Finite pion width effects on the rho-meson and di-lepton spectra

Hendrik van Hees[1] and Jörn Knoll [2]

Gesellschaft für Schwerionenforschung (GSI)
Planckstr. 1, D-64291 Darmstadt, Germany

Abstract

Within a field theoretical model where all damping width effects are treated self-consistently we study the changes of the spectral properties of ρ-mesons due to the finite damping width of the pions in dense hadronic matter at finite temperature. The corresponding effects in the di-lepton yields are presented. Some problems concerning the self consistent treatment of vector or gauge bosons are discussed.

1 Introduction

The properties of vector mesons in a dense hadronic medium have attracted much attention in recent times. Measurements of di-leptons in nuclear collisions promise to access such properties experimentally. Recent experiments by the CERES and DLS collaborations [1, 2, 3] show interesting features in the low lepton pair mass spectrum between 300 to 600 MeV. Various effects which change the mass and/or the width, or brief the spectral properties of the vector mesons in dense matter have been explored to explain the observed enhancement seen in heavy projectile-target collisions compared to proton-proton data. High resolution experiments with the upgrade of CERES and the new di-lepton project HADES at GSI will sharpen the view on the spectral information of vector mesons.

In most of the theoretical investigations the damping width attained by the asymptotically stable particles in the dense matter environment has been ignored sofar. In this contribution we study the in-medium properties of the ρ-meson due to the damping width of the pions in a dense hadron gas within a self consistent scheme.

2 The model

In order to isolate the pion width effects we discard baryons and consider a purely mesonic model system consisting of pions, ρ-mesons, and for curiosity also the chiral partner of the ρ-, the a_1-meson with the interaction Lagrangian

$$\mathscr{L}^{\text{int}} = g_{\rho\pi\pi}\rho_\mu \pi^* \overleftrightarrow{\partial}^\mu \pi + g_{\pi\rho a_1}\pi\rho_\mu a_1^\mu + \frac{g_{\pi 4}}{8}(\pi^*\pi)^2. \tag{1}$$

The first two coupling constants are adjusted to provide the corresponding vacuum widths of the ρ- and a_1-meson at the nominal masses of 770 and 1200 MeV of $\Gamma_\rho = 150\,\text{MeV}$ and $\Gamma_{a_1} = 400\,\text{MeV}$, respectively. The four-pion interaction is used as a tool to furnish additional collisions among the pions. The idea of this term is to provide pion damping widths of 50 MeV or more as they would

[1]e-mail:h.vanhees@gsi.de
[2]e-mail:j.knoll@gsi.de

occur due to the strong coupling to the NN^{-1} and ΔN^{-1} channels in an environment at finite baryon density.

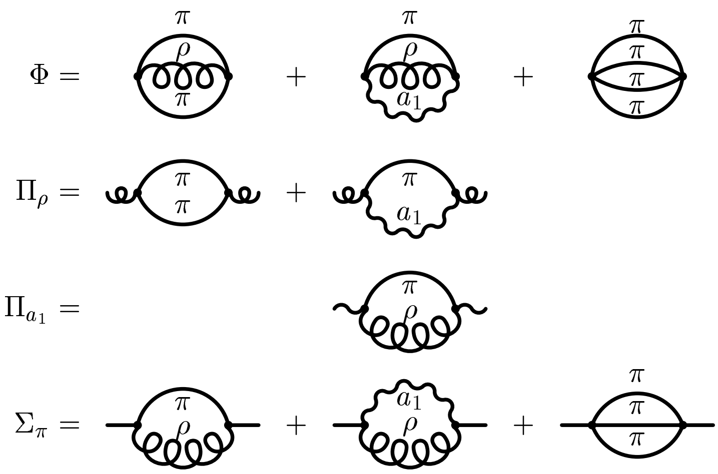
The Φ-functional method originally proposed by Baym[4] provides a self-consistent scheme applicable even in the case of broad resonances. It bases on a re-summation for the partition sum [5, 6]. Its two particle irreducible part $\Phi[G]$ generates the irreducible self-energy $\Sigma(x,y)$ via a functional variation with respect to the propagator $G(y,x)$, i.e.

$$-\mathrm{i}\Sigma(x,y) = \frac{\delta \mathrm{i}\Phi}{\delta \mathrm{i}G(y,x)}. \tag{2}$$

Thereby Φ solely depends on fully re-summed, i.e. self-consistently generated propagators $G(x,y)$. In graphical terms, the variation (2) with respect to G is realized by opening a propagator line in all diagrams of Φ. Further details and the extension to include classical fields or condensates into the scheme are given in ref. [7].

