) colleagues, A. Andronic, P. Braun-Munzinger, B. Friman, C. Greiner, H. van Hees and J. Wambach for clarifying discussions, suggestions and encouragements on various occasions.

Further thanks address S. Pratt and Y. Sinyukov, who emphasized the importance of adiabatic behaviors in the context of HBT and the here addressed decoupling phenomena during some discussions with me already more than a dozen years ago.

As senior scientist *i. R.* I gratefully acknowledge the research options provided by my former research institute GSI.

Data availability

No data was used for the research described in the article.

References

- [1] H. Barz, B. Friman, J. Knoll, H. Schulz, Nucl. Phys. A 484 (1988) 661 [\(Erratum\).](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib50C3642A64B47F33D9C895AB999CDA7Es1)
- [2] H. Barz, B. [Friman,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibF9E404E4732739C256FFAE428B32B553s1) J. Knoll, H. Schulz, Nucl. Phys. A 519 (1990) 831.
- [3] A. Andronic, P. [Braun-Munzinger,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib4736B1730B576C850D563287601BBE39s1) K. Redlich, J. Stachel, Nature 561 (2018) [321–330.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib4736B1730B576C850D563287601BBE39s1)
- [4] M. Juric, et al., Nucl. Phys. B 52 [\(1973\)](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibDB213CBE67F28CC702100839E25534DAs1) 1.
- [5] Y.M. [Sinyukov,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib49C65B10BA0D0DF54BB84160F0097BECs1) S.V. Akkelin, Y. Hama, Phys. Rev. Lett. 89 (2002) 052301.
- [6] J. Knoll, Nucl. Phys. A 821 (2009) 235, [arXiv:0803.2343.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib7054172903890462BDBBBCB0362A12A2s1)
- [7] B. Friman, C. Höhne, J. Knoll, S. Leupold, J. [Randrup,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib471FA33ABCF6572CA2B1A5791824DE62s1) et al., The CBM Physics Book: Compressed Baryonic Matter in Laboratory [Experiments,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib471FA33ABCF6572CA2B1A5791824DE62s1) vol. 814, Springer Berlin [Heidelberg](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib471FA33ABCF6572CA2B1A5791824DE62s1) New York, 2011, Part III, Sect. 5.8.
- [8] V. Mukhanov, Physical [Foundations](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibF5F3196CD6C8B924599AC264A6E4CDD3s1) of Cosmology, Cambridge University Press, Oxford, [2005.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibF5F3196CD6C8B924599AC264A6E4CDD3s1)
- [9] O. Sackur, Ann. Phys. 41 (1911) [958–980.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib64DAF7964112253F9909722EA2BCD30Fs1)
- [10] H. Tetrode, Ann. Phys. 38 (1912) 434-442.
- [11] Y. Ivanov, J. Knoll, D. [Voskresensky,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibE08B64F58994BC3B629E17F8E906F0C9s1) Nucl. Phys. A 672 (2000) 313, arXiv:nucl-th/ [9905028.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibE08B64F58994BC3B629E17F8E906F0C9s1)
- [12] D. [Oliinychenko,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibCEAC34BC119E3348C4EC8CA39AF3C9E3s1) L.-G. Pang, H. Elfner, V. Koch, Phys. Rev. C 99 (2019) 044907, [arXiv:1809.03071.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibCEAC34BC119E3348C4EC8CA39AF3C9E3s1)
- [13] H. van Hecke, H. Sorge, N. Xu, Phys. Rev. Lett. 81 (1998) 5764, [arXiv:nucl-th/](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib01F0E62E6BC48F315777A9AC18C3A6BBs1) [9804035.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib01F0E62E6BC48F315777A9AC18C3A6BBs1)
- [14] A. Förster, et al., Phys. Rev. C 75 (024906) (2007), [arXiv:nucl-ex/0701014.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibF3A4262847633D3A3BF5D9CD02269054s1)