Truncating Φ to a limited subset of diagrams, while preserving the variational relation (2) between $\Phi^{(\mathrm{appr.})}$ and $\Sigma^{(\mathrm{appr.})}(x,y)$ defines an approximation with built-in consistency. Baym[4] showed that such a scheme is conserving at the expectation value level of conserved currents related to global symmetries of the original theory, that its physical processes fulfill detailed balance and unitarity and that at the same time the concept is thermodynamically consistent. However symmetries and conservation laws may no longer be maintained on the correlator level, a draw-back that will lead to problems for the self-consistent treatment of vector and gauge particles on the propagator level, as discussed in sect. 3.

Interested in width effects, we drop changes in the real parts of the self energies. This entitles to drop tadpole contributions for the self energies. For our model Lagrangian (1) one obtains the following diagrams for Φ at the two-point level which generate the subsequently given three self energies Π_ρ, Π_{a_1} and Σ_π



(3)

They are the driving terms for the corresponding three Dyson equations, which have to be solved self consistently. The above coupled scheme pictorially illustrates the concept of Newton's principle of *actio* = *reactio* and detailed balance provided by the Φ-functional. If the self energy of one particle is modified due to the coupling to other species, these other species also obtain a complementary

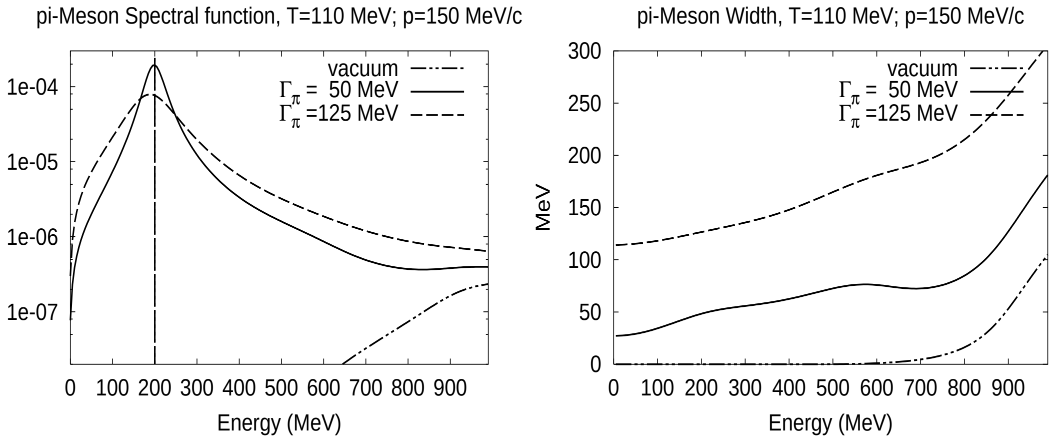


Figure 1: *Spectral function (left) and decay width (right) of the pion as a function of the pion energy at a pion momentum of* 150 *MeV/c in the vacuum and for two self consistent cases discussed in the text.*

term in their self energy. In vacuum the ρ- and a_1-meson have the standard thresholds at $\sqrt{s} = 2m_\pi$ and at $3m_\pi$ respectively. For the pion as the only stable particle in vacuum with a pole at m_π a decay channel opens at $\sqrt{s} = 3m_\pi$ due to the first and last diagram of Σ_π. Correspondingly the vacuum spectral function of the pion shows already some spectral strength for $\sqrt{s} > 3m_\pi$, c.f. fig. 1 (left).

Self consistent equilibrium calculations are performed keeping the full dependence of all two-point functions on three momentum $\vec{p}$ and energy p_0, and treating all propagators with their dynamically determined widths. For simplicity the real parts of the self energies were dropped and all time components of the polarization tensors Π_ρ and Π_{a_1} were put to zero for reasons discussed in sect. 3. The examples shown refer to a temperature of $T = 110\,\text{MeV}$ appropriate for the CERES data. We discuss three different settings. In case (a) the ρ-meson polarization tensor is calculated simply by the perturbative pion loop, i.e. with vacuum pion propagators and thermal Bose-Einstein weights (no self consistent treatment). The two other cases refer to self consistent solutions of the coupled Dyson scheme, where the four-π interaction is tuned such that the sun-set diagram provides a moderate pion damping width of about 50 MeV (case (b)) and a strong one of 125 MeV (case (c)) around the peak in the spectral function, c.f. fig. 1. Since in the thermal case any excitation energy is available, though with corresponding thermal weights, all thresholds disappear and the spectral functions show strength at *all* energies[3]! The pion functions shown in Fig. 1 are plotted against energy in order to illustrate that there is significant strength in the space-like region (below the light cone at 150 MeV) resulting from π-π scattering processes.

As an illustration we display a 3-d plot of the rho-meson spectral function as a function of p_0 and $|\vec{p}|$ in Fig. 2, top left. The right part shows the spectral function as a function of invariant mass at fixed three momentum of $150\,\text{MeV}/c$ in vacuum and for the self consistent cases (a) to (c). The minor changes at the low mass side of the ρ-meson spectral function become significant in the

[3]In mathematical terms: all cuts go from $-\infty$ to $+\infty$ in energy, and the physical sheet of the retarded functions are completely separated from the physical sheet of the corresponding advanced function by these cuts.