- [15] S.V. Akkelin, Y. Hama, I.A. [Karpenko,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibA989094F02B94222D492B7723C4A78B2s1) Y.M. Sinyukov, Phys. Rev. C 78 (2008) 034906, [arXiv:0804.4104.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibA989094F02B94222D492B7723C4A78B2s1)
- [16] S. Pratt, Phys. Rev. Lett. 102 (2009) [232301.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibC6209BE8E38595ED3F6A404402E11762s1)
- [17] Y. Pan, S. Pratt, Phys. Rev. C 89 (2014) [044911.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibB45EB85E14010F60FBFF17FC0BF99ECCs1)
- [18] R. Rapp, E.V. Shuryak, Phys. Rev. Lett. 86 (2001) 2980, [arXiv:hep-ph/0008326.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibA733909F30A84DD3207C8E384603C5CBs1)
- [19] C. Greiner, S. Leupold, J. Phys. G 27 (2001) L95, [arXiv:nucl-th/0009036.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibD78526D890F86609375E0D1507F7F0EBs1)
- [20] S. Acharya, et al., ALICE, [arXiv:2311.11758,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibD0784A7FD386301A3F8A266075D5E4A2s1) 2023.
- [21] H.L. [Anderson,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib2649BA35248615F63D64FC3FA687BC64s1) E. Fermi, E.A. Long, D.E. Nagle, Phys. Rev. 85 (1952) 936.
- [22] K.-J. Sun, R. Wang, C.M. Ko, Y.-G. Ma, C. Shen, Nat. [Commun.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib7008FCC90B9A1960BCEEAB430AD1BAA1s1) 15 (2024) 1074, [arXiv:2207.12532.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib7008FCC90B9A1960BCEEAB430AD1BAA1s1)
- [23] J. Staudenmaier, D. Oliinychenko, J.M. [Torres-Rincon,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibA5A5C91F91443E5F6F8DFFBFBF74E2CFs1) H. Elfner, SMASH, Phys. Rev. C 104 (2021) 034908, [arXiv:2106.14287.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibA5A5C91F91443E5F6F8DFFBFBF74E2CFs1)
- [24] V. Vovchenko, K. Gallmeister, J. [Schaffner-Bielich,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibEE59269C18F3131EE197F398B5FF8EEDs1) C. Greiner, Phys. Lett. B 800 (2020) 135131, [arXiv:1903.10024.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibEE59269C18F3131EE197F398B5FF8EEDs1)
- [25] T. Neidig, K. [Gallmeister,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib4A294B5053635CF3957948AB67458BE5s1) C. Greiner, M. Bleicher, V. Vovchenko, Phys. Lett. B 827 (2022) [136891.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib4A294B5053635CF3957948AB67458BE5s1)
- [26] G. Röpke, L. [Münchow,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib557071B81C045F3E378D977B8AF4875Ds1) H. Schulz, Nucl. Phys. A 379 (1982) 536.
- [27] G. Röpke, M. Schmidt, L. [Münchow,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib4E62A052A3FB7DC6E1245492FAFA67C7s1) H. Schulz, Nucl. Phys. A 399 (1983) 587.
- [28] S. Typel, G. Röpke, T. Klahn, D. [Blaschke,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibF37470679F2D76FBB6D4EAD4A717FBB1s1) H.H. Wolter, Phys. Rev. C 81 (2010) 015803, [arXiv:0908.2344.](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibF37470679F2D76FBB6D4EAD4A717FBB1s1)
- [29] E. Beth, G. [Uhlenbeck,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib06668D0AE914AD7028ED8E4C0280AD46s1) Physica 4 (1937) 915.
- [30] N.L.S. Carnot, Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance, [English translation of Sadi Carnot's book published in 1824, together with the clarifying work by William Thomson (Lord Kelvin)].
- [31] P.J. [Siemens,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibD3F5ED8BD121A0871FFB0383AF85321Bs1) J.I. Kapusta, Phys. Rev. Lett. 43 (1979) 1486.
- [32] Y. Nambu, Phys. Rev. 117 (1960) 648-663.
- [33] J. [Goldstone,](http://refhub.elsevier.com/S0370-2693(24)00649-X/bib0D0B3F3D530C7320F396912145B7E4BCs1) Nuovo Cimento 19 (1961) 154–164.
- [34] G.N. Lewis, Proc. Am. Acad. Arts Sci. 43 [\(1907\)](http://refhub.elsevier.com/S0370-2693(24)00649-X/bibD3B2F6AFF87396D5C6651783833948F5s1) 259.
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