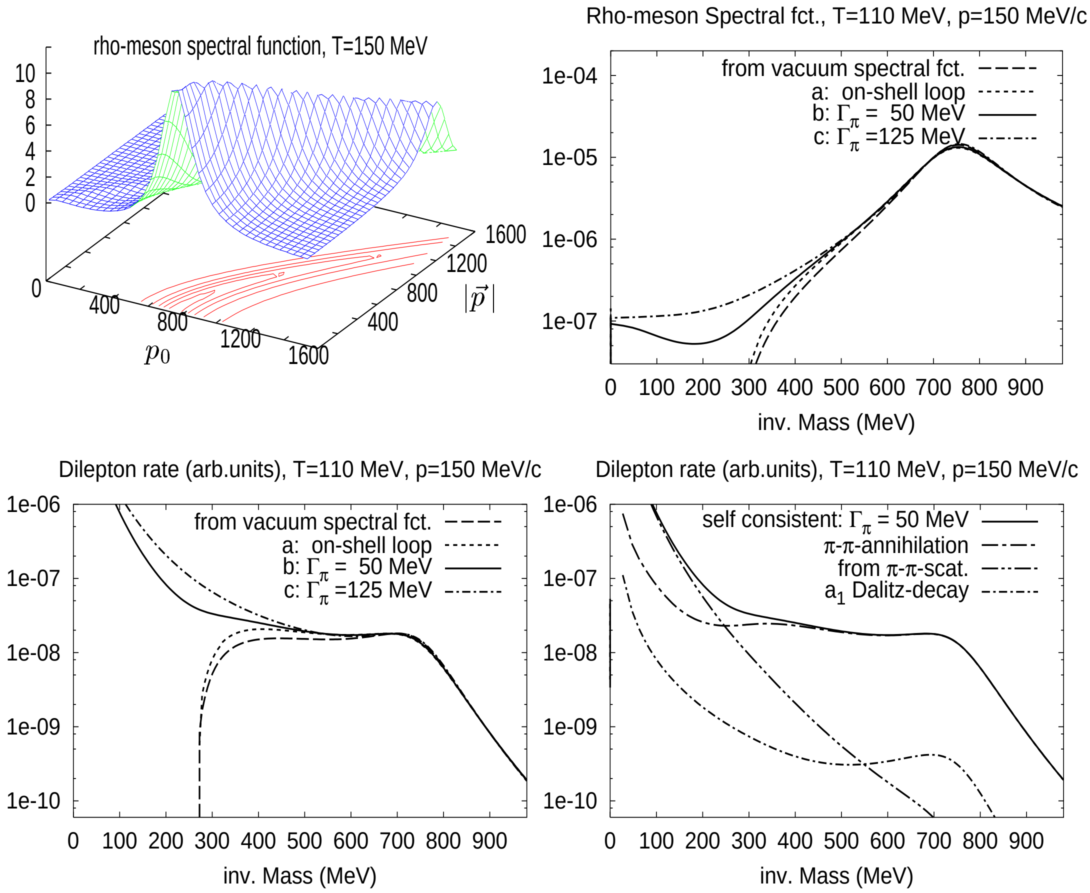


Figure 2: *top: ρ-meson spectral function, bottom: thermal di-lepton rate.*

di-lepton yields given in the left bottom panel. The reason lies in the statistical weights together with additional kinematical factors $\propto m^{-3}$ from the di-lepton decay mechanism. For the moderate damping case ($\Gamma_\pi = 50\,\text{MeV}$) we have decomposed the di-lepton rate into partial contributions associated with π-π bremsstrahlung, π-π annihilation and the contribution from the a_1-meson, which can be interpreted as the a_1 Dalitz decay.

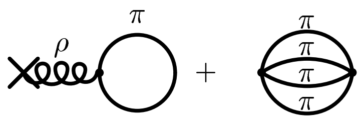
The low mass part is completely dominated by pion bremsstrahlung contributions (like-charge states in the pion loop). This contribution, which vanishes in perturbation theory is *finite* for pions with finite width. It has to be interpreted as bremsstrahlung, since the finite width results from collisions with other particles present in the heat bath. Compared to the standard treatment, where the bremsstrahlung is calculated independently of the π-π annihilation process, this self-consistent treatment has a few advantages. The bremsstrahlung is calculated consistently with the annihilation process, it appropriately accounts for the Landau-Pomeranchuk suppression at low invariant masses [8] and at the same time includes the in-medium pion electromagnetic form-factor for the bremsstrahlung part. As a result the finite pion width adds significant strength to the mass region below 500 MeV compared to the trivial treatment with the vacuum spectral function.

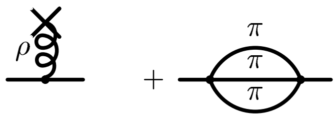
Therefore the resulting di-lepton spectrum essentially shows no dip any more in this low mass region already for a moderate pion width of 50MeV. The a_1 Dalitz decay contribution given by the partial ρ-meson width due to the π-a_1 loop in Π_ρ is seen to be unimportant at all energies. The present calculations have not included any medium modification of the masses of the mesons. The latter can be included through subtracted dispersion relations within such a consistent scheme.

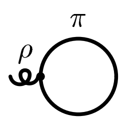
3 Symmetries and gauge invariance

While scalar particles and couplings can be treated self-consistently with no principle problems at any truncation level, considerable difficulties and undesired features arise in the case of vector particles. The origin lies in the fact that, though in Φ-derivable Dyson re-summations symmetries and conservation laws are fulfilled at the expectation value level, they are generally no longer guaranteed at the correlator level. In the case of local gauge symmetries the situation is even worse, because the symmetry of the quantized theory is not the original one but the non-linear BRST symmetry [9, 10]. Contrary to perturbation theory, where the loop expansion corresponds to a strict power expansion in $\hbar$ and symmetries are maintained order by order, partial re-summations mix different orders thus violating the corresponding symmetries. It is obvious that the scheme discussed above indeed violates the Ward identities on the correlator level and thus the vector meson propagators are no longer 4-dimensionally transverse. This means that unphysical states are propagated within the internal lines of the Φ-derivable approximation scheme which lead to a number of difficulties in the numerical treatment of the problem. In the above calculations we have worked around this problem by putting the temporal components of the ρ-meson polarization tensor to zero, an approximation, which is exact for $\vec{p}_\rho = 0$.

Is there a self-consistent truncation scheme, where gauge invariance is maintained also for the internal dynamics, i.e. for the dynamical quantities like classical fields and propagators which enter the self-consistent set of equations? The answer is definitely yes. However one has to restrict the coupling of the gauge fields to the expectation values of the vector currents. This in turn implies that gauge fields are treated on the classical field level only, a level that is presently explored in all hard thermal loop (HTL) approaches [11, 12, 13, 14]. In the case of a π-ρ-meson system the corresponding Φ-derivable scheme is then given by (again omitting the tadpole term)

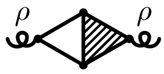
$\Phi\{G_\pi, \rho\} =$  (4)

$\Sigma_\pi =$  (5)

$\left(\partial^\nu \partial_\nu - m^2\right) \rho^\mu = j^\mu =$  (6)

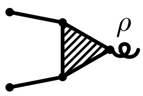
Here full lines represent the self-consistent pion propagators and curly lines with a cross depict the classical ρ-meson field, governed by the classical field equations (6). Since Φ is invariant with respect to gauge transformations of the classical vector field ρ^μ, the resulting equations of motion are gauge covariant. This also holds for fluctuations $\rho^\mu + \delta\rho^\mu$ around mean field solutions of (5 - 6). In this background field method one can define a gauge covariant *external* polarization tensor via variations with respect to the background field $\delta\rho^\mu$

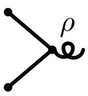
$$\Pi^{\text{ext}}_{\mu\nu}(x_1, x_2) = \frac{\delta}{\delta\rho^\mu(x_2)} \left[\frac{\delta\Phi[G_\pi, \rho]}{\delta\rho^\nu(x_1)}\right]_{G_\pi = G_\pi[\rho]} = \tag{7}$$

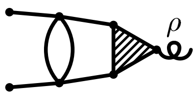


as a response to fluctuations around the mean field. In order to access this tensor one has to solve a corresponding three-point-vertex equation

$$\frac{\delta G_\pi}{\delta\rho^\mu} = \quad = \quad + \tag{8}$$

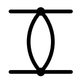






In order to maintain all symmetries and invariances the Bethe-Salpeter Kernel in this equation has to be chosen consistently with the Φ-functional (4), i.e.

$$K_{1234} = \frac{\delta^2\Phi}{\delta G_{12}\delta G_{34}} = \tag{9}$$



Thereby the pion propagator entering the ladder resummation (9) is determined by the self-consistent solution of the coupled Dyson and classical field equations (5 - 6). Thereby the ladder re-summation also accounts for real physical scattering processes. This phenomenon was already discussed in [8] for the description of Bremsstrahlung within a classical transport scheme (Landau-Pomeranchuk-Migdal effect). From this point of view one clearly sees that the pure Φ-functional formalism without the vertex corrections provided by (8) describes only the "decay of states" due to collision broadening. Thus the *internal* polarization tensor given in the Φ-Dyson scheme (3) has a *time-decaying* behavior, with the 00-component approximately behaving like

$$\Pi^{00}_\rho(\tau, \vec{p} = 0) \propto e^{-\Gamma\tau} \tag{10}$$

in a mixed time-momentum representation. This clearly violates charge conservation, since $\partial_0 \Pi^{00}_\rho(\tau, \vec{p})|_{\vec{p}=0}$ does not vanish! Accounting coherently for the multiple scattering of the particles through the vertex re-summation (8) keeps track of the "charge flow" into other states and thus restores charge conservation. Within classical considerations the ladder re-summation (7) indeed yields

$$\Pi^{00}_\rho(\tau, \vec{p} = 0) \propto \sum_n \frac{(\Gamma\tau)^n}{n!}\, e^{-\Gamma\tau} = 1 \tag{11}$$

confirming charge conservation. For further details c.f. ref. [8].

Unless one solves the exact theory there seems to be no obvious self consistent alternative to the above scheme where vector particles are treated dynamically and which at the same time complies with gauge invariance also for the internal propagation. All this seems to defer a dynamical treatment of vector particles on the propagator level. The numerical implementation of the above vertex corrections is in progress. The problem of renormalization omitted here has been investigated using subtracted dispersion relations. Thus for vector particles a fully self-consistent scheme with all the features of the Φ-functional, especially to ensure the consistency of dynamical and thermodynamical properties of the calculated propagators together with the conservation laws on both the expectation value and the correlator level remains an open problem.

The equilibrium calculations presented also serve the goal to gain experience about particles with broad damping width with the aim towards a transport scheme for particles beyond the quasi-particle limit [15], see also [16, 17].

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